We point out that inflaton decays can be a copious source of stable or long-lived particles $\chi$ with mass exceeding the reheat temperature $T_R$. Once higher order processes are included, this statement is true for any $\chi$ particle with renormalizable (gauge or Yukawa) interactions. This contribution to the $\chi$ density often exceeds the contribution from thermal $\chi$ production, leading to significantly stronger constraints on model parameters than those resulting from thermal $\chi$ production alone. For example, we all but exclude models containing stable charged particles with mass less than half the mass of the inflaton.

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According to inflationary models [1], which were first considered to address the flatness, isotropy, and (depending on the particle physics model of the early Universe) monopole problems of the hot Big Bang model, the Universe has evolved through several stages. During inflation, the energy density of the Universe is dominated by the potential energy of the inflaton and the Universe experiences a period of superluminal expansion. Immediately after inflation, coherent oscillations of the inflaton, which behave like non-relativistic matter, dominate the energy density of the Universe. At some later time these coherent oscillations decay, and their energy density is transferred to relativistic particles; this reheating stage results in a radiation-dominated Friedmann–Robertson–Walker (FRW) Universe, as in the hot Big Bang model.

Initially reheating was treated as the perturbative, one particle decay of the inflaton with decay rate $\Gamma_d$ depending on the microphysics. This gives $T_R \sim (\Gamma_d M_P)^{1/2}$ for the reheat temperature [2], where $M_P = 2.4 \times 10^{18}$ GeV is the reduced Planck mass. $T_R$ should be low enough so that the GUT symmetry is not restored and the monopole problem is avoided. In many supersymmetric models there is an even stricter bound, $T_R < 10^7 - 10^9$ GeV, in order to avoid gravitino overproduction which would destroy the successful predictions of nucleosynthesis by its late decay [3].

Later it has been noticed that the initial stage of inflaton decay might occur through a non-perturbative process called parametric resonance, leading to a distribution of final state particles with energies considerably higher than the inflaton mass [4]. However, this does not result in a complete decay of the energy density stored in the inflaton oscillations. It rather brings degrees of freedom which are coupled to the inflaton to local equilibrium with it and among themselves [5]. After some time the energy density of the Universe will then again be dominated by non-relativistic particles. It is therefore believed that in a wide range of realistic inflationary models the final stage of inflaton decay occurs in the perturbative regime and that the reheat temperature is determined therein. Here we will make the (usual) additional assumption that inflaton decay can be described by perturbation theory in a trivial vacuum. This requires the density $n$ of particles produced from inflaton decay to be less than the particle density in a thermal plasma with the same energy density $\rho$, $n^{1/3} < \rho^{1/4}$.

Even before all inflatons decay, their decay products form a plasma which, upon a very quick thermalization, has the instantaneous temperature [6] $T \sim (g^* H T d M^2_P)^{1/4}$, where $H$ is the Hubble parameter and $g^*$ denotes the number of relativistic degrees of freedom in the plasma. This temperature reaches its maximum $T_{\text{max}}$ soon after the inflaton field $\phi$ starts to oscillate, which happens for a Hubble parameter $H_t \leq m_\phi$, with $m_\phi$ being the frequency of inflaton oscillations about the global minimum of the potential. Our assumption that all inflaton decays can be described perturbatively implies $T_{\text{max}} < m_\phi/2$. However, $T_{\text{max}}$ can be much larger than $T_R$. As long as $T > T_R$ the energy density of the Universe is still dominated by the (non-relativistic) inflatons that haven’t decayed yet. The Universe remains in this phase as long as $H > H_d$. During that epoch particles $\chi$ with mass $T_{\text{max}} > m_\chi > T_R$ can be produced copiously from the thermal plasma [7–9]. Here we point out that $\chi$ particles can also be produced directly in inflaton decays. We will show that the $\chi$ abundance from inflaton decay often exceeds that from thermal production, even if the branching ratio for $\phi \rightarrow \chi$ decays is very small.

We begin our argument by pointing out that $T_{\text{max}}$ is frequently well below $m_\phi$. This is important, since thermal production is obviously only efficient if $m_\chi \lesssim T_{\text{max}}$, while inflaton decay can produce pairs of $\chi$ particles as long as $m_\chi < m_\phi/2$. For perturbative inflaton decay thermalization increases the number density and reduces the mean energy of the decay products. Complete thermalization (i.e. both chemical and kinetic) therefore requires $2 \rightarrow N$ reactions, which change the number of particles, to be in equilibrium. Since the rate for higher
order processes is suppressed by powers of the relevant coupling constant $\alpha$, the most important reactions are those with $N = 3$. These reactions have recently been studied in Ref. [10] where the scattering of two matter fermions with energy $\sim m_\phi/2$ (from inflaton decay) to two fermions, plus one gauge boson with typical energy $E \ll m_\phi$, has been considered. The rate for these reactions can be large due to the $t$–channel pole of the scattering matrix element. This pole will be regulated by a cut–off on the exchanged momentum, naturally taken to be the inverse of the average separation between two particles in the plasma [10]. It turns out that the largest possible $T_{\text{max}}$ is given by [11]

$$
T_{\text{max}} \sim T_R \left( \frac{g^3}{3} \frac{g^2}{m_\phi^1/3 T_R^{2/3}} \right)^{3/8}.
$$

Even if $m_\phi$ is near its COBE–derived upper bound of $\sim 10^{13}$ GeV [12], for a chaotic inflation model, and $T_R$ is around $10^9$ GeV (which saturates the gravitino bound) $T_{\text{max}}$ will exceed $T_R$ if the coupling $\alpha^3 \gtrsim 10^{-8}$. This is easily accommodated for particles with gauge interactions. Recall that we assume perturbative inflaton decays, which requires $T_{\text{max}} < m_\phi/2$. Together with eq.(1), taking $\alpha \lesssim 0.1$, this gives $T_{\text{max}} \lesssim 10^{11} (10^9)$ GeV for $T_R = 10^9 (1)$ GeV. This implies in particular that there will be no “wimpzilla” production [7] from thermalized inflaton decay products, since in this case $m_\chi > T_{\text{max}}$.

On the other hand, for $m_\chi \lesssim 20 T_R$ the standard calculation [6] of the density of stable relics applies. Scenarios with $T_{\text{max}} \gtrsim m_\chi \gtrsim 20 T_R$ have only been investigated relatively recently in refs. [7–9], which studied $\chi$ production from the thermal plasma with $T > T_R$. If the $\chi$ density was always well below the equilibrium density, one finds

$$
\Omega_\chi^\text{therm} h^2 \sim \left( \frac{200}{g_*} \right)^{3/2} \frac{\alpha^2}{m_\chi^2} \left( \frac{2000 T_R}{m_\chi} \right)^7.
$$

Here $\Omega_\chi$ is the $\chi$ mass density in units of the critical density and $h$ is the Hubble constant in units of $100$ km/(s–Mpc). We have taken the cross section for $\chi$ pair production or annihilation to be $\sigma \sim \alpha^2/m_\chi^2$. Note that $\Omega_\chi$ is only suppressed by $(T_R/m_\chi)^7$ rather than by exp($-m_\chi/T_R$). A stable particle with mass $m_\chi \sim 2000$ $T_R \cdot \alpha^2/7$ might thus act as the Dark Matter in the Universe (i.e. $\Omega_\chi \simeq 0.3$). However, eq.(1) with $\alpha = 0.05$ implies that $T_{\text{max}} \gtrsim 100 T_R$ is only possible if $T_R < 2 \cdot 10^{-12} M_p$. Eq.(2) is no longer applicable [8] if the coupling $\alpha_\chi$ is so large that $\chi$ reached chemical equilibrium; however, it can then still be used as an upper bound on $\Omega_\chi^\text{therm}$.

We now discuss the direct production of $\chi$ particles in inflaton decay. Most inflatons decay at $T \sim T_R$; moreover, the density of $\chi$ particles produced in earlier inflaton decays will be greatly diluted. Since inflaton decay conserves energy, the density of inflatons can be estimated as $n_\phi \sim 0.3 g_* T_R^4/m_\phi$. Let us denote the branching ratio for $\phi \to \chi$ decays (more accurately, the average number of $\chi$ particles produced in each $\phi$ decay) by $B(\phi \to \chi)$. We translate the $\chi$ density at $T = T_R$ into the present $\chi$ relic density using the relation [6]

$$
\Omega_\chi h^2 = 6.5 \cdot 10^{-7} \cdot \frac{200}{g_*} \frac{m_\chi n_\chi(T_R)}{T_R T_{\text{now}}}.
$$

The $\chi$ density from $\phi$ decay can then be estimated as:

$$
\Omega_\chi^{\text{decay}} h^2 \simeq 2 \cdot 10^5 B(\phi \to \chi) m_\chi T_R / m_\phi 1 \text{ GeV}.
$$

Eq.(4) holds if the $\chi$ annihilation rate is smaller than the Hubble expansion rate at $T \simeq T_R$, which requires

$$
\frac{m_\phi}{M_P} > 5 B(\phi \to \chi) \alpha_\chi^2 \left( \frac{T_R}{m_\chi} \right)^2 \left( \frac{g_*}{200} \right)^{1/2}.
$$

This condition will be satisfied in chaotic inflation models with $m_\phi \sim 10^{-5} M_P$, if $m_\chi$ is large enough to avoid overclosure from thermal $\chi$ production alone. It might be violated in models with light inflatons. In that case the true $\chi$ density at $T_R$ can be estimated by equating the annihilation rate with the expansion rate:

$$
\Omega_\chi^{\text{max}} h^2 \sim \frac{5 \cdot 10^7}{g_*^2} \frac{m_\chi^3}{M_P^2 T_R} \left( \frac{200}{g_*} \right)^{1/2}.
$$

This maximal density violates the overclosure constraint $\Omega_\chi h^2 < 1$ badly for the kind of weakly interacting ($\alpha_\chi \lesssim 0.1$), massive ($m_\chi \gg T_R$ and $m_\chi \gtrsim 1 \text{ TeV}$) particles we are interested in. For the remainder of this article we will therefore estimate the $\chi$ density from inflaton decay using eq.(4).

Our remaining task is to estimate $B(\phi \to \chi)$. This quantity is obviously model dependent, so we have to investigate several scenarios. The first, important special case is where $\chi$ is the lightest supersymmetric particle (LSP). If $m_\phi$ is large compared to typical visible–sector superparticle masses, $\phi$ will decay into particles and superparticles with approximately equal probability [13,11]. Moreover, all superparticles will quickly decay into one $\chi$ particle and some standard particle(s). As long as $m_\chi > T_R$, the time scale for these decays will be shorter than the superparticle annihilation time scale even if $\alpha_\chi \simeq 0.1$. As a result, if $\chi$ is the LSP, then $B(\phi \to \chi) \simeq 1$, independently of the nature of the LSP.

Another possibility is that the inflaton couples to all particles with more or less equal strength, e.g. through non–renormalizable interactions. In that case one expects $B(\phi \to \chi) \sim 1/g_* \sim 1/200$. However, even if $\phi$ has no direct couplings to $\chi$, the rate (4) can be large. The

*Eq.(6) describes the maximal $\chi$ density if $\chi$ decouples at $T \sim T_R$. It is not applicable to WIMP’s decoupling at $T < T_R$. 


key observation is that $\chi$ can be produced in $\phi$ decays that occur in higher order in perturbation theory whenever $\chi$ can be produced from annihilation of particles in the thermal plasma. In most realistic cases, $\phi \rightarrow f f \bar{\chi} \bar{\chi}$ decays$^1$ will be possible if $\chi$ has electroweak gauge interactions, where $f$ stands for some gauge non-singlet with tree-level coupling to $\phi$. A diagram contributing to this decay is shown in Fig. 1. Note that the part of the diagram describing $\chi \bar{\chi}$ production is identical to the diagram describing $\chi \bar{\chi} \leftrightarrow f f$ transitions. This leads to the following estimate:

$$B(\phi \rightarrow \chi \chi) \sim \frac{C_4 \alpha_2^2}{96 \pi^3} \left(1 - \frac{4m_f^2}{m_{\phi}^2}\right)^2 \left(1 - \frac{2m_{\chi}}{m_{\phi}}\right)^{\frac{3}{2}},$$

where $C_4$ is a multiplicity (color) factor. The phase space factors have been written in a fashion that reproduces the correct behavior for $m_{\chi} \rightarrow m_{\phi}/2$ as well as for $m_{\chi} \rightarrow 0$. Occasionally one has to go to even higher order in perturbation theory to produce $\chi$ particles from $\phi$ decays. For example, if $\chi$ has only strong interactions but $\phi$ only couples to $SU(3)$ singlets, $\phi \rightarrow f f q \bar{q} \chi \chi$. A representative diagram can be obtained from the one shown in Fig. 1 by replacing the $\chi$ lines by quark lines, attaching an additional virtual gluon to one of the quarks which finally splits into $\chi \bar{\chi}$. The branching ratio for such six body decays can be estimated as

$$B(\phi \rightarrow \chi \chi \chi) \sim \frac{C_6 \alpha_2^2 \alpha_W^2}{1.1 \cdot 10^7} \left(1 - \frac{4m_f^2}{m_{\phi}^2}\right)^4 \left(1 - \frac{2m_{\chi}}{m_{\phi}}\right)^{\frac{3}{2}}.$$  

Another example where $\chi \bar{\chi}$ pairs can only be produced in $\phi$ decays into six body final states occurs if the inflaton only couples to fields that are singlets under the SM gauge group, e.g. right–handed (s)neutrinos $\nu_R$ [14]. These (s)neutrinos can emit a virtual Higgs boson, which can split into a top quark-antiquark pair; one of which can emit a virtual gluon, which in turn splits into a (strongly interacting) $\chi \bar{\chi}$ pair. In this scenario the factor $\alpha_W^2$ in eq.(8) would have to be replaced by the combination of Yukawa couplings $\lambda_R^2 \chi^2/(16\pi^2)$. If $2m_{\chi} < m_{\nu_R}$, $\chi \bar{\chi}$ pairs can already be produced in four body final states from $\nu_R$ decay. The effective $\phi \rightarrow \chi$ branching ratio would then again be given by eq.(7), with $m_{\phi}$ replaced by $m_{\nu_R}$ in the kinematical factors.

Finally, in supergravity models there in general exists a coupling between $\phi$ and either $\chi$ itself or, for fermionic $\chi$, to its scalar superpartner, of the form $a (m_{\phi} m_{\chi}/M_{\text{Planck}}) \phi \chi \chi + h.c.$ in the scalar potential$^2$.

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$^1$ $\chi = \chi$ if $\chi$ is a Majorana particle.

$^2$ This term also induces an $A$-term from supersymmetry breaking by the inflaton energy density.

The production of $\chi$ particles from inflaton decay will be important for large $m_{\chi}$ and large ratio $m_{\chi}/T_R$, but tends to become less relevant for large ratio $m_{\phi}/m_{\chi}$. Even if $m_{\chi} < T_{\text{max}}$, $\chi$ production from the thermal plasma (2) will be subdominant if

$$B(\phi \rightarrow \chi) > \frac{\alpha^2}{100 m_T} \left(\frac{100 m_T}{m_{\chi}}\right)^6 \frac{m_{\phi} \cdot 1 \text{ TeV}}{m_{\chi}}.$$  

The first factor on the r.h.s. of (10) must be $\lesssim 10^{-6}$ in order to avoid over–production of $\chi$ from thermal sources alone. Even if $\phi \rightarrow \chi$ decays only occur in higher orders of perturbation theory, the l.h.s. of (10) will be of order $10^{-4} (10^{-10})$ for four (six) body final states, see eqs.(7), (8); if $\phi \rightarrow \chi \bar{\chi}$ decays at tree–level, the l.h.s. of (10) will usually be bigger than unity. We thus see that even for $m_{\phi} \sim 10^{13}$ GeV, as in chaotic inflation models, and for $m_{\chi} \sim 10^3 T_R$, $\chi$ production from decay will dominate if $m_{\chi} > 10^7 (10^{10})$ GeV for four (six) body final states. As a second example, consider LSP production in models with very low reheat temperature. The LSP mass should lie within a factor of five or so of 200 GeV. Recall that in this case $B(\phi \rightarrow \chi) = 1$. Taking $\alpha_{\chi} \sim 0.01$, we see that $\chi$ production from decay will dominate over production from the thermal plasma if $m_{\phi} < 6 \cdot 10^7$ GeV for $T_R = 1$ GeV; this statement will be true for all $m_{\phi} \lesssim 10^{13}$ GeV if $T_R \lesssim 100$ MeV.

Let us now assume that eq. (4) indeed gives the dominant contribution to $\chi$ production in the early Universe, and investigate the resulting constraints on model parameters. As well known, any stable particle must satisfy $\Omega_{\chi} h^2 < 1$, since otherwise it would “overclose” the Universe. For example, in case of a neutral LSP with $m_{\chi} \approx 200$ GeV, eq.(4) with $B(\phi \rightarrow \chi) = 1$ implies $m_{\phi}/T_R > 4 \cdot 10^{10}$. Such a large ratio $m_{\phi}/T_R$ in turn requires $\Gamma_d < 10^{-21} m_{\phi}^2 / M_{\text{Pl}}$, which indicates that $\phi$ would have to decay through higher dimensional operators. Of
course, this constraint is no longer valid if $\chi$ reaches equilibrium with the plasma at temperatures $\lesssim T_R$.

Another Dark Matter candidate is a very massive particle, with $m_{\chi} \sim 10^{12}$ GeV; decays of this particle could give rise to the observed very energetic cosmic rays [16] if their lifetime is $\gtrsim 10^8$ times the age of the Universe. We noted above that such massive particles cannot be produced thermally in any realistic model of inflation. On the other hand, eq.(4) shows that inflaton decays might very easily produce too many of such particles. Taking $m_{\chi} = 10m_T = 10^{13}$ GeV, we see that we need a branching ratio as small as $5 \cdot 10^{-8}$ GeV/$T_R$, which implies quite a severe upper bound on $T_R$ even if $\chi$ pairs can only be produced in six body decays of the inflaton. Even taking $T_R = 1$ MeV, the lowest value compatible with successful nucleosynthesis, this requires $B(\phi \rightarrow \chi) < 10^{-4}$. Finally, if $\chi$ is produced only through $M_P$ suppressed interactions, eq.(9) implies $a^2 < 3.5 \cdot 10^{-6}$ GeV $\cdot M_P T_R/m_{\chi}^2$, which again gives a very tight constraint if $m_{\chi} \sim 10^{12}$ GeV.

In some cases other considerations give an even stronger constraint on $\Omega_{\chi}$. For example, the abundance of charged stable particles is severely constrained from searches for exotic isotopes in sea water [9], e.g.

$$\Omega_{\chi} h^2 \leq 10^{-20} \quad (11)$$

for $100$ GeV $\lesssim m_{\chi} \lesssim 10$ TeV; for heavier particles this bound becomes weaker. This bound imposes very severe constraints on supersymmetric models with stable charged LSP. Fixing again $m_{\chi} = 200$ GeV from considerations of naturalness, (11) implies $m_{\phi}/T_R > 4 \cdot 10^{30} B(\phi \rightarrow \chi)$. This is clearly incompatible with the limits $T_R \gtrsim 1$ MeV, $m_{\phi} \lesssim 10^{13}$ GeV, even if $\phi \rightarrow \chi$ decays require six body final states, see eq.(8). We saw above that arranging $\chi$ to have been in equilibrium at $T_R$ does not help. Finally, the relic density of charged LSPs that were in thermal equilibrium at $T < T_R$ violates the constraint (11) by more than ten orders of magnitude. Eq.(4) shows that the situation for larger $m_{\chi}$ would be even worse. We thus conclude that in models where at least a significant fraction of the present entropy of the Universe originates from inflaton decay, a stable charged LSP can only lead to an acceptable cosmology if it is too massive to be produced in inflaton decays.

Our calculation is also applicable to entropy–producing particle decays that might occur after all inflatons have decayed. If $\chi$ is lighter than this additional $\phi'$ particle [13], all our expressions go through with the obvious replacement $\phi \rightarrow \phi'$ everywhere. More generally our result holds if $\phi$ decays result in a radiation dominated era with $T_R > m_{\phi'}$. If $\phi'$ is sufficiently long–lived, the Universe will eventually enter a second matter–dominated epoch. $\phi'$ decays then give rise to a second epoch of reheating, leading to a radiation–dominated Universe with final reheating temperature $T_{R_f}$, and increasing the entropy by a factor $m_{\phi'}/T_{R_f}$. This could be incorporated into eq.(4) by replacing $T_R \rightarrow T_R T_{R_f}/m_{\phi'} > T_{R_f}$. Our result regarding a stable charged LSP would remain valid in such a scenario even if $m_{\chi} > m_{\phi'}$, since the lower bound of $\sim 1$ MeV which we used now applies to $T_{R_f}$. The only way out would be to allow $\phi'$ to be essentially the only decay product of $\phi$, where $\phi'$ itself does not have renormalizable interactions with (MS)SM particles (so that higher order $\phi$ decays are negligible) and $2m_{\chi} > m_{\phi'}$. However, there is presently no motivation for considering such baroque models.

$\chi$ particles can also be produced in scattering reactions of very energetic inflaton decay products before the latter are thermalized; however, we checked through explicit calculation that these additional sources of $\chi$ particles are always subdominant to direct inflaton decays [11].

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