Abstract

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Hadronic decays of $X_{b1}$ into light bottom squarks
I. INTRODUCTION

The possible existence of a light bottom squark $\tilde{b}$ with mass similar to or less than that of the bottom quark $b$ is investigated in several theoretical papers [1, 2, 3, 4, 5, 6, 7]. This hypothesis is not inconsistent with direct experimental searches and indirect constraints from other observables [8, 9, 10]. If the mass $m_1$ is less than half of that of the $P$-wave $\chi_b$ resonances, then the direct decay $\chi_b \to \tilde{b}\tilde{b}^*$ could proceed with sufficient rate for observation, particularly in data being accumulated at the Cornell CESR facility [11]. In this paper, we compute the expected rates for decays of the three $\chi_{b,J}$ states into a pair of bottom squarks as a function of the masses of the bottom squark and the gluino. The mass of the gluino $\tilde{g}$ enters because the gluino is exchanged in the relevant decay subprocesses. The $\tilde{g}$ and the $\tilde{b}$ are the spin-1/2 and spin-0 supersymmetric (SUSY) partners of the gluon ($g$) and bottom quark. This paper extends previous work of one of us [7] on decays of the Upsilon states, $\Upsilon(nS) \to \tilde{b}\tilde{b}^*, n = 1 - 4$.

In the remainder of this introduction, we summarize the phenomenological motivation and status of the hypothesis of a light bottom squark along with a light gluino and then present an outline of the remainder of the paper. A recent presentation of the experimental properties of the $\chi_{b,J}$ states may be found in Ref. [12].

The hypothesis of a relatively light color-octet gluino $\tilde{g}$ (mass $\simeq$ 12 to 16 GeV) that decays with 100% branching fraction into a bottom quark $b$ and a light color-triplet bottom squark $\tilde{b}$ (mass $\simeq$ 2 to 5.5 GeV) is proposed in Ref. [1] in order to explain the observed excess cross section for production of bottom quarks at hadron colliders. In this scenario $\tilde{b}$ is the lightest SUSY particle (LSP), and the masses of all other SUSY particles are arbitrarily heavy, i.e., of order the electroweak scale or greater. Improved agreement is obtained with the magnitude and shape of the transverse momentum distribution of bottom-quark production at hadron colliders. The proposal is consistent with measurements of the time-averaged $B^0\bar{B}^0$ mixing probability [1]. The $\tilde{b}$ either picks up a light quark, becoming a spin-1/2 “mesino”, perhaps living long enough to escape from a typical collider detector, or it decays promptly via baryon-number R-parity violation into a pair of hadronic jets [10, 13].

There are important restrictions on the existence and couplings of bottom squarks from precise measurements of $Z^0$ decays. A light $\tilde{b}$ would be ruled out unless its coupling to the $Z^0$ is very small. The squark couplings to the $Z^0$ depend on the mixing angle $\theta_Z$. If the
light bottom squark (\( \tilde{b}_1 \)) is an appropriate mixture of left-handed and right-handed bottom squarks, its lowest-order (tree-level) coupling to the \( Z^0 \) can be arranged to be small [2] if \( \sin^2 \theta_b \sim 1/6 \). The exclusion by the CLEO collaboration [8] of a \( \tilde{b} \) with mass 3.5 to 4.5 GeV does not apply since their analysis focuses only on the leptonic decays \( \tilde{b} \rightarrow c \ell \nu \) and \( \tilde{b} \rightarrow c \ell \). The \( \tilde{b} \) need not decay leptonically nor into charm. The search by the DELPHI collaboration [9] for long-lived squarks in their \( \gamma \gamma \) event sample is not sensitive to \( m_{\tilde{b}} < 15 \) GeV. Bottom squarks make a small contribution to the ratio \( R \) of the inclusive cross section for \( e^+ e^- \rightarrow \) hadrons divided by that for \( e^+ e^- \rightarrow \mu^+ \mu^- \), requiring an accuracy for detection at the level of 2% or so, much greater than that of current measurements in the 6 to 7% range [14]. In \( e^+ e^- \) production, resonances in the \( \tilde{b} \tilde{b}^* \) system are likely to be impossible to extract from backgrounds [15], but \( \gamma \gamma \) collisions may be more promising. The angular distribution of hadronic jets produced in \( e^+ e^- \) annihilation can be examined in order to bound the contribution of scalar-quark production. Spin-1/2 quarks and spin-0 squarks emerge with different distributions \( \hat{s}_q (1 \pm \cos^2 \theta) \), respectively. The angular distribution measured by the CELLO collaboration [16] is consistent with the production of a single pair of charge-1/3 squarks along with five flavors of quark-antiquark pairs.

The possibility that the gluino may be much less massive than most other supersymmetric particles is intriguing from different points of view [17, 18, 19, 20]. An early study by the UA1 Collaboration [21] excludes \( \tilde{g} \)’s in the mass range \( 4 < m_{\tilde{g}} < 53 \) GeV, but it starts from the assumption that there is a light neutralino \( \tilde{\chi}_1^0 \) whose mass is less than the mass of the gluino. The conclusion is based on the absence of the expected decay \( \tilde{g} \rightarrow q + \bar{q} + E_T \), where \( E_T \) represents the missing energy associated with the \( \tilde{\chi}_1^0 \). In the scenario discussed here, this decay process does not occur since the bottom squark is the LSP and the \( \tilde{\chi}_1^0 \) mass is presumed to be large (i.e., > 50 GeV). An analysis of 2- and 4-jet events by the ALEPH collaboration [22] disfavors \( \tilde{g} \)’s with mass \( m_{\tilde{g}} < 6.3 \) GeV but not \( \tilde{g} \)’s in the mass range relevant for the SUSY interpretation of the bottom squark production cross section. A similar analysis is reported by the OPAL collaboration [23]. A light \( \tilde{b} \) is not excluded by the ALEPH analysis. A very precise measurement of the \( \beta \) function in QCD is potentially the most sensitive probe of the existence of a light gluino since the color-octet gluino makes a contribution to \( \beta \) equivalent to that of 3 new flavors of quarks. However, the analysis of data must take into account the effects of SUSY-QCD production processes. The combined ranges of \( \tilde{b} \) and \( \tilde{g} \) masses proposed in Ref. [1] are compatible with renormalization group
equation\footnote{2} constraints and the absence of color and charge breaking minima in the scalar potential \[3\].

In Sec. II, we begin with a discussion of factorization. The $\chi_b$ is treated as a $\bar{b}\bar{b}$ bound system and described non-perturbatively. The transition of the (on-shell) $\bar{b}\bar{b}$ system into the observed final state is calculated in QCD perturbation theory. In Sec. II, we also outline the projection of the on-shell $\bar{b}\bar{b}$ system into the states of spin, parity, and angular momentum of interest. We present our explicit calculation of $\chi_b \rightarrow \bar{b}\bar{b}^*$ in Sec. III. We include both color-singlet and color-octet configurations. Results and conclusions are summarized in Sec. IV. When compared with the leading-order expectation in the standard model, the bottom squark contribution can increase the predicted hadronic width of the $\chi_{b0}$ by as much as 33% for small bottom squark masses, $O(2\text{ to } 4\text{ GeV})$, and gluino masses in the range of 12 GeV. However, bottom squark decays make a negligible contribution in comparison with the standard model value in the $\chi_{b1}$ and $\chi_{b2}$ cases. The color-octet contribution to bottom squark decays is insignificant for the $\chi_{b0}$ but can dominate in the $\chi_{b2}$ case if current lattice estimates are used for the color-octet matrix element. Data from decays of the $\chi_b$ states could provide significant new bounds on the existence and masses of supersymmetric particles.

II. FACTORIZATION AND CONVENTIONAL DECAYS OF THE $\chi_{bJ}$

To compute the decay of a heavy quarkonium state, we begin with the statement of factorization. The initial $\chi_b$ is taken to be a bound state $\bar{b}\bar{b}$ of bottom quarks. To the extent that the $b$ quarks are heavy enough, the bound state aspects may be described with non-relativistic quantum chromodynamics (NRQCD) \[24\]. The decay dynamics are assumed to be described by short-distance perturbative QCD, in turn also justified in part by the large mass of the $b$ quark. The decay rate of the heavy quarkonium state is then expressed in the factored form \[25, 26\]

$$\Gamma(\chi_{bJ}) = \sum_n C_n <\chi_{bJ}|\mathcal{O}_n|\chi_{bJ}>.$$  \hspace{1cm} (1)

The summation index $n$ stands for the spectroscopic state $^{2S+1}L^{(1,8)}_J$ of the $\bar{b}\bar{b}$ pair. The superscript to the right $(1,8)$ stands for the color quantum number, singlet \[27, 28\] and octet \[25\], respectively. The short-distance coefficient $C_n$, describes the inclusive transition of an on-mass-shell $\bar{b}\bar{b}$ system into the relevant final state, and it is calculable perturbatively.
in a series in the strong coupling strength \( a_s(m_b) \). The non-perturbative matrix element \( \langle \chi_{bJ} | O_n | \chi_{bJ} \rangle \) represents the probability that a \( \chi_{bJ} \) meson evolves into a free \( b \bar{b} \) pair with quantum numbers \( n \). The four quark operator \( O_n \) is defined in Ref. [26].

Although the recent CDF measurement of the polarization of prompt \( J/\psi \)'s presents a serious challenge for the NRQCD factorization formalism for inclusive charmonium production [29], factorization in the bottomonium case is believed to be safe since \( m_b \) is well separated from the long distance scale \( m_b v_b^2 \). For example, the recent CDF measurement of \( \Upsilon(nS) \) polarization [30] agrees with the NRQCD prediction [31] based on matrix elements fitted to the Run IB data [31]. Furthermore, factorization is safer in the decay process than in production. A recent review can be found in chapter 9 of Ref. [32].

An alternative, not unrelated physical picture begins with a Fock state expansion of the \( \chi_b \) into the minimal \( |b\bar{b}\rangle \) color-singlet component and higher components such as \( |b\bar{q}g\rangle \), with the \( b\bar{b} \) pair in a color-octet configuration, \( |b\bar{q}q\rangle \), \( |b\bar{q}g\rangle \), and so forth.

A velocity scaling rule in NRQCD [26] allows one to order the contributions to the decay rate Eq. (1) and to truncate the series on the right hand side of Eq. (1) in a power series in \( v_b \), the mean velocity of the \( b \) in the \( \chi_b \) rest frame. For \( \chi_{bJ} \), there are two potentially equally important contributions to the decay rate. One is the color-singlet spin-triplet \( P \)-wave state that scales as \( \langle \chi_{bJ} | O^3(3P_j^{(1)}) | \chi_{bJ} \rangle / m_b^2 \sim m_b^3 v_b^2 \). The other is the color-octet spin-triplet \( S \)-wave state that is proportional to \( \langle \chi_{bJ} | O^3(3S_1^{(8)}) | \chi_{bJ} \rangle \sim m_b^3 v_b^2 \), where we keep the mass dimension while suppressing the \( v_b \) dependence of the heavy quark fields and the quarkonium state. The scaling factor \( v_b^2 \) of the color-octet matrix element arises from the chromoelectric dipole transition rate from a color-singlet \( P \)-wave state into a color-octet spin-triplet state. This color-octet channel was introduced to resolve the infrared problem in the next-to-leading order QCD corrections to light-hadron decays of \( P \)-wave quarkonium [25].

In the case of \( \Upsilon \) decay to bottom quarks treated in Ref. [7], the leading color-singlet contribution is proportional to \( a_s^2 v_b^0 \). The leading contribution in the standard model decay of \( \Upsilon \) to light hadrons is of order \( a_s^2 v_b^0 \), with one more power of \( a_s \) than in the SUSY case. The SUSY rate is suppressed, however, by the mass of the exchanged gluino in the amplitude and the non-zero bottom squark masses. The leading color-octet contribution in \( \Upsilon \) decay is of order \( a_s^2 v_b^4 \).

In conventional QCD, the \( \chi_{bJ} \) states are assumed to decay via the transition of the \( b\bar{b} \) system into a pair of massless gluons that, in turn, materialize as light hadrons [27]. The
short-distance coefficients $C_n$ are known in next-to-leading order in $\alpha_s$ for the color-singlet states $n = q P_l^{(1)}$ [33, 34] and for the color-octet state $n = q S_l^{(8)}$ [34, 35]. We quote the leading-order result in $\alpha_s v^2$ [25, 26, 27, 36]:

$$\Gamma_{\mathrm{SM}}(\chi_{i0}) = \frac{4\pi\alpha_s^2 H_1}{3 m_b^3} \left( 1 + \frac{n f m_b^2 H_S}{4 H_1} \right),$$  

(2a)

$$\Gamma_{\mathrm{SM}}(\chi_{i1}) = \frac{4\pi\alpha_s^2 H_1}{3 m_b^3} \left( 0 + \frac{n f m_b^2 H_S}{4 H_1} \right),$$  

(2b)

$$\Gamma_{\mathrm{SM}}(\chi_{i2}) = \frac{4\pi\alpha_s^2 H_1}{3 m_b^3} \left( \frac{4}{15} + \frac{n f m_b^2 H_S}{4 H_1} \right).$$  

(2c)

The superscript SM designates the standard model contribution without SUSY terms. The number of flavors $n_f = 4$. The matrix elements in Eq. (2) are the color-singlet term $H_1 = \langle h_i | O(1 P_l^{(1)}) | h_i \rangle$ and the color-octet term $H_S = \langle h_i | O(1 S_l^{(8)}) | h_i \rangle$; $h_i$ is the spin-singlet $P$-wave bottomonium state. Heavy quark spin symmetry is used in the approximations $\langle \chi_{i j} | O(3 P_l^{(1)}) | \chi_{i j} \rangle = H_1 + O(v_b^2)$, and $\langle \chi_{i j} | O(3 S_l^{(8)}) | \chi_{i j} \rangle = H_S + O(v_b^2)$. The color-singlet matrix element can be expressed in terms of the derivative of the radial wave function at the origin, $H_1 = \frac{3 N_c}{2} |R(0)|^2 + O(v_b^2)$. Numerical values of these matrix elements are available from lattice measurements [37, 38]:

$$H_1 = 2.7 \text{ GeV}^5, \quad H_S/H_1 = 2.275 \times 10^{-3} \text{ GeV}^{-2}. \quad (3)$$

As is evident in Eq. (2b), the decay rate for the $\chi_{i1}$ is purely color octet in nature at leading order in $\alpha_s$ because the transition of the $J^{PC} = 1^{++}$ color-singlet state into two massless gluons is forbidden. Since $v_b^2 \sim 0.1$, the color-octet matrix element $H_S$ is small for bottomonium (for charmonium $v_c^2 \sim 0.3$ is not negligible). The color-octet contributions in Eq. (2) are estimated to be at the 5% level in the $\chi_{i0}$ case and at the 20% level in the $\chi_{i2}$ case. The numerical values of the predicted SM hadronic widths in Eq. (2) are approximately 0.8 MeV, 0.04 MeV, and 0.2 MeV for the $\chi_{i0}$, $\chi_{i1}$, and $\chi_{i2}$, respectively.

A. Derivation of the Short-Distance Term

A practical procedure to extract the short-distance coefficient $C_n$ in Eq. (1) begins from a calculation of the free $b\bar{b}$ amplitude with specified spin, color, and orbital angular momentum. We use $P$ and $q$ to denote the total and the relative momenta of the pair. The heavy quark
momentum $p_b$ and its counterpart $p_{\bar{b}}$ are
\[ p_b = \frac{P}{2} + q, \quad p_{\bar{b}} = \frac{P}{2} - q. \] (4)
In the rest frame of the $b\bar{b}$ pair, the components of the vectors become $P = (2E_q, 0)$, $q = (0, q)$, $p_b = (E_q, q)$, and $p_{\bar{b}} = (E_q, -q)$. In the non-relativistic limit, the invariant mass of the pair can be expanded as $2E_q = 2\sqrt{m_b^2 + q^2} = 2m_b + O(q^2)$.

The amplitude for transition of the free $b\bar{b}$ system into a final state is denoted $M_{b\bar{b}}$. It includes a spinor factor $u^i_{\alpha}(p_b, s)\bar{\sigma}_i^j(p_{\bar{b}}, \bar{s})$ where $(s, \alpha, i)$ and $(\bar{s}, \bar{\alpha}, j)$ are the (spin, spinor index, color index) of the $b$ and $\bar{b}$, respectively. The spin-triplet state of the pair can be projected from the outer product of the heavy quark spinors upon multiplying with Clebsch-Gordan coefficients [27, 28, 39, 40].

\[
\mathcal{M}_{b\bar{b}} = \mathcal{M}^{i\bar{j}}_{j\bar{s}}(p_b, p_{\bar{b}}) \sum_{s, \bar{s}} \langle 1\lambda|s, \bar{s}\rangle u^i_{\alpha}(p_b, s)\bar{\sigma}_i^j(p_{\bar{b}}, \bar{s}),
\]
(5a)
\[
\sum_{s, \bar{s}} \langle 1\lambda|s, \bar{s}\rangle u^i_{\alpha}(p_b, s)\bar{\sigma}_i^j(p_{\bar{b}}, \bar{s}) = \Lambda^{i\bar{j}}_{\alpha\bar{\beta}}(0|\chi^\dagger\sigma^k\psi|\bar{b}\bar{b}) + \Lambda^{i\bar{j}}_{\alpha\bar{\beta}}(0|\chi^\dagger T^a\sigma^k\psi|\bar{b}\bar{b}),
\]
(5b)
\[
\Lambda^{i\bar{j}}_{\alpha\bar{\beta}} = \frac{\delta^{ij}}{N_c} \sum_{s, \bar{s}} \left[ \begin{array}{c} x_b \gamma_\mu \nonumber \end{array} \right]_{\alpha\bar{\beta}} L^\mu_k,
\]
(5c)
\[
\Lambda^{i\bar{j}}_{\alpha\bar{\beta}} = 2T^a_{ij} \sum_{s, \bar{s}} \left[ \begin{array}{c} x_b \gamma_\mu \nonumber \end{array} \right]_{\alpha\bar{\beta}} L^\mu_k,
\]
(5d)
where $L^\mu_k$ is the boost matrix from the $b\bar{b}$ rest frame to the frame in which the $b\bar{b}$ pair has momentum $P$. The operators $\psi$ and $\chi^\dagger$ are the annihilation operators for the non-relativistic $b$ and $\bar{b}$ states, respectively. The projection operator Eq. (5) is valid up to terms of $O(q^2)$. The projection operator valid to all orders in $q^n$ is derived in Ref. [40]. The heavy quark state in Eq. (5b) is specified with the non-relativistic normalization $\langle b(p)|\bar{b}(q)\rangle = (2\pi)^3\delta^{(3)}(p - q)$, whereas the Dirac spinors have relativistic normalization $\bar{u}_u = \bar{v}_v = 2m_b$.

Substituting the projection operator Eq. (5b) into Eq. (5a) and expanding with respect to $q$ up to $O(q^3)$, we can express the amplitude $M_{b\bar{b}}$ as a linear combination of a color-singlet $P-$wave term and a color-octet $S-$wave term.

\[
\mathcal{M}_{b\bar{b}} = \mathcal{A}_{ij}(0|\chi^\dagger q^i\sigma^j\psi|\bar{b}\bar{b}) + \mathcal{B}_{ij}(0|T^a\sigma^i\chi^\dagger\psi|\bar{b}\bar{b}).
\]
(6)
The $P$-wave amplitude can be decomposed further into the total angular momentum components, $J = 0, 1, 2,

\[
\mathcal{M}_{b\bar{b}} = \mathcal{A}_0(0|\chi^\dagger\mathcal{K}^{(1)}(p_{0})\psi|\bar{b}\bar{b}) + \mathcal{A}_1(0|\chi^\dagger\mathcal{K}^{(1)}(p_{1})\psi|\bar{b}\bar{b}) + \mathcal{A}_2(0|\chi^\dagger\mathcal{K}^{(1)}(p_{2})\psi|\bar{b}\bar{b}) + \mathcal{B}_0(0|\chi^\dagger\mathcal{K}^{(1)}(S)\psi|\bar{b}\bar{b}).
\]
(7)
The coefficients in Eq. (7) are $A_0 = \mathcal{A}^{ij}/\sqrt{3}$, $A_1 = \epsilon^{ijk} A^{jk}/\sqrt{2}$, and $A_2 = (A^{ij} + A^{ji})/2 - \delta^{ij} A^{kk}/3$. The corresponding operators are $\mathcal{K}(^3P_0) = \mathbf{q} \cdot \mathbf{\sigma}/\sqrt{3}$, $\mathcal{K}(^3P_1) = (\mathbf{q} \times \mathbf{\sigma})^i/\sqrt{2}$, and $\mathcal{K}^{ij}(^3P_2) = (\mathbf{q}^i \mathbf{\sigma}^j + \mathbf{q}^j \mathbf{\sigma}^i)/2 - \delta^{ij} \mathbf{\sigma} \cdot \mathbf{q}/3$.

The squared and spin-averaged amplitude for the $^3P_J$ state is

$$|\mathcal{M}_{bb}|^2 = \sum_{J=0}^{2} |A_J|^2 \langle b\bar{b}| \mathcal{O}(^3P_J^{(1)}) | b\bar{b} \rangle + |B|^2 \langle b\bar{b}| \mathcal{O}(^3S_1^{(8)}) | b\bar{b} \rangle. \quad (8)$$

The coefficients $|A_J|^2$ and $|B|^2$ are obtained by contracting to rotationally symmetric tensors:

$$|A_1|^2 = \frac{\delta^{ij}}{3} A_1^i A_1^j, \quad (9a)$$

$$|A_2|^2 = \frac{1}{3} \left[ \frac{1}{2} \left( \delta^{ik} \delta^{jl} + \delta^{ij} \delta^{lk} \right) - \frac{1}{3} \delta^{ij} \delta^{kl} \right] A_2^i A_2^k, \quad (9b)$$

$$|B|^2 = \frac{\delta^{ij}}{3} B^i B^j. \quad (9c)$$

The coefficient $B^{ai}$ of the color-octet term in the amplitude Eq. (7) has a free color index. However, owing to color conservation, the squared term has the form $B^{ai} B^{aj} = \delta^{ab} B^i B^j$, and the color index of the color-octet matrix element is contracted with the factor $\delta^{ab}$.

The last step to obtain the short distance coefficients is to introduce the normalization factor $(2M_{\chi_b})/(2E_q)^2 = 1/m_Q + O(q^2)$. This factor appears since we use non-relativistic normalization in the matrix elements for the $b\bar{b}$ and $\chi_{bJ}$ states. The short-distance coefficients are therefore expressed as

$$C(\chi^{P_J}) = \frac{\Phi}{4m_b^2} |A_J|^2, \quad C(\chi^{S_1^{(8)}}) = \frac{\Phi}{4m_b^2} |B|^2. \quad (10)$$

The non-relativistic zero-binding energy approximation $m_{\chi_{bJ}} = 2m_b$ is used. For a final state composed of two particles with identical mass $m_J$, the phase space factor $\Phi = \frac{1}{8\pi} (1 - m_J^2/m_b^2)^{1/2}$.

### III. DECAY INTO BOTTOM SQUARKS

In this section we treat the inclusive decay of a $\chi_b$ into a pair of bottom squarks $\bar{b}$ of mass $m_{\tilde{b}}$ carrying four-momenta $k_1$ and $k_2$ respectively. The relevant short-distance perturbative subprocess is

$$b + \bar{b} \rightarrow b + \bar{b}. \quad (11)$$
The Feynman diagrams for this subprocess are shown in Fig. 1. In Fig. 1(a), a $t$-channel gluino $\tilde{g}$ of mass $m_{\tilde{g}}$ is exchanged. The subprocess sketched in Fig. 1(b) in which the $\tilde{b}\tilde{b}$ pair annihilates through an intermediate gluon into a $\tilde{b}\tilde{b}^*$ pair is absent in the color-singlet approximation because the initial state is a color singlet. Both Fig. 1(a) and Fig. 1(b) contribute to the color-octet amplitude. Figure 1(b) is independent of the gluino mass, and its contribution survives therefore even in the limit of gluinos with very large mass. Figure 1(a) contributes to both the $3S^{(1)}_1$ and $3P^{(1)}_j$ channels whereas the analogous SM process in which gluons are produced and a $b$ quark is exchanged contributes only to $3P^{(1)}_{0,2}$.

A. Bottom Squark Mixing and Couplings

The mass eigenstates of the bottom squarks, $\tilde{b}_1$ and $\tilde{b}_2$, are mixtures of left-handed (L) and right-handed (R) bottom squarks, $\tilde{b}_L$ and $\tilde{b}_R$. The mixing is expressed as

$$|	ilde{b}_1\rangle = \sin \theta_1 |\tilde{b}_L\rangle + \cos \theta_1 |\tilde{b}_R\rangle,$$

$$|	ilde{b}_2\rangle = \cos \theta_1 |\tilde{b}_L\rangle - \sin \theta_1 |\tilde{b}_R\rangle.$$  

In our notation, the lighter mass eigenstate is denoted $\tilde{b}_1$. For the case under consideration, the mixing of the bottom squark $\tilde{b}$ is determined by the condition that the coupling of the lighter $\tilde{b}$ to the $Z$ boson be small [2], namely $\sin^2 \theta_1 \simeq 1/6$.

We may also express the mass eigenstate $\tilde{b}_1$ in terms of states of definite parity, the $J^P = 0^+$ scalar and $J^P = 0^-$ pseudo-scalar. Starting from the relationships

$$|0^+\rangle = \frac{1}{\sqrt{2}}(|\tilde{b}_R\rangle + |\tilde{b}_L\rangle),$$

$$|0^-\rangle = \frac{1}{\sqrt{2}}(|\tilde{b}_R\rangle - |\tilde{b}_L\rangle).$$
we obtain

\[ \hat{b}_1 = \frac{1}{\sqrt{2}} (\cos \theta_b + \sin \theta_b) \hat{0}^+ + \frac{1}{\sqrt{2}} (\cos \theta_b - \sin \theta_b) \hat{0}^- . \]  

(16)

The \( \hat{b}_1 \) is a pure \( J^P = 0^+ \) scalar only if \( \sin \theta_b = \cos \theta_b = \frac{1}{\sqrt{2}} \).

The coupling at the three-point vertex in which a \( b \) quark enters and a \( \hat{b}_1 \) squark emerges (the upper vertex in Fig.1(a)) is

\[ i g_s \sqrt{2} T^a_{\beta \alpha} [\cos \theta_b P_L - \sin \theta_b P_R] , \]  

(17)

where \( i \) and \( k \) are the color indices of the incident \( b \) and final \( \hat{b} \), respectively, and \( a \) labels the color of the exchanged gluino. Here, \( P_L = (1 - \gamma_5)/2 \) and \( P_R = (1 + \gamma_5)/2 \). At the lower vertex where an antiquark enters and \( \hat{b}_1 \) emerges, the coupling is

\[ i g_s \sqrt{2} T^a_{j \ell} [\cos \theta_b P_L - \sin \theta_b P_R] , \]  

(18)

where \( j \) and \( \ell \) are the color indices of the incident \( \bar{b} \) and final \( \bar{b}^* \), respectively.

### B. Subprocess Amplitude

The amplitude of the process \( b \bar{b} \rightarrow \hat{b} \hat{b} \), where \( \lambda(\lambda) = -1 \) for \( \hat{b}_L(\hat{b}_R) \) and \( \lambda = 1 \) for \( \hat{b}_R(\hat{b}_L) \), respectively, is

\[ \mathcal{M}_{\hat{b}_a}(\lambda, \lambda) = \left[ \frac{i g_s \lambda}{\sqrt{2}} T^a_{jj} (1 - \lambda \gamma_5) \times \frac{i}{P - m_{\tilde{g}} - m_{\tilde{g}} - m_{\tilde{b}} - m_{\tilde{b}}^2} \right]_{\beta \alpha} \]

\[ + \left( -i g_s T^a_{j \ell} \gamma_5 \right)_{\beta \alpha} \times \left[ -i g_s T^a_{k \ell} (k_1 - k_2) \right]_{\ell \ell} . \]  

(19)

Momentum labels are specified in Fig.1; \( P \) is the four-momentum of the \( \tilde{b}_L \). As remarked earlier, \( \alpha \) and \( \beta \) are the spinor indices of the \( b \) and \( \bar{b} \). The first term in Eq. (19) represents the diagram shown in Fig. 1(a), and the other term is for the diagram of Fig. 1(b). We do not simplify the factors in Eq. (19) so that the Feynman rules can be identified in this expression.

The relatively large masses of the exchanged gluino Fig.1(a) and of the \( \chi_3 \) should justify the use of simple perturbation theory to compute the decay amplitude; \( g_s \) is the strong coupling, \( \alpha_s = g_s^2/4\pi \).

Using the mixing relation Eq. (13), we can display the explicit mixing angle dependence of the amplitude for the light bottom squark pair final state in the form

\[ \mathcal{M}_{\hat{b}_a} = \sin^2 \theta \mathcal{M}_{\hat{b}_a}(+, +) + \cos^2 \theta \mathcal{M}_{\hat{b}_a}(-, -) + \sin \theta \cos \theta (\mathcal{M}_{\hat{b}_a}(+, -) + \mathcal{M}_{\hat{b}_a}(-, +)) . \]
Substituting $\mathcal{M}_{j\alpha}^{\tilde{r}}$ into Eq. (5a) and using the projection operator Eq. (5b), we derive an expression in the form of Eq. (6). Useful color factor formulas are

$$[T_{kl}^a T_{ji}^a] \delta_{ij} = \frac{N_c^2 - 1}{2N_c} \delta_{kl}, \quad (20a)$$

$$[T_{kl}^a T_{ji}^a] T_{ij}^b = -\frac{1}{2N_c} T_{kl}^b. \quad (20b)$$

The partial-wave amplitudes in Eq. (7) are found to be

$$A_0 = \frac{32\pi \alpha_s}{9\sqrt{3}} \delta_{k\ell} \frac{6 \sin \theta_b \cos \theta_b m_\ell (m_\ell^2 + m_\ell^2 - m_\ell^2) - m_i (m_i^2 + 3m_i^2 - m_i^2)}{(m_\ell^2 + m_\ell^2 - m_\ell^2)^2}, \quad (21a)$$

$$A_1^i = \frac{64\pi \alpha_s}{9\sqrt{2}} \delta_{k\ell} k_{i\mu} L_{i\mu} \left(\cos^2 \theta_b - \sin^2 \theta_b \right), \quad (21b)$$

$$A_2^{ij} = \frac{64\pi \alpha_s}{9} \delta_{k\ell} \frac{m_i k_{i\mu} k_{j\nu} L_{i\mu} L_{j\nu}}{(m_b^2 + m_b^2 - m_b^2)^2}, \quad (21c)$$

$$B_{ai} = \frac{4\pi \alpha_s}{3} \frac{m_i^2}{m_b} \left(\frac{2m_b^2}{m_b^2 + m_b^2 - m_b^2} \right), \quad (21d)$$

Using the polarization summation relations in Eq. (9), we obtain the short-distance factors of Eq. (10). Substituting these short-distance coefficients into the factorization formula Eq. (1), we derive

$$\Gamma(\eta_b \to \bar{b}_1 \bar{b}_1') = 0, \quad (22a)$$

$$\Gamma(\gamma \to \bar{b}_1 \bar{b}_1') = \frac{32\pi \alpha_s^2}{81} \frac{\mathcal{G}_1}{1 - \frac{m_b^2}{m_b^2}} \left(1 - \frac{m_b^2}{m_b^2}\right)^{3/2} \frac{m_b^2}{(m_b^2 + m_b^2 - m_b^2)^2}, \quad (22b)$$

$$\Gamma(\chi_{b,b} \to \bar{b}_1 \bar{b}_1') = \frac{4\pi \alpha_s^2}{3m_b^4} \left(1 - \frac{m_b^2}{m_b^2}\right)^{1/2} \left(D_J + D_8 \frac{m_b^2}{m_b^2} \right), \quad (22c)$$

where $\mathcal{G}_1 = \langle \gamma | \mathcal{O}^{(3) \bar{S}_1^{(1)}} | \gamma \rangle = \frac{N_c}{2\pi} |\tilde{R}(0)|^2$, and $\tilde{R}(0)$ is the $\gamma$ wave function at the origin.

The coefficients are

$$D_0 = \frac{8}{27} m_b^4 \left[6m_\ell (m_\ell^2 + m_\ell^2 - m_\ell^2) \sin \theta_b \cos \theta_b - m_i (m_i^2 + 3m_i^2 - m_i^2) \right]^2, \quad (23a)$$

$$D_1 = \frac{16}{27} \left(1 - \frac{m_b^2}{m_b^2}\right) \frac{m_b^4 \left[\cos^2 \theta_b - \sin^2 \theta_b \right]^2}{(m_b^2 + m_b^2 - m_b^2)^4}, \quad (23b)$$

$$D_2 = \frac{64}{135} \left(1 - \frac{m_b^2}{m_b^2}\right)^2 \frac{m_b^8}{(m_b^2 + m_b^2 - m_b^2)^4}, \quad (23c)$$

$$D_8 = \frac{1}{144} \left(1 - \frac{m_b^2}{m_b^2}\right) \left(3 + \frac{2m_b^2}{m_b^2 + m_b^2 - m_b^2}\right)^2. \quad (23d)$$
Equation (22b) was derived previously in Ref. [7]. Equation (22a) indicates that $\eta_k$ decay is not allowed in lowest-order. In the color-singlet case, reduction of the relevant trace provides a lowest-order amplitude proportional to $(\cos^2 \theta_k - \sin^2 \theta_k) p_n \cdot (p_n - 2k_1)$, where $p_n$ and $k_1$ are the four-vector momenta of the $\eta_k$ and one of the final $\tilde{b}$'s. Evaluation in the $\eta_k$ rest frame shows that this amplitude is zero for a two-body final state.

A symmetric $S$-wave two-body $\tilde{q}\tilde{q}^*$ system can be constructed with one of the pair in a $J^P = 0^-$ state and the other a $J^P = 0^+$ state. Under charge-conjugation $C$, a squark $\tilde{q}$ is transformed into an anti-squark $\tilde{q}^*$ with no overall phase. The two-body $\tilde{q}\tilde{q}^*$ system will have $J^{PC} = O^{--}$, the same quantum numbers as the $\eta_k$. Therefore, there appears to be no overall symmetry that would forbid the exclusive lowest-order process $\eta_k \to \tilde{b}_l\tilde{b}_l^*$, and Eq. (22a) is not true more generally than in the lowest-order model we use.

As remarked above, the pure color-octet Fig. 1(b) term is independent of the gluino mass, and its contribution survives therefore even in the limit of gluinos with very large mass. Equation (21d) is written in a form that makes this fact transparent; the solitonic “3” represents the contribution in the limit that only Fig. 1(b) contributes. If $m_1^2 \ll m_3^2$ and $m_1^2 \ll m_3^2$, the color-singlet coefficients $D_J$, Eqs. (23a - 23c), vanish in proportion to $m_J^{-n}$, $n = 2^{J+1}$, but the color-octet coefficient $D_8$ remains finite. As $m_3 \to \infty$,

$$\Gamma (\chi_{ij} \to \tilde{b}_l\tilde{b}_l^*) \rightarrow \frac{4\pi \alpha_s^2 \mathcal{H}_8}{3m_3^2} \frac{1}{16} \left(1 - \frac{m_1^2}{m_3^2}\right)^{3/2} = \Gamma_{\text{SM}} (\chi_{ij}) \frac{1}{4m_3} \left(1 - \frac{m_1^2}{m_3^2}\right)^{3/2}.$$  (24)

The momentum of a final state bottom squark in the rest frame of the $\chi_b$ is $|k| = \frac{1}{2} \sqrt{m_{\chi_b}^2 - m_3^2} \approx m_3 \sqrt{1 - \frac{m_{\chi_b}^2}{m_3^2}}$. We observe that $\Gamma \propto |k|^{\ell+1}$, with $\ell = 1$, in the case of $\Upsilon$ decay, Eq. (22b), as expected because the decay of the $\Upsilon$ produces a pair of scalars in a $P$-wave. In the $\chi_J$ case, combining the overall phase-space factor $\propto |k|$ in Eq. (22c), with those $\propto |k|^{2J}$ in the expressions for $D_J$ in Eqs. (23a - 23c), we note that the color-singlet terms have the threshold behaviors expected for decays into $S$, $P$, and $D$ wave systems of two spin-0 particles. The overall color-octet coefficient Eqs. (22c) and (23d) is proportional to $|k|^3$.

The dependences on the mixing angle $\theta_\ell$ in Eqs. (23a - 23d) can be understood if we recast the results in terms of production of left-handed and right-handed bottom squarks, $\tilde{b}_L$ and $\tilde{b}_R$. In the $\chi_{i0}$ case, the left-left and right-right combinations are produced with equal rates, leading to a term in Eq. (23a) proportional to $(\cos^2 \theta_\ell + \sin^2 \theta_\ell) = 1$. In addition, the left-right and right-left combinations are produced with equal rates, leading to the term
proportional to $\sin \theta_b \cos \theta_b$. For $\chi_{b1}$, the left-left and right-right combinations are produced with equal but opposite rates, leading to a dependence of $(\cos^2 \theta_b - \sin^2 \theta_b)$. The left-right and right-left combinations do not contribute. In the $\chi_{b2}$ case, the the left-left and right-right combinations are produced with equal rates, resulting in a dependence of $(\cos^2 \theta_b + \sin^2 \theta_b)$, and the left-right and right-left combinations do not contribute.

We can obtain a rough estimate of the relative size of the color-octet and color-singlet contributions in Eq. (22c) by beginning with the simplifications $(m^2_1 - m^2_b) \ll m^2_\tilde{g}$ and $m^2_\tilde{g} \ll m^2_\tilde{g}$, both reasonably fair approximations for the masses of interest. Doing so, and defining $x = m^2_1/m^2_\tilde{g}$, we find $D_8 : D_0 : D_1 : D_2 \simeq 1 : 512x \cos^2 \theta_b \sin^2 \theta_b/3 : 256x^2(\cos^4 \theta_b - \sin^2 \theta_b)^2/27 : 1024x^4/135$. Adopting the lattice estimate $m^2_\tilde{g}/\mathcal{H}_8/\mathcal{H}_1 \simeq 0.05$ (c.f., Eq. (3)) along with the typical value $x \simeq 1/10$ for the range of sparticle masses under consideration, we conclude that the color-octet contributions to bottom quark pair production should be roughly a 2% effect for the $\chi_{b0}$, approximately 50% of the answer for $\chi_{b1}$, and dominant in the $\chi_{b2}$ case. Nevertheless, as shown in the next section, bottom squark decays are expected to make a small overall contribution to the hadronic widths of the $\chi_{b1}$ and $\chi_{b2}$.

IV. PREDICTIONS AND DISCUSSION

The total hadronic decay rates of the $\chi_{bJ}$ states are obtained after adding the contributions from conventional two-gluon decay and those for bottom squark decay, Eqs. (2) and (22c). These final decay rates are shown in the first rows of Figs. 2 and 3 and are compared with the standard model expectations. The left-hand sides of the figures show results for the choice $m_{\tilde{g}} = 12$ GeV, and the right-hand sides for $m_{\tilde{g}} = 16$ GeV. Results are displayed as a function of the bottom squark mass in the range $2$ GeV $< m_{\tilde{b}} < m_{\tilde{g}}$. The choices of $m_{\tilde{g}}$ and $m_{\tilde{b}}$ are guided by the results in Ref. [1]. We employ the lattice estimate for the ratio of color-singlet and color-octet components, $\mathcal{H}_8/\mathcal{H}_1$, listed in Eq. (3). For the mixing angle, strong coupling strength, and $b$ quark mass, we use $\sin^2 \theta = 1/6$, $\alpha_s = 0.2$, and $m_b = 4.75$ GeV. The sign of the product $\sin \theta \cos \theta$ is not constrained by experiment. We consider both possibilities; results for $\sin \theta \cos \theta > 0$ are shown in Fig. 2, and those for $\sin \theta \cos \theta < 0$ in Fig. 3. The substantial differences in the $\chi_{b0}$ case between Fig. 2 and Fig. 3 arise from the cancellation in the numerator of Eq. (23a) when $\sin \theta \cos \theta < 0$. Owing to the phase space and angular momentum threshold suppression factors $|k|^2J^2J+1$, the SUSY contributions vanish as
\( m_\tilde{b} \to m_\tilde{b} \left( |k| \to 0 \right) \) for all \( \chi_{kJ} \) (as well as for the \( \Gamma \)).

The ratios of the decay rates through the bottom squark final state to the standard model prediction are shown in the second row of Figs. 2 and 3. The dependence on the overall factor \( \frac{4\pi m_\tilde{b}^2 \mathcal{H}_1}{3m_\tilde{b}^4} \) cancels in this ratio, removing all dependence on the strong coupling strength \( \alpha_s(m_\tilde{b}) \) (at the order in perturbation theory at which we work [35]) and much of the dependence on the \( b \) quark mass \( m_b \) and on the matrix element \( \mathcal{H}_1 \). The contributions of bottom squarks decays to the hadronic widths are small in comparison to the standard model values except for the \( \chi_{10} \). As seen in Fig. 2, the SUSY contribution can be as large as 33\% of the standard model value for small \( m_\tilde{b}, m_\tilde{\beta} = 12 \text{ GeV} \), and positive \( \sin \theta \cos \theta \). The fraction decreases as \( m_\tilde{b} \) and \( m_\tilde{\beta} \) increase, and is reduced if \( \sin \theta \cos \theta < 0 \). By contrast, for small \( m_\tilde{b} \), the SUSY contributions are at most 15\% and 1\% of the standard model values in the \( \chi_{11} \) and \( \chi_{12} \) cases. Inspection of Eqs. (2c) and (22c) provides an easy explanation for the small role of the SUSY contribution in the \( \chi_{12} \) case. We observe that \( \Gamma_{\text{SM}} \sim (4/15)\Gamma_8 \), whereas \( \Gamma_{\text{SUSY}} \sim D_8 m_\tilde{b}^3 \Gamma_8 \mathcal{H}_8 / \mathcal{H}_1 \sim (1/20) D_8 \Gamma_8 \). Since \( D_8 \sim 1/16 \), we obtain \( \Gamma_{\text{SUSY}} / \Gamma_{\text{SM}} \sim 1\% \).

We comment on results obtained if the color-octet configuration is absent or, equivalently, the matrix element \( \mathcal{H}_8 \) is much smaller than estimated in Eq. (3). The conclusions regarding \( \chi_{10} \) are essentially unchanged since the color-singlet contribution is dominant for both the SM and bottom squark decays. For \( \chi_{11} \), the SM color-singlet term is absent at leading order; the SUSY decay mode would therefore dominate at this order. In the \( \chi_{12} \) case, the color-singlet rate for decay into bottom squarks is negligible compared to the SM color-singlet rate for \( gg \) decay. Overall, in the absence of color-octet contributions the predictions for the ratio \( \Gamma_{\text{SUSY}} / \Gamma_{\text{SM}} \) would be qualitatively similar in the \( \chi_{10} \) and \( \chi_{12} \) cases to those in Figs. 2 and 3, but very different for the \( \chi_{11} \).

A ratio \( R \) [25] may be formed in which the color-octet contributions cancel:

\[
R = \frac{\Gamma(\chi_{10}) - \Gamma(\chi_{11})}{\Gamma(\chi_{12}) - \Gamma(\chi_{11})},
\]

(25)

This ratio depends only on the short-distance coefficients. Results are shown in the third row of Figs. 2 and 3. The ratio is enhanced significantly by the SUSY contribution for small \( m_\tilde{\beta}, m_\tilde{\beta} = 12 \text{ GeV} \), and positive \( \sin \theta \cos \theta \).

Direct observation of \( \chi_{\tilde{b}} \) decay into bottom squarks requires an understanding of the ways that bottom squarks may manifest themselves [1, 10]. If the \( \tilde{b} \) lives long enough, it
FIG. 2: Hadronic decay rates of the $\chi_b$ states as functions of the bottom squark mass are shown in the first row for $\sin \theta_b \cos \theta_b > 0$. The left-hand column shows results for $m_{\tilde{b}} = 12$ GeV and the right-hand column for $m_{\tilde{b}} = 16$ GeV. Ratios of the bottom squark decay rates to the standard model predictions are shown in the second row. The third row shows the ratio $R$ defined in Eq. (25).
Fig. 3: As in Fig. 2 but with $\sin \theta_3 \cos \theta_2 < 0$.

will pick up a light quark and turn into a $J = 1/2$ $B$-mesino, $\bar{B}$, the superpartner of the $B$ meson. The mass of the mesino could fall roughly in the range 3 to 6 GeV for the interval of $\bar{b}$ masses favored by the analysis of the bottom quark cross section. Charged $B$-mesino
signatures in $\chi_{tJ}$ decay include single back-to-back equal momentum tracks in the center-of-mass; measurably lower momentum than lepton pairs; an angular distribution consistent with the decay of a state of spin $J$ into two fermions; and ionization, time-of-flight, and Cherenkov signatures typical of a particle whose mass is heavier than that of a proton. On the other hand, possible baryon-number-violating R-parity-violating decays of the bottom squark lead to $u + s$, $c + d$, and $c + s$ final states. These final states of four light quarks may be indistinguishable from conventional hadronic final states mediated by the two-gluon intermediate state. Since the bottom squark decay mode is predicted to be substantial for $\chi_{t0}$'s, decays of the $\chi_{t0}$ primarily via the $\bar{B}$ possibility would offer an excellent opportunity to observe bottom squarks or to place significant limits on their possible masses.

Acknowledgments


[10] For more discussion of phenomenological constraints and bottom squark decays, see


[35] Since the next-to-leading order results in Refs. [33] and [34] do not agree, we quote only leading order results in Eq. (2).


[40] G. T. Bodwin and A. Petrelli (to be published); G. T. Bodwin and J. Lee (to be published).