Polarization of the Microwave Background Due to Primordial Gravitational Waves

Robert Crittenden, Richard L. Davis, and Paul J. Steinhardt

Department of Physics, University of Pennsylvania, Philadelphia, PA 19104

Abstract: The contribution of gravitational wave (tensor metric) and energy density (scalar metric) fluctuations to the cosmic microwave background polarization is computed. We find that the tensor contribution is significant only at large angular scales (multipoles $\ell \lesssim 40$). For standard recombination, the tensor contribution can dominate at $\ell \lesssim 40$; however, the effect would be difficult to detect since the total (scalar plus tensor) polarization is $< 1\%$. For models with late reionization, the total large angular scale polarization is large ($\sim 7 – 9\%$), but the tensor fraction is negligibly small. Hence, polarization may be useful for discriminating ionization history, but is less promising as a means for detecting tensor fluctuations.

Subject headings: cosmology: cosmic microwave background — cosmology: observations

The cosmic microwave background (CMB) temperature anisotropy may be induced by a combination of energy density (Bardeen et al. 1983) (scalar metric) fluctuations and gravitational wave (Starobinsky 1985, Abbott and Wise 1984) (tensor metric) fluctuations. Distinguishing the scalar and tensor components is important for testing cosmological models (Davis et al. 1992, Krauss and White 1992, Salopek 1992, Sahni and Souradeep 1992, Liddle and Lyth 1992, Lucchin et al. 1992, Lidsey and Coles 1992), since only scalar metric fluctuations can grow into non-linear structures, such as galaxies, while tensor fluctuations red shift and disperse upon entering the horizon. One suggested approach for determining the tensor contribution is to compare measurements of the CMB anisotropy on scales ranging from large ($\lesssim 1^\circ$) to small ($\gtrsim 1^\circ$) and take advantage of the fact that unpolarized scalar and tensor fluctuations vary differently with angular scale (Davis et al. 1992, Crittenden et al. 1993). Another suggestion has been to measure the CMB polarization since tensor fluctuations might induce a polarization significantly greater than a spectrum of purely energy density perturbations (Polnarev 1985, Ng and Ng 1993).
In this Letter we report on a computation of the polarization induced by a primordial spectrum of energy density and gravitational wave perturbations. By solving numerically the radiation transfer equations, we find that tensor perturbations significantly enhance the net polarization at large angular scales (multipoles $\ell \lesssim 40$). For example, in the case where there are equal scalar and tensor contributions to the $\Delta T/T$ (unpolarized) quadrupole moment, the tensor contribution can boost the polarization quadrupole by a factor of four or more compared to models with scalar fluctuations only. Nevertheless, we find the prospects for detecting primordial gravitational radiation using polarization are not promising: For standard recombination the net polarization on large-angular ($\gtrsim 1^\circ$) scales is less than 1% of $\Delta T/T$, too small to be detected with current technology. On small angular scales, the net polarization rises, but the tensor fraction drops to undetectable values. The fractional decrease occurs because the tensor contribution to $\Delta T/T$ on subdegree scales diminishes rapidly with increasing $\ell$ whereas the scalar anisotropy rises (Starobinsky 1985, Crittenden et al. 1993). Changing the Hubble constant and/or introducing a cosmological constant does not significantly alter the conclusions. For nonstandard reionization histories, the net polarization can be much higher ($\sim 7 - 9\%$). Hence, polarization may be a useful tool for testing ionization history. However, now the tensor fraction does not dominate on any angular scale, so that polarization is still not useful discriminating tensor fluctuations.

Our results apply to the general class of flat, Friedmann-Robertson-Walker universes with a gaussian distribution of adiabatic initial fluctuations, assuming power-law spectra and cold dark matter. An important subset of these models have strong theoretical motivation from inflationary cosmology. Inflation predicts a nearly scale-invariant (spectral index $n \approx 1$) spectrum of both scalar and tensor fluctuations. (The scalar fluctuations are created by quantum fluctuations of the inflaton field which drives the transition from the false to true vacuum phase (Bardeen et al. 1983). Quantum fluctuations in the inflaton on microscopic scales are stretched by inflation into a spectrum of energy density perturbations spanning cosmological wavelengths. Similarly, tensor fluctuations are produced by quantum fluctuations of the graviton field which are inflated to cosmological wavelengths (Starobinsky 1985, Abbott and Wise 1984). The relative contributions of scalar and tensor fluctuations to the CMB anisotropy depends upon the details of the inflationary potential. Likewise, the spectral tilt, $1 - n$, is model dependent. However, there is a nearly model-independent relation between the two:

$$C_2^{(T)}/C_2^{(S)} \approx 7(1 - n) \, ,$$

where $C_2^{(T)}$ and $C_2^{(S)}$ are the $\ell = 2$ (quadrupole) moments of the power spectrum (Davis
et al. 1992, Sahni and Souradeep 1992, Liddle and Lyth 1992, Lucchin et al. 1992, Lidsey and Coles 1992). (The only known exceptions to Eq. (1) require artificial, exponential fine-tuning of inflationary potential parameters.) A key test for inflation is to measure independently the ratio of tensor to scalar contributions to the CMB anisotropy and the tilt, $1 - n$, and determine if they satisfy the relation above.

As indicated in the preceding, there is strong motivation for investigating whether polarization measurements can be useful in extracting the tensor component of the CMB anisotropy. In order to compute the scalar- and tensor-induced polarization we evolve the photon distribution function, $f(x, q, t)$, for photons at position $x$ at time $t$ with momentum $q$, using first-order perturbation theory of the general relativistic Boltzmann equation for radiative transfer (Bond and Efstathiou 1984, Bond and Efstathiou 1987) with a Thomson scattering source term, in the synchronous gauge. Photon polarization is included by making $f$ a 4-dimensional vector with components related to the Stokes parameters ($f_s$ with $s = t, p, u, v$ correspond to the usual $I, Q, U, V$ Stokes notation), where we use Chandrasekhar’s treatment of the scattering source term for Rayleigh (and thus Thomson) scattering in a plane parallel atmosphere (Chandrasekhar 1960). In the scalar case, only $f_t$ and the ‘polarization’ $f_p$ are needed, so two transfer equations are required (Bond and Efstathiou 1984). In the tensor case $f_u$ also does not vanish, but it is related to $f_p$, so again only two perturbed transfer equations turn out to be required, the second of which describes the polarization. The full radiation transfer equations (coupled equations for the unpolarized and polarized anisotropy) are given in Bond and Efstathiou 1984 for the scalar case and Crittenden et al. 1993 for the tensor case. By numerically solving these equations, we have extracted the tensor and scalar multipole moments, $C_\ell^{(T)}$, $C_\ell^{(T)P}$, for both unpolarized and polarized measurements. The sum is the total multipole moment, $C_\ell$, which describes the temperature autocorrelation function:

$$C(\alpha) = \left< \frac{\Delta T}{T}(\mathbf{q}) \frac{\Delta T}{T}(\mathbf{q}') \right> = \sum_\ell (2\ell + 1) C_\ell P_\ell (\cos \alpha),$$

where $\mathbf{q} \cdot \mathbf{q}' = \cos \alpha$.

Fig. 1 shows $C_\ell^{P}/C_\ell$, the ratio of polarized to unpolarized moments, vs. $\ell$. Since moment $\ell$ is dominated by fluctuations on angular scales $\approx \pi/\ell$, we can use the square root of this ratio to estimate the polarization-to-anisotropy ratio on angular scale $\approx \pi/\ell$. The figure corresponds to the inflationary prediction, Eq. 1, with $n = 0.85$, for which tensor and scalar contributions to the CMB quadrupole are equal. The Hubble constant is $H_0 = 100h$ km/sec-Mpc with $h = 0.5$ and the baryon density is fixed by nucleosynthesis
(Walker et al. 1991), $\Omega_B = 0.125 h^{-2}$. As stated above, we assume that $\Omega_{total} = 1$, where the missing mass is cold dark matter. The results given here are relatively insensitive to variations in $h$ or the cosmological constant $\Lambda$, or a possible hot component. Quantitatively, the polarization changes by $\ll 1\%$ for variations $0.5 \leq h \leq 1.0$ and $0.0 \leq \Omega_\Lambda \leq 0.8$.

In Fig. 1, the tensor contribution to the polarization significantly dominates the scalar contribution for $\ell \lesssim 40$ corresponding to length scales that were outside the horizon at decoupling. However, over this range, the net polarization-to-anisotropy ratio is less than $1\%$. Since the CMB anisotropy is $\Delta T/T \sim O(10^{-5})$, a polarization experiment with sensitivity $< O(10^{-7})$ is needed to detect the tensor-induced polarization over this range. The requisite sensitivity is at the limit of the most optimistic estimates for near-future detectors (Smoot 1993). At subdegree scales ($l > 100$), the polarization-to-anisotropy ratio increases, but, for such $\ell$'s, the tensor contribution to the polarization is negligible. Polarization and anisotropy at subdegree scales is dominated by wavelengths inside the horizon at decoupling. For these wavelengths the gravitational wave contribution has red-shifted and dispersed whereas the scalar contribution has grown, as shown in Fig. 2. Consequently, the tensor fraction of the polarization (and anisotropy) is negligible. Polarization does not appear to be a promising approach for detecting the gravitational wave contribution predicted by inflation and standard recombination.

The total polarization is enhanced if the universe is reionized at some time after decoupling (Polnarev 1985). Fig. 3 shows the results for the case of no recombination. All other spectral and cosmological parameters are the same as in Fig. 1. The polarization from both scalar and tensor components (and the total polarization) have increased on large angular scales to $\sim 8\%$ (see also Bond and Efstathiou 1984 and Nasel’skii and Polnarev 1987), levels that may be detectable in the near future. It appears that polarization may be useful for discriminating the ionization history. However, we also note that the tensor fraction of the polarization for no recombination is smaller than the scalar even for small $\ell$. We have also considered models with recombination followed by reionization at some red shift $z_r$. Changing $z_r$ can alter the total polarization (Stark 1981). For $h = 0.5$ and $\Omega_B = 0.5$, the polarization rises to $\sim 9\%$ for $z_r \sim 200$, corresponding to two or three optical depths between reionization and the present. Despite modest differences in total polarization, the situation for no recombination and the non-standard reionization histories is the same in terms of the tensor mode: distinguishing the tensor contribution requires sensitivity to better than 1.
Our calculations apply to a larger class of models than those predicted by inflation. If we relax the inflationary predictions relating the gravitational wave background to the spectral index, and were to assume \textit{a priori} a primordial stochastic background of gravitational radiation, polarization is still not a practical method for extracting the tensor contribution to CMB anisotropy. Fig. 4 illustrates the extreme limits of pure tensor and pure scalar fluctuations. (In the tensor curve, the polarization does not diminish at large \( \ell \), as it does in Fig. 1, because the scalar anisotropy, which normally dominates at large \( \ell \) has, been set to zero.) The polarization-to-anisotropy ratio is only weakly \( n \)-dependent. For tensor fluctuations, the only discernible \( n \)-dependence is on large angular scales; for scalar fluctuations, there is no discernible difference on large or small scales for the range of \( n \) illustrated in the figure. The weak \( n \)-dependence makes it easy estimate the polarization for a wide variety of models. As in earlier examples, the tensor polarization dominates the scalar at large angular scales, but the magnitude is too small to be detected in either case. Now we observe that the pure scalar and pure tensor polarizations are quite similar at small angular scales, too, where the polarization might be large enough to be detected. Only around 1\({}^\circ\), for \( \ell \approx 100 \), does there appear to be a narrow window in which the predictions from these \textit{extreme} models approach measurable difference. This difference disappears if a scalar contribution greater than \( \approx 10\% \) is added to the pure tensor fluctuations.

Actual experiments measure a wide range of angular scales and measure many multipoles at the same time. The experimental sensitivity to different \( \ell \)’s can be expressed in terms of a filter function, \( W_\ell \), where:

\[
C(\alpha) = \sum_\ell (2\ell + 1)C_\ell W_\ell P_\ell(\cos \alpha). \tag{3}
\]

For the purposes of illustration, we will compute the polarization measured for the models in Figs. 1 and 3 for a hypothetical single beam anisotropy experiment. We shall suppose that the experiment measures with equal sensitivity \( (W_\ell = 1) \) all multipoles ranging from \( \ell \approx 5 \) (lower \( \ell \)’s excluded by limited sky coverage) to some \( \ell_{max} = (50, 100, 300, 1000) \) (fixed by the beam size). With these windows, we find the rms total polarization-to-anisotropy ratio \( [C_p(\alpha = 0)/C(\alpha = 0)]^{1/2} \) for the standard recombination model with \( n = 0.85 \) and \( C_2^{(T)}/C_2^{(S)} = 1 \) in Fig. 1 is \((.5\%, \ 1.0\%, \ 1.3\%, \ 5.4\%)\), to be compared with \((.4\%, \ 1.0\%, \ 1.4\%, \ 6.0\%)\) for a model with \( n = 0.85 \) but purely scalar fluctuations. For the non-standard recombination model in Fig. 3, the result for \( n = 0.85 \) and \( C_2^{(T)}/C_2^{(S)} = 1 \) is \((7.4\%, \ 7.4\%, \ 7.4\%, \ 7.4\%)\), to be compared with \((7.9\%, \ 7.8\%, \ 7.8\%, \ 7.8\%)\) for a model with same ionization history and \( n = 0.85 \) but purely scalar fluctuations. Distinguishing a tensor
contribution in either example requires sensitivity at the level $\ll 1\%$. In addition, we find that the polarization can change by more than $1\%$ if $h$, $\Omega_B$, or $z_r$ are varied within present observational limits. Hence, discriminating a tensor contribution is further complicated by uncertainties in these cosmological parameters.

In sum, polarization may be a useful discriminant for determining the ionization history of the universe. However, the polarization enhancement due to gravitational waves is too small to be resolved unless there is more than an order of magnitude improvement in experimental sensitivity. Our conclusions apply to a large class of cosmological models and cosmological parameters. If inflation is correct, or if a primordial background of gravitational radiation exists for some other reason, then it appears that the most promising method for separating the scalar and tensor contributions to the CMB anisotropy is to combine small- and large-angular scale unpolarized measurements, as described by Davis et al. 1992 and Crittenden et al. 1993.

We thank J.R. Bond, G. Efstathiou, and G. F. Smoot for useful contributions to this work. This research was supported by the DOE at Penn (DOE-EY-76-C-02-3071).
REFERENCES


Chandrasekhar, S. 1960, Radiation Transfer (Dover: New York), 1


Ng, K. L. and Ng, K. W. 1993, Institute of Physics, Academica Sinica Preprint IP-ASTP-08-93

Polnarev, A. G. 1985, Soviet Ast., 29, 607


Smoot, G. F., private communication


This manuscript was prepared with the AAS \LaTeX{} macros v3.0.
Fig. 1.— Ratio of polarization multipole to (unpolarized) anisotropy multipole vs. moment $(\ell)$ predicted by an inflationary model with $n = 0.85$, $h = 0.5$, cold dark matter, and standard recombination. (Inflation predicts equal scalar and tensor contributions to the unpolarized quadrupole.) The ratio above represents, roughly, the square of the polarization-to-anisotropy ratio on angular scale $\approx \pi/\ell$.

Fig. 2.— The total, tensor and scalar contributions to the unpolarized anisotropy for the model in Fig. 1. Although the scalar and tensor contributions to the quadrupole $(\ell = 2)$ moment are equal, the tensor contribution drops to insignificant values at subdegree $(\ell > 100)$ scales, which explains why the tensor contribution to the polarization is small at subdegree scales in Fig. 1.

Fig. 3.— Same as Fig. 1, except with no recombination (i.e., extended ionization). The net polarization is greatly enhanced on large angular scales, but the tensor contribution is subdominant for all $\ell$.

Fig. 4.— Ratio of polarization moments to anisotropy moments for pure tensor and pure scalar spectra, with $h = 0.5$, cold dark matter, and standard recombination. There is no discernible variation with $n$ for the pure scalar case and only differences at small $\ell$ for the tensor case.