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Aspects of supersymmetry breaking in open-string models

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Abstract: In these notes we review some recent work on non-supersymmetric type I vacua. After presenting the main features of models with brane supersymmetry breaking, with some emphasis also on their low-energy dynamics and the structure of the vacuum, we turn to compactifications on magnetized orbifolds, to conclude with a particular realization of the Scherk-Schwarz mechanism in orientifold constructions.

1 Introduction

Type I models have become the subject of an intense activity during the last few years, since their perturbative definition offers interesting new possibilities for low-energy phenomenology. Their consistency and a number of their most amusing features may be traced to the relation to suitable “parent” models of oriented closed strings, from which their spectra can be derived [1]. In this procedure, a special role is played by “tadpole conditions” for R-R and NS-NS states. While the former are to be regarded as *global* neutrality conditions for R-R charges, and are usually linked to gauge and gravitational anomalies [2], the latter simply imply that the configurations of D-branes and O-plane be *globally* massless.

The explicit study of type I vacua, however, has revealed an unexpected difficulty: in some interesting chiral four-dimensional models it is apparently impossible to satisfy some of the tadpole conditions [3]. This peculiar phenomenon can often be traced to sign flips of some crosscap contributions or, in more suggestive space-time language, to the reversal of some orientifold charges and tensions [4]. Actually, this difficulty can be evaded if one relaxes the condition that O-plane and D-brane configurations be massless and supersymmetric, thus allowing for the simultaneous presence of branes and antibranes. The emergence of a non-vanishing NS-NS tadpole in the low-energy effective action is also crucial in order to have consistent couplings between the supersymmetric bulk and the non-supersymmetric branes [5].

The paper is organized as follows. In section 2 we review the construction of ten-dimensional type I models with and without brane supersymmetry breaking. We introduce the main ingredients in the orientifold constructions and discuss the role of NS-NS and R-R tadpole conditions. Section 3 is devoted to a similar analysis in the context of six-dimensional $T^4/\mathbb{Z}_2$ orbifolds where, at times, brane supersymmetry breaking is actually demanded by the very consistency of the construction. Section 4 is devoted to the main

features of the low-energy action of models with a non-vanishing dilaton tadpole. We show how a potential term for the dilaton is required in order to couple consistently the supersymmetric bulk modes to the non-supersymmetric brane matter, and how it modifies the vacuum of the model. In section 5 we study the propagation of open strings in non-trivial gauge backgrounds and in particular we review the role of magnetized branes in the orientifold constructions with and without supersymmetry. Finally, section 6 presents an alternative way to break supersymmetry in open-string vacua. It corresponds to a string-theory realization of the Scherk-Schwarz mechanism where the opposite phenomenon of brane supersymmetry breaking emerges: supersymmetry is completely broken in the bulk while the massless open-string modes are supersymmetric.

More details about orientifold constructions with and/or without supersymmetry can be found in the review paper [6], and references therein.

2 The type I superstring: $SO(32)$ vs $USp(32)$

This model contains a single sector, corresponding to the (super)character $(V_8 - S_8)$, and is thus rather simple to build. The starting point is the type IIB superstring, whose excitations are encoded in the torus amplitude

$$\mathcal{T} = |V_8 - S_8|^2, \quad (1)$$

and comprise the graviton, the dilaton and a two-index antisymmetric tensor from the NS-NS sector, an additional scalar, an additional two-form and a four-form with self-dual field strength from the R-R sector, as well as two left-handed gravitini and two right-handed spinors from the NS-R and R-NS sectors. Altogether these massless modes fill up the $\mathcal{N} = (2, 0)$ supergravity multiplet. The orientifold construction then associates to $\mathcal{T}$ the amplitudes

$$\mathcal{K} = \frac{1}{2}(V_8 - S_8), \quad (2)$$

$$\mathcal{A} = \frac{N^2}{2}(V_8 - S_8), \quad (3)$$

$$\mathcal{M} = \frac{\epsilon N}{2}(\hat{V}_8 - \hat{S}_8), \quad (4)$$

where ϵ is a sign. The Klein-bottle projection symmetrizes the NS-NS sector, thus eliminating from the massless spectrum the two-form, and antisymmetrizes the R-R sector, thus eliminating the second scalar and the self-dual four-form. Since the complete projection leaves only one combination of each pair of fermion modes, the resulting massless spectrum corresponds to the minimal $\mathcal{N} = (1, 0)$ ten-dimensional supergravity, and comprises a graviton, a two-form, now from the R-R sector, a dilaton, a left-handed gravitino and a right-handed spinor. In a similar fashion, the massless open sector is a $(1, 0)$ super Yang-Mills multiplet for the group $SO(N)$ if $\epsilon = -1$ or $USp(N)$ if $\epsilon = +1$.

One can also write the corresponding transverse-channel amplitudes

$$\tilde{\mathcal{K}} = \frac{2^5}{2}(V_8 - S_8), \quad (5)$$

$$\tilde{\mathcal{A}} = \frac{2^{-5}}{2}N^2(V_8 - S_8), \quad (6)$$

$$\tilde{\mathcal{M}} = \epsilon N(\hat{V}_8 - \hat{S}_8), \quad (7)$$

and the tadpole condition

$$\frac{2^5}{2} + \frac{2^{-5} N^2}{2} + 2 \frac{\epsilon N}{2} = \frac{2^{-5}}{2} (N + 32\epsilon)^2 = 0, \quad (8)$$

that applies both to the NS-NS and to the R-R sector, selects uniquely the $SO(32)$ gauge group ($N = 32$, $\epsilon = -1$).

This cancellation can be given a suggestive space-time interpretation: the world-sheet boundaries traced by the ends of the open strings are mapped to extended objects, D9-branes, that fill the whole of space-time, while the crosscaps are mapped to corresponding non-dynamical objects, the orientifold O9-planes. In general, both Dp -branes and Op -planes have tensions and carry R-R charges with respect to $(p+1)$ -form potentials C_{p+1} [7]. For D-branes, tension and charge are both positive while, as we shall soon see, two types of O-planes can be present in perturbative type-I vacua: those with negative tension and negative charge, here denoted O_+ -planes, and those with positive tension and positive charge, here denoted O_- -planes [8]. In addition, there are of course anti-D-branes and anti-O-planes, with identical tensions and opposite R-R charges. If these results are combined with non-perturbative string dualities, a rich zoo of similar extended objects emerges [9]. Let us stress that the NS-NS and R-R tadpole conditions are conceptually quite different and play quite distinct roles: while the latter reflect the need for overall charge neutrality, consistently with the Gauss law for C_{p+1} if its Faraday lines are confined to a compact space, and are related to space-time anomalies [2], the former give rise to a dilaton-dependent correction to the vacuum energy that, in principle, can be non-vanishing. This will be loosely referred to as a dilaton tadpole.

In the type I $SO(32)$ superstring, NS-NS and R-R tadpoles, related by supersymmetry, cancel at the same time, and therefore this vacuum configuration involves D9-branes and corresponding $O9_+$ -planes. However, it is simple to generalize this model to a non-supersymmetric configuration with a residual dilaton tadpole. To this end, let us begin by assigning different reflection coefficients to the NS-NS and R-R states flowing in $\tilde{\mathcal{A}}$ and $\tilde{\mathcal{M}}$, so that

$$\tilde{\mathcal{A}} = \frac{2^{-5}}{2} \left[(n_+ + n_-)^2 V_8 - (n_+ - n_-)^2 S_8 \right], \quad (9)$$

$$\tilde{\mathcal{M}} = \epsilon_{\text{NS}}(n_+ + n_-) \hat{V}_8 - \epsilon_{\text{R}}(n_+ - n_-) \hat{S}_8. \quad (10)$$

In the direct channel, these expressions become

$$\mathcal{A} = \frac{1}{2} \left[(n_+^2 + n_-^2)(V_8 - S_8) + 2n_+n_-(O_8 - C_8) \right], \quad (11)$$

$$\mathcal{M} = \frac{1}{2} \left[\epsilon_{\text{NS}}(n_+ + n_-) \hat{V}_8 - \epsilon_{\text{R}}(n_+ - n_-) \hat{S}_8 \right]. \quad (12)$$

Therefore, the model involves two types of Chan-Paton charges, of multiplicities n_+ and n_- , and two distinct sectors: strings with like charges at their ends have the standard GSO projection, that involves the vector and the S spinor, while the additional modes with unlike charges have the opposite projection, that involves the tachyon and the C spinor, as stressed in [10].

The two tadpole conditions

$$\text{NS-NS:} \quad 32 + \epsilon_{\text{NS}}(n_+ + n_-) = 0, \quad (13)$$

$$\text{R-R:} \quad 32 + \epsilon_{\text{R}}(n_+ - n_-) = 0, \quad (14)$$

have a single positive integer solution for the multiplicities, $\epsilon_{\text{NS}} = \epsilon_{\text{R}} = -1$, $n_+ = 32$, $n_- = 0$, that corresponds again to the $\text{SO}(32)$ superstring, but relaxing the NS-NS tadpole allows an infinity of solutions, with, say, $\epsilon_{\text{R}} = -1$ and $n_+ - n_- = 32$. From the transverse amplitudes one can read the relative values of tensions and charges for the D-branes and O-planes now present. In particular, from the R-R couplings one can easily see that n_+ and n_- count, respectively, the numbers of D9-branes and D9-antibranes, while the two choices $\epsilon_{\text{NS}} = \epsilon_{\text{R}} = \pm 1$ correspond to $\text{O}_{\mp}$ planes. $\mathcal{M}$ encodes rather neatly this last property, since its NS contribution is sensitive to the relative signs of D-brane and O-plane tensions. All configurations with $n_- \neq 0$ have tachyonic instabilities, introduced by the simultaneous presence of branes and antibranes [10], that reflect their mutual attraction. Some definite progress has been made recently in connection with these “brane” tachyons [11], while the current understanding of “bulk” tachyons is still far more primitive. For $n_+ = 0$ there is a new possibility, a non-supersymmetric model first discovered in [12], with $\epsilon_{\text{NS}} = \epsilon_{\text{R}} = +1$, involving D9-antibranes and O_- planes. The resulting gauge group is $\text{USp}(32)$, but its massless spinors are still in the antisymmetric representation.

3 The $T^4/\mathbb{Z}_2$ orbifold

We can now move to study lower dimensional models, where the breaking of supersymmetry is, at times, demanded by the very consistency of the construction. To this end, let us consider the open descendants of the $T^4/\mathbb{Z}_2$ orbifold, allowing for the simultaneous presence of branes and antibranes of the same type. For the sake of clarity, let us begin from the torus amplitude

$$\mathcal{T} = \frac{1}{4} \left\{ |Q_o + Q_v|^2 \Lambda^4 + |Q_o - Q_v|^2 \left| \frac{2\eta}{\vartheta_2} \right|^4 + 16|Q_s + Q_c|^2 \left| \frac{\eta}{\vartheta_4} \right|^4 + 16|Q_s - Q_c|^2 \left| \frac{\eta}{\vartheta_3} \right|^4 \right\}, \quad (15)$$

and its Klein bottle projection

$$\mathcal{K} = \frac{1}{4} \left\{ (Q_o + Q_v)(P^4 + W^4) + 2 \times 16\epsilon (Q_s + Q_c) \left(\frac{\eta}{\vartheta_4} \right)^2 \right\}, \quad (16)$$

where we have used the standard $\mathbb{Z}_2$ decompositions

$$\begin{aligned} Q_o &= V_4 O_4 - C_4 C_4, & Q_v &= O_4 V_4 - S_4 S_4, \\ Q_s &= O_4 C_4 - S_4 O_4, & Q_c &= V_4 S_4 - C_4 V_4. \end{aligned} \quad (17)$$

We have written $\mathcal{K}$ in this form, since one can actually modify the Klein bottle projection for twisted closed states, as in [13]. Multiple Klein-bottle projections, first discussed in [14], have also interesting physical applications to tachyon-free non-supersymmetric open-string models [15]. The choice $\epsilon = +1$ corresponds to the usual $T^4/\mathbb{Z}_2$ orientifold, and gives a closed unoriented spectrum containing $\mathcal{N} = (1, 0)$ supergravity coupled to 20 hyper multiplets and 1 tensor multiplet, while the choice $\epsilon = -1$ gives $\mathcal{N} = (1, 0)$ supergravity coupled to 4 hyper multiplets and 17 tensor multiplets.

From the Klein-bottle contribution to the massless tadpoles

$$\tilde{\mathcal{K}}_0 = \frac{2^5}{4} \left\{ \left(\sqrt{v} + \frac{\epsilon}{\sqrt{v}} \right)^2 Q_o + \left(\sqrt{v} - \frac{\epsilon}{\sqrt{v}} \right)^2 Q_v \right\}, \quad (18)$$

one can anticipate the need for a net number of 32 D9 and 32 (anti)D5-branes (for $\epsilon = \pm 1$, respectively) in order to cancel the R-R tadpoles.

Proceeding as in the previous sections, the annulus amplitude in the presence of branes and antibranes is

$$\begin{aligned}
\mathcal{A} = & \frac{1}{4} \left\{ (V_4 O_4 + O_4 V_4 - C_4 C_4 - S_4 S_4) \left[(N_+^2 + N_-^2) P^4 + (D_+^2 + D_-^2) W^4 \right] \right. \\
& + 2(O_4 O_4 + V_4 V_4 - S_4 C_4 - C_4 S_4) \left[N_+ N_- P^3 P_{1/2} + D_+ D_- W^3 W_{1/2} \right] \\
& + (V_4 O_4 - O_4 V_4 - C_4 C_4 + S_4 S_4) \left(\frac{2\eta}{\vartheta_2} \right)^2 (R_{N_+}^2 + R_{N_-}^2 + R_{D_+}^2 + R_{D_-}^2) \\
& + 2(O_4 C_4 + V_4 S_4 - S_4 O_4 - C_4 V_4) \left(\frac{\eta}{\vartheta_4} \right)^2 (N_+ D_+ + N_- D_-) \\
& + 2(O_4 S_4 + V_4 C_4 - C_4 O_4 - S_4 V_4) \left(\frac{\eta}{\vartheta_4} \right)^2 (N_+ D_- + N_- D_+) \\
& + 2(O_4 C_4 - V_4 S_4 - S_4 O_4 + C_4 V_4) \left(\frac{\eta}{\vartheta_3} \right)^2 (R_{N_+} R_{D_+} + R_{N_-} R_{D_-}) \\
& \left. + 2(-O_4 S_4 + V_4 C_4 - C_4 O_4 + S_4 V_4) \left(\frac{\eta}{\vartheta_3} \right)^2 (R_{N_+} R_{D_-} + R_{N_-} R_{D_+}) \right\}, \quad (19)
\end{aligned}$$

where $N_{\pm}$ ($D_{\pm}$) refer to the D9 and D $\bar{9}$ (D5 and D $\bar{5}$) branes, respectively, and the R 's are orbifold-induced breaking terms. In order to lift the tachyons, following [16] we have inserted the D5 and D $\bar{5}$ -branes at different fixed points of the orbifolds, and we have added Wilson lines for the D9-branes along a different coordinate.

Finally, the Möbius amplitude

$$\begin{aligned}
\mathcal{M} = & -\frac{1}{4} \left\{ (\hat{V}_4 \hat{O}_4 + \hat{O}_4 \hat{V}_4 - \hat{C}_4 \hat{C}_4 - \hat{S}_4 \hat{S}_4) \left[N_+ P^4 + \epsilon D_+ W^4 \right] \right. \\
& + (\hat{V}_4 \hat{O}_4 + \hat{O}_4 \hat{V}_4 + \hat{C}_4 \hat{C}_4 + \hat{S}_4 \hat{S}_4) \left[N_- P^4 + \epsilon D_- W^4 \right] \\
& - (\hat{V}_4 \hat{O}_4 - \hat{O}_4 \hat{V}_4 - \hat{C}_4 \hat{C}_4 + \hat{S}_4 \hat{S}_4) \left(\frac{2\hat{\eta}}{\hat{\vartheta}_2} \right)^2 (\epsilon N_+ + D_+) \\
& \left. - (\hat{V}_4 \hat{O}_4 - \hat{O}_4 \hat{V}_4 + \hat{C}_4 \hat{C}_4 - \hat{S}_4 \hat{S}_4) \left(\frac{2\hat{\eta}}{\hat{\vartheta}_2} \right)^2 (\epsilon N_- + D_-) \right\} \quad (20)
\end{aligned}$$

gives a proper symmetrization of the annulus amplitude, while the tadpole conditions

$$\begin{aligned}
N_+ - N_- = 32, \quad D_+ - D_- = 32\epsilon, \\
R_{N_+} = R_{N_-} = R_{D_+} = R_{D_-} = 0
\end{aligned} \quad (21)$$

require net numbers of 32 D9-branes and 32 D5-branes (antibranes) for $\epsilon = \pm 1$.

The choice $\epsilon = +1$, with O9 $_+$ and O5 $_+$ planes, corresponds to a deformation of the supersymmetric U(16) $\times$ U(16) model [17, 18], and requires the introduction of complex charges, so that

$$\begin{aligned}
N_{\pm} = n_{\pm} + \bar{n}_{\pm}, \quad R_{N_{\pm}} = i(n_{\pm} - \bar{n}_{\pm}), \\
D_{\pm} = d_{\pm} + \bar{d}_{\pm}, \quad R_{D_{\pm}} = i(d_{\pm} - \bar{d}_{\pm}).
\end{aligned} \quad (22)$$

The resulting massless open spectrum can be extracted from

$$\begin{aligned}
\mathcal{A}_0 + \mathcal{M}_0 = & (n_+ \bar{n}_+ + n_- \bar{n}_- + d_+ \bar{d}_+ + d_- \bar{d}_-) (V_4 O_4 - C_4 C_4) \\
& + \frac{1}{2} (n_+^2 + \bar{n}_+^2 + n_-^2 + \bar{n}_-^2 + d_+^2 + \bar{d}_+^2 + d_-^2 + \bar{d}_-^2) (O_4 V_4 - S_4 S_4)
\end{aligned}$$

$$\begin{aligned}
& +(n_+\bar{d}_+ + \bar{n}_+d_+ + n_-\bar{d}_- + \bar{n}_-d_-) (O_4C_4 - S_4O_4) \\
& -(n_+\bar{d}_- + n_-\bar{d}_+ + \bar{n}_+d_- + \bar{n}_-d_+) C_4O_4 \\
& +(n_+d_- + \bar{n}_+\bar{d}_- + n_-d_+ + \bar{n}_-\bar{d}_+) O_4S_4 \\
& -\frac{1}{2}\{(n_+ + \bar{n}_+ + d_+ + \bar{d}_+) (\hat{O}_4\hat{V}_4 - \hat{S}_4\hat{S}_4) \\
& +(n_- + \bar{n}_- + d_- + \bar{d}_-) (\hat{O}_4\hat{V}_4 + \hat{S}_4\hat{S}_4)\} .
\end{aligned} \tag{23}$$

The 99 and 55 sectors are supersymmetric, and comprise vector multiplets in the adjoint of $U(n_+)_9 \times U(n_-)_{\bar{9}} \times U(d_+)_5 \times U(d_-)_{\bar{5}}$, together with hyper multiplets in the antisymmetric and conjugate antisymmetric representations. On the other hand, the $\bar{9}\bar{9}$ and $5\bar{5}$ are not supersymmetric and contain left-handed Weyl fermions in the antisymmetric and conjugate antisymmetric representations, and a quartet of real scalars in the symmetric and conjugate symmetric representations. Finally, the 95 and $\bar{9}\bar{5}$ sectors give hyper multiplets in the $(n_+, \bar{d}_+)$ and $(n_-, \bar{d}_-)$, respectively, while the $9\bar{5}$ and 95 sectors, evidently not supersymmetric, give a quartet of scalars in the (n_+, d_-) , (n_-, d_+) and right-handed Weyl fermions in the $(n_+, \bar{d}_-)$, $(n_-, \bar{d}_+)$. The irreducible gravitational and gauge anomalies vanish as a result of tadpole conditions, whereas the residual anomaly polynomial

$$\mathcal{I}_8 = -\frac{1}{16} \left[\text{tr} R^2 - 2(\text{tr} F_{N_+}^2 - \text{tr} F_{N_-}^2) \right] \left[\text{tr} R^2 - 2(\text{tr} F_{D_+}^2 - \text{tr} F_{D_-}^2) \right] \tag{24}$$

requires a conventional Green-Schwarz mechanism [19] involving the single antisymmetric tensor present in the massless closed spectrum.

The choice $\epsilon = -1$, instead, involves $O9_+$ and $O5_-$ planes. As a result a net number of D5 anti-branes is required, thus leading to the breaking of supersymmetry in the open sector. A consistent particle interpretation of the direct-channel amplitudes then calls for the parametrization

$$\begin{aligned}
N_{\pm} &= n_{\pm} + m_{\pm} , & R_{N_{\pm}} &= n_{\pm} - m_{\pm} , \\
D_{\pm} &= p_{\pm} + q_{\pm} , & R_{D_{\pm}} &= p_{\pm} - q_{\pm} ,
\end{aligned} \tag{25}$$

in terms of real charges. The massless open spectrum can now be read from

$$\begin{aligned}
\mathcal{A}_0 + \mathcal{M}_0 &= \frac{1}{2}(n_+^2 + n_-^2 + m_+^2 + m_-^2 + p_+^2 + p_-^2 + q_+^2 + q_-^2) (V_4O_4 - C_4C_4) \\
&+(m_+n_+ + m_-n_- + p_+q_+ + p_-q_-) (O_4V_4 - S_4S_4) \\
&+(m_+q_+ + n_+p_+ + m_-q_- + n_-p_-) (O_4C_4 - S_4O_4) \\
&+(m_+p_- + n_+q_- + m_-p_+ + n_-q_+) O_4S_4 \\
&-(m_+q_- + n_+p_- + m_-q_+ + n_-p_+) C_4O_4 \\
&-\frac{1}{2}\{(n_+ + m_+ - p_+ - q_+)(\hat{V}_4\hat{O}_4 - \hat{C}_4\hat{C}_4) \\
&+(n_- + m_- - p_- - q_-)(\hat{V}_4\hat{O}_4 + \hat{C}_4\hat{C}_4)\} ,
\end{aligned} \tag{26}$$

and the massless excitations thus comprise gauge bosons in the adjoint of $SO(m_+)_9 \times SO(n_+)_9 \times SO(m_-)_{\bar{9}} \times SO(n_-)_{\bar{9}} \times \text{USp}(p_+)_5 \times \text{USp}(q_+)_5 \times \text{USp}(p_-)_{\bar{5}} \times \text{USp}(q_-)_{\bar{5}}$, left-handed Weyl fermions in the adjoint of $SO(m_+)$, $SO(n_+)$, $\text{USp}(p_+)$ and $\text{USp}(q_+)$, in the symmetric representations of $SO(m_-)$ and $SO(n_-)$ and in the antisymmetric representations of $\text{USp}(p_-)$ and $\text{USp}(q_-)$. They also comprise a full hyper multiplet in the (m_+, n_+) , (m_-, n_-) , (p_+, q_+) , (p_-, q_-) , as well as half hyper multiplets in the (m_+, q_+) , (n_+, p_+) , (m_-, q_-) and (n_-, p_-) . Finally, the $9\bar{5}$ and $5\bar{9}$ sectors comprise complex scalars in the

(m_+, p_-) , (n_+, q_-) , (m_-, p_+) and (n_-, q_+) , and left-handed symplectic Majorana-Weyl spinors in the (m_+, q_-) , (n_+, p_-) , (m_-, q_+) and (n_-, p_+) .

The tadpole conditions eliminate all irreducible gauge and gravitational anomalies, whereas the residual reducible anomaly

$$\begin{aligned}
\mathcal{I}_8 = & \frac{1}{64} \left[2\text{tr}R^2 - \left(\text{tr}F_{m_+}^2 + \text{tr}F_{n_+}^2 - \text{tr}F_{m_-}^2 - \text{tr}F_{n_-}^2 - \text{tr}F_{p_+}^2 - \text{tr}F_{q_+}^2 + \text{tr}F_{p_-}^2 + \text{tr}F_{q_-}^2 \right) \right]^2 \\
& - \frac{1}{64} \left(\text{tr}F_{m_+}^2 + \text{tr}F_{n_+}^2 - \text{tr}F_{m_-}^2 - \text{tr}F_{n_-}^2 + \text{tr}F_{p_+}^2 + \text{tr}F_{q_+}^2 - \text{tr}F_{p_-}^2 - \text{tr}F_{q_-}^2 \right)^2 \\
& + \frac{1}{64} \left(\text{tr}F_{m_+}^2 - \text{tr}F_{n_+}^2 - \text{tr}F_{m_-}^2 + \text{tr}F_{n_-}^2 - \text{tr}F_{p_+}^2 + \text{tr}F_{q_+}^2 + \text{tr}F_{p_-}^2 - \text{tr}F_{q_-}^2 \right)^2 \\
& + \frac{4}{64} \left(\text{tr}F_{m_+}^2 - \text{tr}F_{n_+}^2 + \text{tr}F_{m_-}^2 - \text{tr}F_{n_-}^2 \right)^2 \\
& + \frac{4}{64} \left(\text{tr}F_{p_+}^2 - \text{tr}F_{q_+}^2 + \text{tr}F_{p_-}^2 - \text{tr}F_{q_-}^2 \right)^2 \\
& + \frac{3}{64} \left(\text{tr}F_{m_+}^2 - \text{tr}F_{n_+}^2 - \text{tr}F_{m_-}^2 + \text{tr}F_{n_-}^2 + \text{tr}F_{p_+}^2 - \text{tr}F_{q_+}^2 - \text{tr}F_{p_-}^2 + \text{tr}F_{q_-}^2 \right)^2 \quad (27)
\end{aligned}$$

can be removed by a generalized Green-Schwarz mechanism [20].

In both models a potential is generated for the six-dimensional dilaton and the volume of the internal manifold

$$V_{\text{eff}} \sim e^{-\phi_6} \left[(N_+ + N_- - 32)\sqrt{v} + \frac{(D_+ + D_- - 32\epsilon)}{\sqrt{v}} \right], \quad (28)$$

as a result of uncancelled NS-NS tadpoles. As in the previous model, this stabilizes the internal volume at a local minimum, that in this case is

$$v_0 = \frac{D_+ + D_- - 32\epsilon}{N_+ + N_- - 32}, \quad (29)$$

and gives mass to the corresponding (breathing-mode) field.

4 Properties of models with brane supersymmetry breaking

The emergence of a tree-level dilaton potential, induced by the relaxed NS-NS tadpole condition, naïvely incompatible with bulk supersymmetry, is crucial in order to couple sensibly a non-supersymmetric open-string spectrum to a supersymmetric bulk: on the branes supersymmetry is realized non-linearly, as in [21], and the dilaton tadpole is the leading term in the expansion of the Volkov-Akulov action for the goldstino, a singlet spinor always present in the open sector of models featuring brane supersymmetry breaking. For example in the simplest ten-dimensional model the goldstino corresponds precisely to the singlet in the $496 = 495 + 1$ decomposition for the antisymmetric two-tensor of $\text{USp}(32)$ [5]. Moreover, the presence of the dilaton tadpole is incompatible with a maximally symmetric Minkowski space, and leads to a spontaneous compactification to lower dimensions [22]. The fate of these non-supersymmetric vacua still deserves a fuller investigation since, although in the minimal models tachyons are not present, the net attraction between O-planes and branes would be expected to induce a Coleman-Weinberg-like phenomenon. In the following we shall address these issues for the simplest realization of brane supersymmetry breaking: the ten-dimensional $\text{USp}(32)$ superstring.

4.1 The USp(32) superstring: the low-energy effective action

Following [5], let us briefly review how a single spinor can be treated *à la* Volkov-Akulov as a goldstino of global supersymmetry. For simplicity let us restrict our attention to the ten-dimensional case, considering a Majorana-Weyl fermion θ with the supersymmetry transformation

$$\delta\theta = \epsilon + \frac{1}{4}\bar{\epsilon}\gamma^\mu\theta\partial_\mu\theta. \quad (30)$$

The commutator of two such transformations is a translation,

$$[\delta_1, \delta_2]\theta = \frac{1}{2}(\bar{\epsilon}_2\gamma^\mu\epsilon_1)\partial_\mu\theta, \quad (31)$$

and thus eq. (30) provides a realization of supersymmetry. An invariant Lagrangian is then

$$\mathcal{L} = -\det e, \quad (32)$$

where the one-form

$$e_\mu{}^m = \delta_\mu{}^m + \frac{1}{4}\bar{\theta}\gamma^m\partial_\mu\theta \quad (33)$$

transforms as

$$\delta e^m = -\mathcal{L}_\xi e^m, \quad (34)$$

with $\mathcal{L}_\xi$ the Lie derivative with respect to $\xi_\mu = \frac{1}{4}\bar{\theta}\gamma_\mu\epsilon$.

Using the same techniques, for a generic field A , transforming under supersymmetry as

$$\delta A = \mathcal{L}_\xi A, \quad (35)$$

a globally supersymmetric Lagrangian is determined by the substitution

$$\mathcal{L}(\eta_{\mu\nu}, A, \partial_\mu A) \rightarrow e\mathcal{L}(g_{\mu\nu}, A, D_\mu A) \quad (36)$$

where $g_{\mu\nu} = e_\mu{}^m e_{\nu m}$ and D_μ is the covariant derivative constructed out of $e_\mu{}^m$.

We can now turn to the locally supersymmetric case, starting from $\mathcal{N} = (1, 0)$ supergravity Lagrangian [23] for the closed-string sector, written in the string frame

$$\begin{aligned} e^{-1}\mathcal{L}_{\text{closed}} = & e^{-2\phi}\left[-\frac{1}{2}R + 2(\partial\phi)^2 - \frac{1}{2}\bar{\psi}_\mu\gamma^{\mu\nu\rho}D_\nu\psi_\rho + 4\bar{\lambda}\gamma^\mu D_\mu\lambda \right. \\ & \left. - 2\sqrt{2}\bar{\lambda}\gamma^{\mu\nu}D_\mu\psi_\nu - 2\sqrt{2}\bar{\psi}_\mu\gamma^\nu\gamma^\mu\lambda\partial_\nu\phi - \bar{\psi}_\mu\gamma^\mu\psi_\nu\partial^\nu\phi\right] - \frac{3}{8}H_{\mu\nu\rho}H^{\mu\nu\rho} \\ & - \frac{1}{16}e^{-\phi}H^{\rho\sigma\tau}\left[\bar{\psi}_\mu\gamma^{[\mu}\gamma_{\rho\sigma\tau}\gamma^{\nu]}\psi_\nu - 2\sqrt{2}\bar{\psi}_\mu(\gamma^\mu{}_{\rho\sigma\tau} + 3\delta^\mu_{\rho}\gamma_{\sigma\tau})\lambda - 4\bar{\lambda}\gamma_{\rho\sigma\tau}\lambda\right], \end{aligned} \quad (37)$$

supplemented by the transformation rules

$$\begin{aligned} \delta e_\mu{}^m &= \frac{1}{2}\bar{\epsilon}\gamma^m\psi_\mu, \\ \delta\phi &= \frac{1}{\sqrt{2}}\bar{\epsilon}\lambda \\ \delta B_{\mu\nu} &= -\frac{1}{2}e^{-\phi}\bar{\epsilon}(\gamma_\mu\psi_\nu - \gamma_\nu\psi_\mu - \sqrt{2}\gamma_{\mu\nu}\lambda), \\ \delta\lambda &= \frac{1}{2\sqrt{2}}\gamma^\mu\partial_\mu\phi\epsilon - \frac{1}{8\sqrt{2}}e^\phi\gamma^{\mu\nu\rho}H_{\mu\nu\rho}\epsilon, \\ \delta\psi_\mu &= D_\mu\epsilon + \frac{1}{16}e^\phi\gamma^{\nu\rho\sigma}\gamma_\mu H_{\nu\rho\sigma}\epsilon. \end{aligned} \quad (38)$$

We want now to couple this supergravity Lagrangian to the non-supersymmetric matter originating from the open-string sector of the USp(32) model: gauge bosons in the adjoint representation, a Majorana-Weyl fermion χ in the 495 antisymmetric irreducible representation, and a singlet Majorana-Weyl spinor θ that is to be identified with the

goldstino. At the disk level, the low energy Lagrangian not only contains the kinetic terms familiar from the supersymmetric case. There is also a tree-level dilaton tadpole Λ , as can be extracted from the orientifold partition function, as well as new terms involving the goldstino θ , a novelty in this class of models [5]:

$$\mathcal{L}_{\text{disk}} = e \left[e^{-\phi} \left(-\frac{1}{4} \text{tr} F^2 - \Lambda - \frac{1}{2} \text{tr} \bar{\chi} \gamma^\mu D_\mu \chi \right) + \frac{1}{16} \text{tr} \bar{\chi} \gamma^{\mu\nu\rho} \chi H_{\mu\nu\rho} \right] + \mathcal{L}_\theta. \quad (39)$$

The θ -dependence can then be fixed demanding that the total Lagrangian be invariant under the previous linear transformations for the closed-string fields, supplemented (at first order) by

$$\delta\theta = \epsilon, \quad \delta A = \mathcal{L}_\xi A, \quad \delta\chi = \mathcal{L}_\xi \chi. \quad (40)$$

In analogy to the global case, this can be accomplished if we redefine closed string fields in such a way that their supersymmetry variations are actually general coordinate transformations with parameter ξ_μ . In particular, following (36) the redefinitions

$$\begin{aligned} \hat{e}_\mu{}^m &= e_\mu{}^m + \frac{1}{4} \bar{\theta} \gamma^m D_\mu \theta - \frac{1}{2} \bar{\theta} \gamma^m \psi_\mu + \frac{1}{2} \bar{\theta} S^m{}_n \theta e_\mu{}^n, \\ \hat{\phi} &= \phi - \frac{1}{\sqrt{2}} \bar{\theta} \lambda - \frac{1}{32} e^\phi \bar{\theta} \gamma^{\mu\nu\rho} \theta H_{\mu\nu\rho}, \end{aligned} \quad (41)$$

with $S^{mn} = -\frac{1}{32} e^\phi \left(\gamma^{\mu\nu\rho} H_{\mu\nu\rho} \eta^{mn} - 6 H_{\mu\nu}{}^\rho \gamma^{\mu\nu}{}^{(n} e_{\rho}{}^{m)} \right)$, allow us to rewrite the full Lagrangian in a geometric fashion [5]

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{\text{closed}}(e, \phi, B, \psi, \lambda) + \mathcal{L}_{\text{disk}}(\hat{e}, \hat{\phi}; A, \chi) \\ &= \mathcal{L}_{\text{closed}}(e, \phi, B, \psi, \lambda) + \hat{e} e^{-\hat{\phi}} \left(\frac{1}{4} \hat{g}^{\mu\rho} \hat{g}^{\nu\sigma} \text{tr} F_{\mu\nu} F_{\rho\sigma} + \Lambda \right) \\ &\quad + e \left(-\frac{1}{2} e^{-\phi} \text{tr} \bar{\chi} \gamma^\mu D_\mu \chi + \frac{1}{16} \text{tr} \bar{\chi} \gamma^{\mu\nu\rho} \chi H_{\mu\nu\rho} \right). \end{aligned} \quad (42)$$

After expanding $\mathcal{L}$ up to terms quartic in the fermions one indeed finds

$$\mathcal{L} = \mathcal{L}_{\text{closed}}(e, \phi, B, \psi, \lambda) + \mathcal{L}_{\text{disk}}(e, \phi, B, A, \chi) + \mathcal{L}_\theta + \dots \quad (43)$$

that defines $\mathcal{L}_\theta$.

A final comment is in order about the geometrization of the Chern-Simons terms responsible for the cancellation of reducible anomalies. In this formulation of (1,0) supergravity a geometric description of the Chern-Simons terms is not possible. We can, however, resort to the dual formulation in terms of a six-form gauge field and show that the, now Wess-Zumino term, can be conveniently written as

$$\mathcal{L}_{\text{WZ}} \sim \hat{B}_6 \wedge F \wedge F, \quad (44)$$

with $\hat{B}_6$ a properly dressed six-form [24].

4.2 The USp(32) superstring: the vacuum

To complete the analysis of the USp(32) superstring one should study the vacuum of the model [22]. To this end we put to zero all the fermionic fields and look for solutions with a non-vanishing metric and dilaton. The relevant contributions to the low-energy Action are then

$$\mathcal{S} = \int d^{10}x \sqrt{-g} \left[e^{-2\phi} \left(R + 4(\partial\phi)^2 \right) + \Lambda e^{-\phi} \right], \quad (45)$$

while the field equations read

$$\begin{aligned} e^{-2\phi} (8(\partial\phi)^2 - 8\nabla^2\phi - 2R) + \Lambda e^{-\phi} &= 0, \\ e^{-2\phi} (R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\phi) - \tfrac{1}{4}g_{\mu\nu}\Lambda e^{-\phi} &= 0. \end{aligned} \quad (46)$$

It is then evident that the presence of the dilaton tadpole is incompatible with a maximally symmetric Minkowski space. However, the theory admits a background with manifest $SO(1,8)$ symmetry resulting from a warping of the ninth dimension

$$ds^2 = e^{2A(u)}\eta_{mn}dx^m dx^n + \Lambda^{-1}e^{2B(u)}du^2, \quad \phi = \phi(u). \quad (47)$$

Substituting this Ansatz into the eqs. (46), after some algebra one finds the solution

$$e^\phi = e^{\phi_0}|u|^{2/3}e^{3u^2/4}, \quad (48)$$

$$ds^2 = |u|^{4/9}e^{\phi_0/2}e^{u^2/4}\eta_{mn}dx^m dx^n + |u|^{-2/3}e^{-\phi_0}e^{-3u^2/4}dy^2. \quad (49)$$

It displays two time-like singularities, one at the origin, $u = 0$, and one at infinity, $u = +\infty$, so that the effective range of the u coordinate is $0 < u < +\infty$. On the other hand, the dilaton vanishes at $u = 0$ and diverges at $u = +\infty$, where the perturbative analysis fails to apply. Moreover, the solution (48) has a striking feature: though the u coordinate is non-compact, its effective length is actually finite

$$2\pi R_c = e^{-\phi_0/2}\Lambda^{-1/2} \int_0^\infty \frac{du}{u^{1/3}} e^{-3u^2/8} < +\infty. \quad (50)$$

The topology of the solution is thus a ten-dimensional manifold with two boundaries at $u = 0$ and $u = +\infty$.

Another hint in favour of an effective nine-dimensional vacuum comes from the analysis of the lower-dimensional Plank mass and gauge coupling

$$\begin{aligned} M_{\text{Pl}}^7 &= M_s^8 \Lambda^{-1/2} e^{-3\phi_0/4} \int_0^\infty \frac{du}{u^{1/9}} e^{-3u^2/4} < +\infty, \\ \frac{1}{g_{\text{YM}}^2} &= M_s^6 \Lambda^{-1/2} e^{-\phi_0/4} \int_0^\infty du u^{1/9} e^{-u^2/2} < +\infty. \end{aligned} \quad (51)$$

Both of them are finite, thus suggesting that the gravitational and gauge physics are effectively nine-dimensional.

Although the main features of the solutions crucially rely on the presence of the dilaton tadpole, the main characteristic of models with brane supersymmetry breaking, their real persistence after quantum effects are taken into account is an open question. The conformal factor e^{2A} as well as the string coupling vanish at the origin and diverge at $u = +\infty$. We thus expect α' and g_s corrections to be important at the two edges of the interval.

5 Magnetized orbifolds and brane transmutation

Another mechanism to break supersymmetry in the open sector consists in the open-string propagation in the presence of non-trivial gauge configurations [25, 26], that, recently, have triggered a consistent attention [27, 28]. Some intuitive field theory arguments suffice to

expose the essence of the phenomenon, and are well captured by the low-energy effective action for D9 branes in an internal Abelian background [27]¹,

$$\mathcal{S}_9 = -T_{(9)} \int_{\mathcal{M}_{10}} d^{10}x e^{-\phi} \sum_{a=1}^{32} \sqrt{-\det(g_{10} + q_a F)} - \mu_{(9)} \sum_{p,a} \int_{\mathcal{M}_{10}} e^{q_a F} \wedge C_{p+1} + \dots, \quad (52)$$

where a labels the types of Chan-Paton charges that couple to the magnetic fields with strength q_a , and

$$T_{(p)} = \sqrt{\frac{\pi}{2\kappa^2}} (2\pi\sqrt{\alpha'})^{3-p} = |\mu_{(p)}|. \quad (53)$$

Here T and μ are the tension and the R-R charge for a type I Dp brane [7], while κ defines the ten-dimensional Newton constant $G_N^{(10)} = \kappa^2/8\pi$. To illustrate the phenomenon, it suffices to consider the geometry $\mathcal{M}_{10} = \mathcal{M}_6 \times T^2 \times T^2$ with constant Abelian magnetic fields H_1 and H_2 lying in the two internal tori. These are monopole fields, and thus satisfy the Dirac quantization conditions

$$q H_i v_i = k_i \quad (i = 1, 2), \quad (54)$$

where, aside from powers of 2π , $v_i = R_i^{(1)} R_i^{(2)} / \alpha'$ are the dimensionless volumes of the two tori of radii $R_i^{(1)}$ and $R_i^{(2)}$, k_i are the degeneracies of the corresponding Landau levels and q is the elementary electric charge for the system. As anticipated, we forego the restriction in [25, 26] and actually pick up a pair of Abelian fields aligned within the same U(1) subgroup. This affects the Wess-Zumino coupling, giving rise to an effective D5 charge, so that

$$\begin{aligned} \mathcal{S}_9 = & -T_{(9)} \int_{\mathcal{M}_{10}} d^{10}x e^{-\phi} \sqrt{-g_6} \sum_{a=1}^{32} \sqrt{(1 + q_a^2 H_1^2)(1 + q_a^2 H_2^2)} \\ & - 32 \mu_{(9)} \int_{\mathcal{M}_{10}} C_{10} - (2\pi\sqrt{\alpha'})^4 \mu_{(9)} v_1 v_2 H_1 H_2 \sum_{a=1}^{32} q_a^2 \int_{\mathcal{M}_6} C_6, \end{aligned} \quad (55)$$

where g_6 denotes the six-dimensional space-time metric, and where for simplicity we have chosen an identity metric in the internal space. In particular, if the two internal fields have identical magnitudes, for the resulting (anti)self-dual configuration the action becomes

$$\begin{aligned} \mathcal{S}_9 = & -32 \int_{\mathcal{M}_{10}} \left(d^{10}x \sqrt{-g_6} T_{(9)} e^{-\phi} + \mu_{(9)} C_{10} \right) \\ & - \sum_{a=1}^{32} \left(\frac{q_a}{q} \right)^2 \int_{\mathcal{M}_6} \left(d^6x \sqrt{-g_6} |k_1 k_2| T_{(5)} e^{-\phi} + k_1 k_2 \mu_{(5)} C_6 \right). \end{aligned} \quad (56)$$

Therefore, not only the Dirac quantization conditions (54) have compensated the integration over the internal tori, but in the second line of (56) the additional powers of α' have nicely converted $T_{(9)}$ and $\mu_{(9)}$ into $T_{(5)}$ and $\mu_{(5)}$. As a result, a D9 brane on a magnetized $T^2 \times T^2$ indeed mimics a D5 brane or a D5 anti-brane according to whether the orientations of H_1 and H_2 , reflected by the relative sign of k_1 and k_2 , are identical or opposite.

We can now extend the analysis to string theory, following [1]. A precise control of the conformal field theory exhibits very nicely several properties of the effective action, including new couplings to twisted states of orbifold models [27]. As we anticipated,

¹As in [27], the (dimensionless) magnetic fields differ from the conventional ones by a $2\pi\alpha'$ rescaling.

supersymmetry is generically broken, but supersymmetric vacuum configurations can be obtained starting from an orbifold that normally requires the introduction of D5 branes. The simplest such instance is the six-dimensional compactification on $(T^2 \times T^2)/\mathbb{Z}_2$ with Klein-bottle projection

$$\mathcal{K} = \frac{1}{4} \left\{ (Q_o + Q_v)(0; 0) [P_1 P_2 + W_1 W_2] + 16 \times 2(Q_s + Q_c)(0; 0) \left(\frac{\eta}{\vartheta_4(0)} \right)^2 \right\}, \quad (57)$$

that corresponds to the introduction of $O9_+$ and $O5_+$ planes, and thus to a projected $\mathcal{N} = (1, 0)$ supersymmetric closed spectrum with one tensor multiplet and 20 hyper multiplets. For the sake of brevity, in this section we shall confine our attention to the models of [27] without D5 branes, where the $O5$ charge is fully compensated by the magnetized D9 branes. The characters used in eq. (57) and in the following are in general combinations of theta-functions with non-vanishing arguments, that extend the standard $(1, 0)$ supersymmetric combinations, and are described in detail in [27].

As in our field theory considerations, we introduce a pair of internal magnetic fields aligned with the same $U(1)$ subgroup of $SO(32)$, and we restrict our attention to the maximal residual gauge group, $U(m) \times U(n)$, with $m + n = 16$. In writing the direct-channel annulus amplitude, let us begin by recalling [29] that a uniform magnetic field with components H_1 and H_2 in the two internal tori alters the boundary conditions for open strings, shifting their mode frequencies by

$$z_i^{L,R} = \frac{1}{\pi} \left[\tan^{-1}(q_L H_i) + \tan^{-1}(q_R H_i) \right], \quad (58)$$

where q_L (q_R) denotes the charge of the left (right) end of the open string with respect to the $U(1)$ field H_i . A further novelty [29] is displayed by “dipole” strings, with opposite end charges, whose oscillator modes are unaffected, but whose world-sheet coordinates undergo a complex “boost”, so that their Kaluza-Klein momenta m_i are rescaled according to

$$m_i \rightarrow \frac{m_i}{\sqrt{1 + q_a^2 H_i^2}}. \quad (59)$$

The direct-channel annulus amplitude is

$$\begin{aligned} \mathcal{A} = & \frac{1}{4} \left\{ (Q_o + Q_v)(0; 0) \left[(m + \bar{m})^2 P_1 P_2 + 2n\bar{n} \tilde{P}_1 \tilde{P}_2 \right] \right. \\ & - 2(m + \bar{m})(n + \bar{n})(Q_o + Q_v)(z_1\tau; z_2\tau) \frac{k_1\eta}{\vartheta_1(z_1\tau)} \frac{k_2\eta}{\vartheta_1(z_2\tau)} \\ & - (n^2 + \bar{n}^2)(Q_o + Q_v)(2z_1\tau; 2z_2\tau) \frac{2k_1\eta}{\vartheta_1(2z_1\tau)} \frac{2k_2\eta}{\vartheta_1(2z_2\tau)} \\ & - \left[(m - \bar{m})^2 - 2n\bar{n} \right] (Q_o - Q_v)(0; 0) \left(\frac{2\eta}{\vartheta_2(0)} \right)^2 \\ & - 2(m - \bar{m})(n - \bar{n})(Q_o - Q_v)(z_1\tau; z_2\tau) \frac{2\eta}{\vartheta_2(z_1\tau)} \frac{2\eta}{\vartheta_2(z_2\tau)} \\ & \left. - (n^2 + \bar{n}^2)(Q_o - Q_v)(2z_1\tau; 2z_2\tau) \frac{2\eta}{\vartheta_2(2z_1\tau)} \frac{2\eta}{\vartheta_2(2z_2\tau)} \right\}, \quad (60) \end{aligned}$$

while the corresponding Möbius amplitude is

$$\mathcal{M} = -\frac{1}{4} \left\{ (\hat{Q}_o + \hat{Q}_v)(0; 0) [(m + \bar{m}) P_1 P_2] \right.$$

$$\begin{aligned}
& - (n + \bar{n})(\hat{Q}_o + \hat{Q}_v)(2z_1\tau; 2z_2\tau) \frac{2k_1\hat{\eta}}{\hat{\vartheta}_1(2z_1\tau)} \frac{2k_2\hat{\eta}}{\hat{\vartheta}_1(2z_2\tau)} \\
& - (m + \bar{m})(\hat{Q}_o - \hat{Q}_v)(0; 0) \left(\frac{2\hat{\eta}}{\hat{\vartheta}_2(0)} \right)^2 \\
& - (n + \bar{n})(\hat{Q}_o - \hat{Q}_v)(2z_1\tau; 2z_2\tau) \frac{2\hat{\eta}}{\hat{\vartheta}_2(2z_1\tau)} \frac{2\hat{\eta}}{\hat{\vartheta}_2(2z_2\tau)} \Big\} .
\end{aligned} \tag{61}$$

The arguments z_i ($2z_i$) are associated to strings with one (two) charged ends while, for the sake of brevity, both the imaginary modulus $\frac{1}{2}it$ of $\mathcal{A}$ and the complex modulus $\frac{1}{2} + \frac{1}{2}it$ of $\mathcal{M}$ are denoted by the same symbol τ , although the proper “hatted” contributions to the Möbius amplitude are explicitly indicated. P_i and W_i are conventional momentum and winding sums for the two-tori, while a “tilde” denotes a sum with momenta “boosted” as in (59). Finally, m and n (together with their conjugates $\bar{m}$ and $\bar{n}$) are Chan-Paton multiplicities for the D9 brane, and several terms with opposite z_i arguments have been grouped together, using the symmetries of the Jacobi theta-functions.

For generic magnetic fields, the open spectrum is indeed non-supersymmetric and develops Nielsen-Olesen instabilities [30]. As emphasized in [26], the emergence of these tachyonic modes can be ascribed to the magnetic couplings of the internal components of gauge fields. For instance, small magnetic fields affect the mass formula for the untwisted string modes according to

$$\Delta M^2 = \frac{1}{2\pi\alpha'} \sum_{i=1,2} [(2n_i + 1)|(q_L + q_R)H_i| + 2(q_L + q_R)\Sigma_i H_i] , \tag{62}$$

where the first term originates from the Landau levels and the second from the magnetic moments of the spins Σ_i . For the internal components of the vectors, the magnetic moment coupling generally overrides the zero-point contribution, leading to tachyonic modes, unless $|H_1| = |H_2|$, while for spin- $\frac{1}{2}$ modes it can at most compensate it. Moreover, if $H_1 = H_2$ the supersymmetry charge, that belongs to $C_4 C_4$, is also unaffected. On the other hand, if $H_1 = -H_2$ one obtains models with “brane supersymmetry breaking”, similar in spirit to those of [31, 32, 33, 4, 34]. However, in this case supersymmetry is broken on the whole magnetized D9 brane, since we are working effectively in the presence of blown-up instantons.

The untwisted R-R tadpole conditions arising from the $C_4 S_2 C_2$ sector read

$$\left[m + \bar{m} + n + \bar{n} - 32 + q^2 H_1 H_2 (n + \bar{n}) \right] \sqrt{v_1 v_2} - \frac{32}{\sqrt{v_1 v_2}} = 0 , \tag{63}$$

aside from terms that vanish after identifying the multiplicities of conjugate representations $(m, \bar{m})$ and $(n, \bar{n})$. The additional (untwisted) R-R tadpole conditions from Q_o and Q_v are compatible with (63) and do not add further constraints. This expression reflects the familiar Wess-Zumino coupling of eq. (52), and therefore the various powers of H correspond to R-R forms of different degrees. In particular, as we anticipated in our field theory discussion, the term bilinear in the magnetic fields has a very neat effect: it charges the D9 brane with respect to the six-form potential. This can be seen very clearly making use of the quantization condition (54), that turns the tadpole conditions (63) into

$$\begin{aligned}
m + \bar{m} + n + \bar{n} &= 32 , \\
k_1 k_2 (n + \bar{n}) &= 32 .
\end{aligned} \tag{64}$$

In a similar fashion, the untwisted NS-NS tadpoles exhibit very nicely their relation to the Born-Infeld term in (52), and can be linked to its derivatives with respect to the corresponding moduli, while the twisted NS-NS tadpoles display new couplings to twisted states present in the effective Lagrangian. A novelty is that some of the tadpoles are *not* perfect squares, as a result of the peculiar behaviour of the internal magnetic deformations under world-sheet time reversal. All these features are described in some detail in [27]. While for generic internal magnetic fields it is impossible to satisfy the NS-NS tadpoles, the supersymmetric choice $H_1 = H_2$ makes them all nicely compatible with the R-R ones.

We can now describe the low-lying spectrum of a model without D5 branes and with $k_1 = k_2 = 2$, the minimal Landau-level degeneracies that on this $\mathbb{Z}_2$ orbifold are compatible with the positivity of the direct channel. Although the closed spectrum is the standard one, and comprises the $\mathcal{N} = (1, 0)$ gravitational multiplet, together with one tensor multiplet and twenty hyper multiplets, the open spectrum is quite different from the familiar one of [17, 18], with gauge group $U(16)|_9 \times U(16)|_5$. Having excluded the D5 branes, the solution of the tadpole conditions yields $m = 12$, $n = 4$, and the result is a rather unusual supersymmetric $\mathbb{Z}_2$ model, with a gauge group of rank 16, $U(12) \times U(4)$, and with charged hyper multiplets in the representations $(66 + \overline{66}, 1)$, in five copies of the $(1, 6 + \overline{6})$, and in four copies of the $(\overline{12}, 4)$. A distinctive feature of this spectrum, that is free of all irreducible gauge and gravitational anomalies, consistently with the vanishing of all R-R tadpole conditions [2], as well as of the similar ones in [27], is that some of the matter occurs in multiple families. This peculiar phenomenon is a consequence of the multiplicities of Landau levels, that in these $\mathbb{Z}_2$ orbifolds with vanishing B_{ab} are multiples of two for each magnetized torus. Notice that, when D5 branes are also present [27], one is led in general to rank reductions, but not simply by powers of two as in the presence of a quantized B_{ab} [35, 36, 32], that can also be introduced in this class of models [37].

6 The one-dimensional shift-orbifold

Until now we have discussed models characterized by a non-supersymmetric open string spectrum coupled to a supersymmetric bulk. The main essence of these phenomena rely on the presence of supersymmetry-preserving O-plane configurations and on the inclusion of anti-branes. One can then expect that in order to break supersymmetry also in the bulk anti-O-planes should be added. Indeed, this is the case, as we shall discuss in the following simple model.

To this end, we now turn to orbifold compactifications where the target-space coordinates are not identified under reflections, as in the previous case, but under discrete, fractional, shifts of the lattice basis vectors or, more generally, under the combined action of shifts and internal symmetries. This combined action is particularly interesting, since it can implement in String Theory [38] the Scherk-Schwarz mechanism [39] to attain the breaking of supersymmetry. In the simplest case of circle compactification it is equivalent to allowing higher-dimensional modes that are periodic around the circle only up to an internal symmetry transformation. The Kaluza-Klein momenta of the various fields are correspondingly shifted proportionally to their charges, with the consequent possibility of introducing mass differences between bosons and fermions.

In field theory, where only Kaluza-Klein excitations are present, the Scherk-Schwarz mechanism can only result from shifts of internal KK momenta. On the other hand, String Theory presents one more possibilities, since one has the option of affecting either momenta or windings. For oriented closed superstrings these two deformations, related by

T-duality, describe essentially the same phenomena. After orientifolding, however, they lead to completely different results. As in [40], we shall refer to these two mechanisms as Scherk-Schwarz and M-theory breaking, since the second can actually be related to conventional deformations along the eleventh coordinate.

As we have emphasized, in string theory one has the additional option of considering winding shifts. A T-duality can turn these into the more conventional momentum shifts, but only at the price of turning the type IIB string into type IIA, so that the dimension of the branes is correspondingly affected. Intuitively, one would then expect that the resulting momentum shifts, *orthogonal* to the D8 branes, be ineffective on their excitations. Although this expectation is not fully realized in this simple setting⁴, the massless brane modes, as we shall see, are indeed supersymmetric [40]. This phenomenon is usually referred to as “brane supersymmetry”, and can be nicely illustrated by the following nine-dimensional example.

The starting point is the partition function for the type IIB superstring subject to winding shifts along a circle of radius R

$$\begin{aligned} T_W = & (V_8 \bar{V}_8 + S_8 \bar{S}_8) \Lambda_{m,2n} + (O_8 \bar{O}_8 + C_8 \bar{C}_8) \Lambda_{m+\frac{1}{2},2n} \\ & - (V_8 \bar{S}_8 + S_8 \bar{V}_8) \Lambda_{m,2n+1} - (O_8 \bar{C}_8 + C_8 \bar{O}_8) \Lambda_{m+\frac{1}{2},2n+1} . \end{aligned} \quad (65)$$

Notice that in this case the torus amplitude (65) develops a tachyonic instability for $R > \sqrt{\alpha'}$, while for $R \rightarrow 0$ supersymmetry is recovered. These properties are shared by the descendants that we are about to describe, and therefore we shall implicitly restrict our analysis to values of R inside the region of stability. This is beyond the usual limit of applicability of field theory considerations and, as we shall see, some surprises are in store.

In this case, new states contribute to the direct-channel Klein bottle amplitude

$$\mathcal{K}_W = \frac{1}{2} (V_8 - S_8) P_m + \frac{1}{2} (O_8 - C_8) P_{m+\frac{1}{2}} , \quad (66)$$

while in the corresponding transverse-channel amplitude

$$\tilde{\mathcal{K}}_W = \frac{2^5}{2} \frac{2R}{\sqrt{\alpha'}} (V_8 W_{4n} - S_8 W_{4n+2}) \quad (67)$$

the only massless contribution originates from the NS-NS character V_8 . As a result, these winding-shift orientifolds involve equal numbers of O9 planes and O9 anti-planes, whose overall R-R charge indeed vanishes.

On the other hand, the transverse-channel annulus amplitude can only accommodate V_8 and S_8 , that have zero-momentum lattice modes, and can be written in the form

$$\begin{aligned} \tilde{\mathcal{A}}_W = & \frac{2^{-5}}{2} \frac{2R}{\sqrt{\alpha'}} \left\{ \left[(n_1 + n_2 + n_3 + n_4)^2 V_8 \right. \right. \\ & - (n_1 + n_2 - n_3 - n_4)^2 S_8 \left. \right] W_{4n} \\ & + \left[(n_1 - n_2 + n_3 - n_4)^2 V_8 \right. \\ & \left. \left. - (n_1 - n_2 - n_3 + n_4)^2 S_8 \right] W_{4n+2} \right\} , \end{aligned} \quad (68)$$

where for later convenience we have distinguished four types of contributions. As usual, from the relative sign of the Chan-Paton multiplicities in the coefficient of S_8 one can see that n_1 and n_2 count the D9 branes, while n_3 and n_4 count the D9 antibranes.

⁴In more complicated models, whole towers of open-string excitations can be supersymmetric [41].

From $\tilde{\mathcal{K}}_W$ and $\tilde{\mathcal{A}}_W$ one can derive as usual the transverse-channel Möbius amplitude

$$\begin{aligned} \tilde{\mathcal{M}}_W = & -\frac{2R}{\sqrt{\alpha'}} \left[(n_1 + n_2 + n_3 + n_4) \hat{V}_8 W_{4n} \right. \\ & \left. - (n_1 - n_2 - n_3 + n_4) \hat{S}_8 W_{4n+2} \right] \end{aligned} \quad (69)$$

and then extract the tadpole conditions

$$n_1 + n_2 + n_3 + n_4 = 32, \quad n_1 + n_2 = n_3 + n_4. \quad (70)$$

However, in the limit $R \rightarrow 0$, that as we have seen is well within the stability region, this model develops the additional tadpoles

$$n_1 + n_3 = n_2 + n_4, \quad n_1 - n_2 - n_3 + n_4 = 32, \quad (71)$$

arising from the sectors with shifted winding sums W_{4n+2} , whose states collapse to zero mass. This is the analogue, in this context, of the phenomenon stressed in [42]. Enforcing all these conditions leads to the unique solution

$$n_1 = 16 = n_4, \quad n_2 = 0 = n_3, \quad (72)$$

Finally, S and P modular transformations yield the direct-channel open string amplitudes

$$\begin{aligned} \mathcal{A}_W = & \frac{1}{2}(n_1^2 + n_4^2) (V_8 - S_8) (P_m + P_{m+\frac{1}{2}}) \\ & + n_1 n_4 (O_8 - C_8) (P_{m+\frac{1}{4}} + P_{m+\frac{3}{4}}) \end{aligned} \quad (73)$$

and

$$\mathcal{M}_W = -\frac{1}{2}(n_1 + n_4) \left[(\hat{V}_8 - \hat{S}_8) P_m + (\hat{V}_8 + \hat{S}_8) P_{m+\frac{1}{2}} \right]. \quad (74)$$

The resulting spectrum is indeed supersymmetric at the massless level, where it contains a vector multiplet for the gauge group $\text{SO}(16) \times \text{SO}(16)$. On the other hand, the massive excitations are not supersymmetric, as a result of the different Möbius projections of Bose and Fermi modes, as well as of the presence of the O_8 and C_8 sectors.

The peculiar result for the gauge group can actually be given an interesting interpretation in terms of M theory [43]. Namely, by a T-duality one can turn these winding shifts into momentum shifts in a direction orthogonal to the branes, that can be identified with the eleventh dimension of M theory. We are thus facing Scherk-Schwarz breakings in the Horava-Witten scenario [44], that here have a neat perturbative description. These interesting issues are further discussed in [40].

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