ABSTRACT

Ioka and Nakamura (2001) proposed a simple jet model which is compatible with the peak luminosity-spectral lag relation, the peak luminosity-variability relation and various other relations in the Gamma-Ray Bursts. If the viewing angle is much larger than the collimation angle of the jet in the model by Ioka and Nakamura, for appropriate model parameters we obtain the observational characteristics of the X-ray flashes such as the peak flux ratio and the fluence ratio between the $\gamma$-ray ($50-300$ keV) and the X-ray band ($2-10$ keV), the X-ray photon index, the typical duration and the event rate $\sim 100$ yr$^{-1}$. In our model if the distance to the X-ray flashes is much larger than $\sim 1$ Gpc (or $z \gg 0.2$) they are too dim to be observed so that the spatial distribution of the X-ray flashes should be homogeneous and isotropic.

Subject headings: gamma rays: bursts — gamma rays: theory
1. INTRODUCTION

Recently, a new class of X-ray transients has been recognized. The Wide Field Cameras (WFCs) on the BeppoSAX in the X-ray range $2 - 25\text{ keV}$ have detected some Fast X-ray Transients (FXTs) with the duration less than $\sim 10^3\text{s}$, which are not triggered and not detected by the Gamma Ray Burst Monitor (GRBM) in the $\gamma$-ray range $40 - 700\text{ keV}$ (Heise et al. 2001; see also Strohmayer et al. 1998). In Heise et al. (2001), these FXTs are defined as X-ray flashes. This definition of X-ray flashes excludes the X-ray counterparts of the typical Gamma-Ray Bursts (GRBs) including X-ray rich GRBs. 17 X-ray flashes have been observed in the WFCs on the BeppoSAX in about 5 years while 49 GRB counterparts have been observed in the same period.

X-ray flashes have the following properties (Heise et al. 2001).

(i) The peak flux of the X-ray flashes ranges between $10^{-8}$ and $10^{-7}\text{ erg s}^{-1}\text{ cm}^{-2}$ (Fig. 2 of Heise et al. 2001). The mean peak flux of the X-ray flashes is about a factor three smaller than that of the GRBs. 9 out of 17 X-ray flashes are detected in either the lowest or the lowest two BATSE energy channels ($25 - 50\text{ keV}$ and $50 - 100\text{ keV}$) (Kippen et al. 2001).

(ii) The ratio of the peak flux and the fluence between the X-ray range ($2 - 10\text{ keV}$) and the $\gamma$-ray range ($50 - 300\text{ keV}$) for 9 X-ray flashes are shown in Fig. 3 of Heise et al. (2001). The peak flux ratio extends up to a factor of 100, and the fluence ratio extends up to a factor of 20.

(iii) The energy spectrum in the range $2 - 25\text{ keV}$ fits with a single power low with the photon index between 1.2 and 3 and the mean of about 2, while the mean photon index of 36 GRBs in the same X-ray band is about 1 with the range between 0.5 and 3.

(iv) The duration of the X-ray flashes ranges between 10 s and 200 s, which is the same order as that of the GRBs.

(v) The event rate of the X-ray flashes is estimated as $\sim 100\text{ yr}^{-1}$ since the WFCs observed $\sim 3\text{ yr}^{-1}$ with the covering $40^\circ \times 40^\circ$ (full width to zero response).

(vi) The sky distribution is consistent with being isotropic. The spatial distribution is consistent with being homogeneous in Euclidean space since $\langle V/V_{max} \rangle = 0.56 \pm 0.12$ (Heise et al. 2000; Schmidt, Higdon & Hueter 1988).
At present, the origin of the X-ray flashes is not known. Heise et al. (2001) have proposed that X-ray flashes could be GRBs at large redshift $z > 5$, when $\gamma$-rays would be shifted into the X-ray range. However, as they have pointed out in their paper, one cannot explain the duration distribution since no time dilation due to cosmological expansion is observed.

Ioka & Nakamura (2001) have proposed that the X-ray flashes could be GRBs observed from the large viewing angle as shown in Figure 1 (see also Nakamura 2000). They computed the kinematical dependence of the peak luminosity, the pulse width and the spectral lag of the peak luminosity on the viewing angle $\theta_v$ of a jet. For appropriate model parameters they obtained the peak luminosity-spectral lag relation similar to the observed one. They suggested that the viewing angle of the jet might cause various relations in GRBs such as the peak luminosity-variability relation and the luminosity-width relation. Very recently several authors have also suggested that the viewing angle is the key parameter to understand the various properties of the GRBs (Zhang & Mészáros 2001; Rossi, Lazzati & Rees 2001; Salmonson & Galama 2001). In this circumstance it is meaningful to study the off-axis GRB model for the X-ray flashes by Ioka & Nakamura (2001) in more details.

In this Letter, we will show that the GRBs observed from the large viewing angle possesses the above mentioned properties (i)-(vi) of the X-ray flashes. In § 2 we will describe a simple jet model for the X-ray flashes. In § 3 we will consider the peak flux ratio and the fluence ratio (property (ii)). In § 4 we will consider the peak flux, the photon index and the event rate (properties (i), (iii) and (v)). § 5 will be devoted to discussion (properties (iv) and (vi)).

2. EMISSION MODEL OF X-RAY FLASHES

We apply a simple jet model by Ioka & Nakamura (2001) to the X-ray flashes. Let us consider an emitting thin shell that is confined to a cone of an opening half-angle $\Delta \theta$ and moves radially outward with a Lorentz factor of $\gamma = 1/\sqrt{1 - \beta^2}$. A general formula to calculate the observed flux from an optically thin material is derived by Granot, Piran & Sari (1999) and Woods & Loeb (1999). Here we adopt their formulations and notations. Let us use a spherical coordinate system $r = (r, \theta, \phi)$ in the lab-frame, where the $\theta = 0$ axis points toward the detector and the central engine is located at the origin. In addition, let also the detector be at a distance $D$ from the source and $\alpha \equiv r \sin \theta / D = r \sqrt{1 - \mu^2} / D$ be the angle that a given ray makes with the $\theta = 0$ axis. Then the observed flux at the observed
time $T$, measured in erg s$^{-1}$ cm$^{-2}$ Hz$^{-1}$, is given by

$$F_\nu(T) = \frac{\nu D}{\gamma \beta} \int_0^{2\pi} d\phi \int_0^{\sin^{-1}(1+\beta)} d\alpha^2 \frac{\nu^2 H(\Delta \theta - |\theta - \theta_v|) H(\cos \phi - \frac{\cos \Delta \theta - \cos \theta_v \cos \theta}{\sin \theta_v \sin \theta})}{(1 - \beta \cos \theta(T))^2},$$

(1)

and $\mu = (1 - \nu' / \gamma \nu) / \beta$, where $\Omega'_d$, $t = T + (r \mu / c)$, $\alpha_m$, and $j'_\nu$ are the direction towards the detector measured in the comoving frame, the lab-frame time, the maximum value of $\alpha$, and the comoving frame emissivity in units of erg s$^{-1}$ cm$^{-3}$ Hz$^{-1}$ sr$^{-1}$. Note here that a prime means the physical quantities in the comoving frame.

There are three timescales that determine the temporal pulse structure of the X-ray flashes: the hydrodynamic timescale $T_{\text{dyn}}$, the cooling timescale $T_{\text{cool}}$, and the angular spreading timescale $T_{\text{ang}}$ (Kobayashi, Piran & Sari 1997; Katz 1997; Fenimore, Madras & Sergei 1996). Since we consider that the X-ray flashes are the GRBs observed from the large viewing angle, we assume $T_{\text{cool}} \ll T_{\text{dyn}} \ll T_{\text{ang}}$ as in the case of the GRBs (e.g., Piran 1999; Sari, Narayan & Piran 1996). Then, we can adopt an infinitesimally thin shell approximation and a delta function time dependence of the emissivity in the comoving frame. Furthermore, if the emission is isotropic in the comoving frame, the emissivity has a functional form of

$$j'_\nu(\Omega'_d, r, t) = A_0 f(\nu') \delta(t - t_0) \delta(r - r_0) \times H(\Delta \theta - |\theta - \theta_v|) H(\cos \phi - \frac{\cos \Delta \theta - \cos \theta_v \cos \theta}{\sin \theta_v \sin \theta}),$$

(2)

where $f(\nu')$ represents the spectral shape and $\theta_v$ is the angle that the axis of the emission cone makes with the $\theta = 0$ axis. The delta functions describe an instantaneous emission at $t = t_0$ and $r = r_0$, and $H(x)$ is the Heaviside step function, which describes that the emission is inside the cone. Then, the flux of a single pulse is given by

$$F_\nu(T) = \frac{2c^2 \beta \gamma^4 A_0 (r_0/c \beta \gamma^2) \Delta \phi(T) f(\nu(1 - \beta \cos \theta(T)))}{D^2 (1 - \beta \cos \theta(T))^2},$$

(3)

where $1 - \beta \cos \theta(T) = (c \beta / r_0)(T - T_0)$ and $T_0 = t_0 - r_0 / c \beta$. For $\Delta \theta > \theta_v$ and $0 < \theta(T) \leq \Delta \theta - \theta_v$, $\Delta \phi(T) = \pi$, otherwise $\Delta \phi(T) = \cos^{-1} \left[ \frac{\cos \Delta \theta - \cos \theta_v \cos \theta}{\sin \theta_v \sin \theta(T)} \right]$. For $\theta_v < \Delta \theta$, $\theta(T)$ varies from 0 to $\theta_v + \Delta \theta$ while from $\theta_v - \Delta \theta$ to $\theta_v + \Delta \theta$ for $\theta_v > \Delta \theta$. In the latter case, $\Delta \phi(T) = 0$ for $\theta(T) = \theta_v - \Delta \theta$. A pulse starts at $T_{\text{start}}$ and ends at $T_{\text{end}}$ where

$$T_{\text{start}} = T_0 + (r_0 / c \beta)(1 - \beta \cos(\max[0, \theta_v - \Delta \theta])),$$

(4)

$$T_{\text{end}} = T_0 + (r_0 / c \beta)(1 - \beta \cos(\theta_v + \Delta \theta)).$$

(5)

The spectrum of the GRBs is well approximated by the Band spectrum (Band et al. 1993). In order to have a spectral shape similar to the Band spectrum, we adopt the following
form of the spectrum in the comoving frame,

\[ f(\nu') = \left( \frac{\nu'}{\nu_0'} \right)^{1+\alpha_B} \left[ 1 + \left( \frac{\nu'}{\nu_0'} \right)^s \right]^{(\beta_B-\alpha_B)/s}, \]

where \( \alpha_B \) (\( \beta_B \)) is the low (high) energy power law index, and \( s \) describes the smoothness of the transition between the high and low energy. In the GRBs, \( \alpha_B \sim -1 \) and \( \beta_B \sim -3 \) are typical values (Preece et al. 2000). Equations (3) and (6) are the basic equations to calculate the flux of a single pulse, which depends on ten parameters for \( \gamma \gg 1, \theta_v \ll 1 \) and \( \Delta \theta \ll 1: \gamma \nu_0', \gamma \theta_v, \gamma \Delta \theta, r_0/c\beta \gamma^2, T_0, \alpha_B, \beta_B, s, D \) and \( \gamma^4 A_0 \).

In order to study the dependence on the viewing angle \( \theta_v \), we fix parameters as \( \gamma \Delta \theta = 10, \alpha_B = -1, \gamma \nu_0' = 300 \text{ keV}, r_0/c\beta \gamma^2 = 10 \text{ s} \) and \( s = 1 \), since the typical GRBs have the break energy of \( \sim 300 \text{ keV} \) (Preece et al. 2000) and the pulse duration of \( \sim 10 \text{ s} \). Other parameters, i.e., the viewing angle \( \gamma \theta_v \), the high energy power law index \( \beta_B \) and the distance \( D \), are varied depending on circumstances. We fix the amplitude \( \gamma^4 A_0 \) so that the isotropic \( \gamma \)-ray energy \( E_{\text{iso}} = 4\pi D^2 S(20 - 2000 \text{ keV}) \) equals \( 10^{53} \text{ erg} \) when \( \beta_B = -3.0 \) and \( \gamma \theta_v = 0 \). Here \( S(\nu_1 - \nu_2) = \int_{T_{\text{end}}}^{T_{\text{start}}} F(T; \nu_1 - \nu_2) \text{dT} \) is the fluence in the energy range \( \nu_1 - \nu_2 \) and \( F(T; \nu_1 - \nu_2) = \int_{\nu_1}^{\nu_2} F_\nu(T) \text{d}\nu \) is the flux in the same energy range. The result is

\[ A_0 = 1.2 \text{ erg cm}^{-2} \text{ Hz}^{-1} \left( \frac{E_{\text{iso}}}{10^{53} \text{ erg}} \right) \left( \frac{r_0/c\beta \gamma^2}{10 \text{ s}} \right)^{-2} \left( \frac{\gamma}{100} \right)^{-4}. \]

Note that when we adopt \( \gamma = 100 \), the opening half-angle of the jet is similar to the observed one \( \Delta \theta \sim 0.1 \) and the total energy corrected for geometry is comparable to the observed value (Frail et al. 2001),

\[ \frac{(\Delta \theta)^2}{2} E_{\text{iso}} \sim 5 \times 10^{50} \text{ erg}. \]

### 3. PEAK FLUX RATIO AND FLUENCE RATIO

In this section, we calculate the peak flux ratio \( R_{\text{peak}} = F_{\text{peak}}(2 - 10 \text{ keV})/F_{\text{peak}}(50 - 300 \text{ keV}) \) and the fluence ratio \( R_{\text{fluence}} = S(2 - 10 \text{ keV})/S(50 - 300 \text{ keV}) \), and compare the results with observations.

Figure 2 shows the peak flux ratio \( R_{\text{peak}} \) and the fluence ratio \( R_{\text{fluence}} \) as a function of the viewing angle \( \gamma \theta_v \). When the viewing angle \( \theta_v \) is larger than the opening half-angle \( \Delta \theta \), both the peak flux ratio \( R_{\text{peak}} \) and the fluence ratio \( R_{\text{fluence}} \) increase as the viewing angle \( \gamma \theta_v \) increases. The ratios, \( R_{\text{peak}} \) and \( R_{\text{fluence}} \), increase as the high energy index \( \beta_B \) decreases.

We can understand these behavior as follows. As shown in § A, the maximum frequency \( \nu_{\text{max}} \) at which most of the radiation energy is emitted is estimated as \( \nu_{\text{max}} \sim \nu_0'/\delta \) where
\[ \delta \equiv \gamma [1 - \beta \cos(\theta_v - \Delta \theta)] \simeq [1 + \gamma^2 (\theta_v - \Delta \theta)^2]/2 \gamma \] is the Doppler factor and \( \theta_v > \Delta \theta \). Thus the maximum frequency \( \nu_{\text{max}} \) decreases as the viewing angle increases. In the following, we consider two observation band: the lower energy band \( \nu_1 - \nu_2 \text{ keV} \), and the higher energy band \( \nu_3 - \nu_4 \text{ keV} \). The maximum frequency \( \nu_{\text{max}} \) is larger than the highest observed energy \( \nu_4 \approx 300 \text{ keV} \) in the present case when

\[ \gamma \theta_v < \gamma \theta_v^{(4)} \equiv \gamma \Delta \theta + \left( \frac{2 \gamma \nu_0}{\nu_4} - 1 \right)^{1/2}. \] (9)

In this case, we observe the low energy part of the Band spectrum in equation (6). Since the low energy power law index is \( \alpha_B = -1 \), the peak flux ratio \( R_{\text{peak}} = F_{\text{peak}}(\nu_1 - \nu_2 \text{ keV})/F_{\text{peak}}(\nu_3 - \nu_4 \text{ keV}) \) and the fluence ratio \( R_{\text{fluence}} = S(\nu_1 - \nu_2 \text{ keV})/S(\nu_3 - \nu_4 \text{ keV}) \) are given by \( R_{\text{peak}} \sim R_{\text{fluence}} \sim (\nu_2/\nu_4)^{2+\alpha_B} \), where \( \alpha_B > -2 \). Similarly, when the maximum frequency \( \nu_{\text{max}} \) is smaller than the lowest observed energy \( \nu_1 = 2 \text{ keV} \), i.e.,

\[ \gamma \theta_v > \gamma \theta_v^{(1)} \equiv \gamma \Delta \theta + \left( \frac{2 \gamma \nu_0}{\nu_1} - 1 \right)^{1/2}, \] (10)

the peak flux ratio and the fluence ratio are given by \( R_{\text{peak}} \sim R_{\text{fluence}} \sim (\nu_1/\nu_3)^{2+\beta_B} \), where \( \beta_B < -2 \).

We compare Figure 2 with observations. Observed peak flux ratios extend up to a factor of 100 and observed fluence ratios extend up to a factor of 20 (Fig. 3 of Heise et al. 2001). One can see that when \( \gamma \Delta \theta = 10 \lesssim \gamma \theta_v \lesssim \gamma \theta_v^{(1)} \sim 3 \gamma \Delta \theta \) and \(-4 \lesssim \beta_B \lesssim -2\), \( R_{\text{peak}} \) and \( R_{\text{fluence}} \) agree with the observational data. Furthermore, Kippen et al. (2002) reported that \( \nu_{\text{max}} \) ranges between about 2 and 90 keV. For our parameters, this can be reproduced if the viewing angle satisfies \( \Delta \theta \lesssim \theta_v \lesssim \theta_v^{(1)} \).

4. PEAK FLUX, PHOTON INDEX AND EVENT RATE

We calculate the peak flux and the photon index in the energy band \( 2 - 25 \text{ keV} \) as a function of the viewing angle \( \gamma \theta_v \), and plot it in the peak flux – photon index plane. Figure 3 show the results for \( \beta_B = -3 \). The distance is varied from \( D = 0.01 \text{ Gpc} \) to \( D = 2.1 \text{ Gpc} \) for our parameters. One can see that the photon index increases and the peak flux decreases as the viewing angle \( \gamma \theta_v \) increases.

As discussed in § 3, we observe the low (high) energy part of the Band spectrum in equation (6) when \( \gamma \theta_v < \gamma \theta_v^{(4)} (\gamma \theta_v > \gamma \theta_v^{(1)}) \) where \( \nu_4 = 25 \text{ keV} \) and \( \nu_1 = 2 \text{ keV} \) in equations (9) and (10). Therefore the photon index in the energy range \( 2 - 25 \text{ keV} \) is nearly equal to the low (high) energy spectral index \( |\alpha_B| = 1 (|\beta_B|) \) when \( \gamma \theta_v < \gamma \theta_v^{(4)} \approx 14.8 (\gamma \theta_v > \gamma \theta_v^{(1)} \approx \)
27.3). With the analytical estimates in § A, we can also find that the peak flux $F_{\text{peak}}$ is approximately given by

$$F_{\text{peak}} \simeq 4.3 \times 10^{-6} \text{erg s}^{-1} \text{cm}^{-2} \left( \frac{D}{1 \text{ Gpc}} \right)^{-2} \left[ 1 + \gamma^2 (\theta_v - \Delta \theta)^2 \right]^{-2+\alpha_B}$$

$$\times \left( \frac{r_0/c\gamma^2}{10 \text{ s}} \right) \left( \frac{\gamma \nu'_0}{300 \text{ keV}} \right)^{-1-\alpha_B} \left( \frac{\gamma^4 A_0}{1.2 \times 10^8 \text{ erg s cm}^{-2}} \right),$$

(11)

when $\Delta \theta \lesssim \theta_v \lesssim \theta_v^{(4)}$ (In practice, the above equation can be applied to more larger viewing angle $\gamma \theta_v \lesssim 30$. We have confirmed that numerical results can be fitted within 5% errors). The peak flux $F_{\text{peak}}$ is smaller for larger viewing angle. However, if the distances to such sources are small, $F_{\text{peak}}$ may be comparable to that of the typical GRBs, which have large distances and small viewing angle.

For comparison, we also plot the observed data in the same figures (Fig. 2 of Heise et al. 2001). One can see that the observed X-ray flashes take places within $\sim 2 \text{ Gpc}$ and have the viewing angle of $\gamma \Delta \theta = 10 \lesssim \gamma \theta_v \lesssim \gamma \theta_v^{(1)} \sim 3 \gamma \Delta \theta$.

The distance to the furthest X-ray flash $D_{\text{XRF}}$ gives the observed event rate of the X-ray flashes. The observed event rate $R_{\text{XRF}}$ can be estimated as

$$R_{\text{XRF}} = r_{\text{GRB}} \cdot n_g \cdot \frac{4\pi}{3} D_{\text{XRF}}^3 \cdot \frac{f_{\text{XRF}}}{f_{\text{GRB}}},$$

(12)

where $r_{\text{GRB}}$ and $n_g$ are the event rate of the GRBs and the number density of galaxies, respectively. $f_{\text{XRF}}$ ($f_{\text{GRB}}$) is the solid angle subtended by the direction to which the source is observed as the X-ray flash (GRB). From previous discussions, one can find that the emitting thin shell with opening half-angle $\Delta \theta$ is observed as the X-ray flash (GRB) when the viewing angle is within $\Delta \theta \lesssim \theta_v \lesssim \theta_v^{(1)} \sim 3 \Delta \theta$. Therefore the ratio of each solid angle is estimated as $f_{\text{XRF}}/f_{\text{GRB}} \sim (3^2 - 1^2)/1^2 = 8$. Using this value, we obtain

$$R_{\text{XRF}} \sim 10^2 \text{ events yr}^{-1} \left( \frac{r_{\text{GRB}}}{5 \times 10^{-8} \text{ events yr}^{-1} \text{ galaxy}^{-1}} \right) \left( \frac{D_{\text{XRF}}}{2 \text{ Gpc}} \right)^3 \times \left( \frac{n_g}{10^{-2} \text{ galaxies Mpc}^{-3}} \right) \left( \frac{f_{\text{XRF}}/f_{\text{GRB}}}{8} \right),$$

(13)

which is comparable to the observation.

5. DISCUSSION

We have shown that the observed data of the X-ray flashes can be reproduced by a simple jet model of the GRBs. This suggests that the X-ray flashes are identical with the
GRBs. We may say that in the context of our model, nearby GRBs are observed as X-ray flashes when we see them from the off-axis viewing angle. If the distance to the X-ray flashes is much larger than a few Gpc, they cannot be observed since the observed flux is low. This is consistent with the observed value of $\langle V/V_{\max} \rangle \sim 0.5$ since the nearby sources distribute homogeneously in Euclidean space.

Our view of the X-ray flashes is different from that of Heise et al. (2001). They have proposed that X-ray flashes could be GRBs at large redshift $z > 5$, when $\gamma$-rays would be shifted into the X-ray range. However, the observed total duration $T_{90}^{(obs)}$ cannot be explained. In our model, $\gamma$-rays are shifted into the X-ray range by the relativistic beaming effect. The total duration is equal to the lifetime of the central engine and thus does not depend on the viewing angle $\theta_v$. Hence the total duration of the X-ray flashes may be similar to that of the GRBs in our model.

We can calculate $T_{90}$, the observed duration of a single pulse in the X-ray band ($2 - 25$ keV). When the viewing angle ranges from $\gamma \theta_v = 10$ to $\gamma \theta_v = 30$, the pulse duration is about

$$T_{90} \sim 30 - 3 \times 10^3 \text{s} \left( \frac{r_0/c\beta\gamma^2}{10 \text{s}} \right).$$

This value is comparable but a little bit inconsistent with the observation since the observed pulse duration $T_{90}$, which is the order of the angular spreading timescale, should be less than the total duration $T_{90}^{(obs)} \sim 10 - 200$ sec, which is the time interval between the first and the last emission. This contradiction can be resolved as follows. So far, we have assumed the isotropic energy of the instantaneous emission $E_{iso} \sim 10^{53}$ erg and the time unit $r_0/c\beta\gamma^2 \sim 10$ s. The effect of changing the values of these two parameters appears only in the flux normalization $(\gamma^4 A_0)(r_0/c\beta\gamma^2)$ in equation (3). However, one can see that from equation (7), if one rescales these parameters as $E_{iso} \rightarrow E_{iso}' = 10^{53} N^{-1}$ erg and $r_0/c\beta\gamma^2 \rightarrow (r_0/c\beta\gamma^2)' = 10N^{-1}$ s, the flux normalization factor is invariant $(\gamma^4 A_0)(r_0/c\beta\gamma^2)' = [(\gamma^4 A_0)(r_0/c\beta\gamma^2)]'$, which implies that the result is unchanged. $N$ means the number of instantaneous emissions, since we fix the total emission energy as $E_{iso}^{(tot)} = 10^{53}$ erg. If we adopt $N \gtrsim 15$, $T_{90}$ of each emission can be less than $T_{90}^{(obs)}$.

Ioka & Nakamura (2001) showed that the variability of GRBs is small for large viewing angle. In addition, our model predicts that the number of pulses of the X-ray flashes is smaller than that of typical GRBs. This can be expected from the following discussions. In this paper, we consider the time averaged emissions, which means that successive emissions from multiple subjets with the opening half-angle $\Delta \theta^{(j)} \sim \gamma^{-1} \sim \Delta \theta/10$ are approximated by one spontaneous emission caused by a single jet with the viewing angle $\theta_v$ and the opening half-angle $\Delta \theta$. Let the viewing angle of each subjet to be $\theta_v^{(j)}$. The observed flux (or fluence) in the X-ray band due to the subjets with $\theta_v^{(j)} \sim \theta_v + \Delta \theta$ are much smaller than that with
\( \theta_v^{(j)} \sim \theta_v - \Delta \theta \), and hence negligible. We have confirmed this in the practical calculation. If \( \theta_v \gtrsim \Delta \theta \), the emissions of subjets with \( \theta_v^{(j)} \sim \theta_v - \Delta \theta \) dominates, while if \( \theta_v \sim 0 \), in the GRB case, the emissions from almost all subjets may be detected.

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A. ANALYTICAL ESTIMATES

(I) \( \nu_{\text{max}} \) and \( (\nu S_{\nu})_{\text{max}} \): In equation (3) the typical value of \( \theta(T) \) is \( \sim (\theta_v - \Delta \theta) \) when \( \theta_v > \Delta \theta \) since the flux peaks soon after the jet edge becomes visible. Since the function \( \nu' f' \nu' \) in equation (6) takes a maximum at \( \sim \nu'_0 \), \( \nu S_{\nu} \) takes a maximum at \( \nu_{\text{max}} \sim \nu'_0 / \delta \propto \delta^{-1} \) where \( \delta \equiv \gamma [1 - \beta \cos(\theta_v - \Delta \theta)] \simeq [1 + \gamma^2 (\theta_v - \Delta \theta)^2] / 2 \gamma \) and \( S_{\nu} = \int_{T_{\text{start}}}^{T_{\text{end}}} F_{\nu}(T) dT \). At \( \nu_{\text{max}} \), \( F_{\nu} \) in equation (3) is proportional to \( \delta^{-2} \) so that we expect \( (\nu S_{\nu})_{\text{max}} \propto \delta^{-3} \) (Ioka & Nakamura 2001). Note here that \( \int \Delta \phi(T) dT \) depends on \( \theta_v \) and \( \delta \) very weakly.

(II) \( T_{\text{ang}} \) and \( \nu F_{\nu}^{\text{peak}} \): The pulse duration \( T_{\text{ang}} \) can be estimated from equations (4) and (5) as \( T_{\text{ang}} \propto (T_{\text{end}} - T_{\text{start}}) \propto \theta_v^2 \propto \delta \) for \( \theta_v \sim \Delta \theta \), and \( T_{\text{ang}} \propto (T_{\text{end}} - T_{\text{start}}) \propto \theta_v \propto \delta^{1/2} \) for \( \theta_v \gg \Delta \theta \). The peak flux \( F_{\nu}^{\text{peak}} \) can be estimated from the relation \( F_{\nu}^{\text{peak}} T_{\text{ang}} \sim S \propto \delta^{-1+\alpha_B (\delta^{-1+\beta_B})} \) when the maximum frequency \( \nu_{\text{max}} \) is higher (lower) than the observed frequency.

REFERENCES


Fig. 1.— Our model is schematically shown. The X-ray flashes are the typical GRBs observed from the large viewing angle.
Fig. 2.— The peak flux ratio $R_{\text{peak}} = F_{\text{peak}}(2 - 10\,\text{keV})/F_{\text{peak}}(50 - 300\,\text{keV})$ (upper panel) and the fluence ratio $R_{\text{fluence}} = S(2 - 10\,\text{keV})/S(50 - 300\,\text{keV})$ (lower panel) as a function of the viewing angle $\gamma\theta_v$. The solid curve shows the case $\beta_B = -3$, and the dashed curves show the other cases, $\beta_B = -2$ and $\beta_B = -4$. We adopt $\gamma\Delta\theta = 10$, $\alpha_B = -1$, $\gamma\nu'_0 = 300\,\text{keV}$ and $s = 1$. The dotted line shows the viewing angle $\gamma\theta_v^{(4)} = 27.3$ ($\gamma\theta_v^{(4)} = 11$) at which the maximum frequency $\nu_{\text{max}}$ equals the lowest (highest) observed energy, i.e., 2 keV (300 keV). Here the maximum frequency $\nu_{\text{max}}$ means the frequency at which most of the radiation energy is emitted. At $\gamma\theta_v < \gamma\theta_v^{(4)}$ the ratios, $R_{\text{peak}}$ and $R_{\text{fluence}}$, nearly equal $(\nu_2/\nu_4)^{2+\alpha_B} = (10/300)$, and at $\gamma\theta_v > \gamma\theta_v^{(1)}$ the ratios, $R_{\text{peak}}$ and $R_{\text{fluence}}$, nearly equal $(\nu_1/\nu_3)^{2+\beta_B} = (2/50)^{-1}$, as shown by the long dashed lines.
Fig. 3.— The photon index in the energy range $2 - 25$ keV as a function of the peak flux in the same energy range by varying the distance $D$. We adopt $\gamma \Delta \theta = 10$, $\alpha_B = -1$, $\beta_B = -3$, $\gamma' = 300$ keV and $s = 1$. The values of the viewing angle $\gamma \theta_v$ are given in parenthesis. The right side (left side) of the two solid curves is $D = 0.01$ Gpc ($D = 2.1$ Gpc). Points which correspond same values of $\gamma \theta_v$ but different $D$ are connected by horizontal dotted lines. The observed data are shown from Heise et al. (2001). Squares (triangles) are those which were (were not) detected by BATSE.