On the probability of major-axis precession in triaxial ellipsoidal potentials

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Abstract

Orbits in triaxial ellipsoidal potentials precess about either the major or minor axis of the ellipsoid. In standard perturbation theory it can be shown that a circular orbit will precess about the minor axis if its angular momentum vector lies in a region bounded by two great circles which pass through the intermediate axis and which are inclined with minimum separation \(i_T\) from the minor axis, where \(i_T = \arctan \left( \frac{(B^2 - C^3)(A^2 - B^2)}{A^2} \right)^{1/2}\) and \(A, B\) and \(C\) are the axis ratios, \(A \geq B \geq C\). We test the accuracy of this formula by performing orbit integrations to determine \(i_s\), the simulated turnover angle corresponding to \(i_T\).

We reach two principal conclusions: (i) \(i_s\) is usually greater than \(i_T\), by as much as 12 degrees even for moderate triaxialities, \(A/1.2 < B < C/0.8\). This reduces the expected frequency of polar rings. (ii) \(i_s\) is not a single, well-defined number but can vary by a few degrees depending upon the initial phase of the orbit. This means that there is a reasonable probability for capture of gas onto orbits which precess about both axes. Interactions can then lead to substantial loss of angular momentum and subsequent infall to the galactic centre.

Key words: Galaxies: peculiar, Galaxies: formation

1 Introduction

Polar ring galaxies are systems in which a ring of gas (and/or young stars) is seen orbit about the major axis of an early-type galaxy. In many cases it has been established that the host galaxy is rotating at right-angles to the ring and this is sometimes taken to be part of the definition of a polar ring system. There are just 6 confirmed polar rings but 27 good candidates and many more possible—see Whitmore et al. (1990) for a review.

The host galaxy in polar ring systems often appears to be an S0 but this leads to theoretical problems: in an oblate axisymmetric system all orbits will precess about the minor axis at a rate which is a function of radius (typically the period is proportional to radius). This differential precession will cause the polar ring to fragment. This can be overcome if a small degree of triaxiality is assumed as orbits whose angular momentum is sufficiently close to the major axis will then precess around the major rather than the minor axis. In general accreted discs of gas will have an arbitrary orientation. In this case differential precession about a symmetry axis will lead to dissipation and collapse into the plane perpendicular to that axis.

The regions of phase space which lead to precession about the major or minor axes can be investigated using perturbation theory in Hamiltonian mechanics. It is assumed that the unperturbed potential is spherically symmetric and that the orbits are circular. The Hamiltonian is then expanded in spherical harmonics and the time-averaged perturbing potential calculated (see, for example, Steiman-Cameron & Durisen, 1984). This results in an expression of the form

\[ < \Phi_1 > \propto C_{20} (3 \sin^2 i - 2) + 6C_{22} \cos 2\Omega \sin^2 i \]

(1)

where \(\Omega\) and \(i\) are the node and inclination of the orbit, as illustrated in Figure 1, and \(C_{20}\) and \(C_{22}\) are constants. Orbits will precess along lines \(< \Phi_1 > = \text{constant}\) as shown in Figure 2. If the angular momentum of the orbit, \(\mathbf{J}\), lies within the shaded region then it will precess about the major axis, otherwise it will precess about the minor axis. The dividing lines between the two regions are great circles which pass through the intermediate axis and which are inclined at an angle

\[ i_T = \arcsin \left( \frac{C_{20} + 2C_{22}}{C_{20} - 2C_{22}} \right)^{\frac{1}{2}} \]

(2)

to the minor axis. Hence the probability that a randomly inclined disk will precess around the major axis is

\[ f = 1 - \frac{2i_T}{\pi} \]
2 INTEGRATION METHOD AND RESULTS

2.1 The integration method

The numerical code used in this paper is that described by Pearce & Thomas (1991). It is a simple predictor-corrector method integrates orbits to high accuracy and we refer the reader to this paper for details.

We use a potential

\[
\phi = \ln \left( \frac{x^2}{A} + \frac{y^2}{B} + \frac{z^2}{C} \right)
\]

where \( A \geq B \geq C \) and without loss of generality we can take \( B = 1 \). The time-averaged first-order perturbation potential takes the form of Equation 1 with

\[
C_{20} = \frac{1}{4\pi} + \frac{1}{\pi} - \frac{2}{A^2}
\]

\[
C_{22} = \frac{1}{2} \left( \frac{1}{A^2} - \frac{1}{B^2} \right)
\]

which gives, on substitution in Equation 2,

\[
\tau_T = \arcsin \left( \frac{1}{x} - \frac{1}{y} \right) \frac{1}{2} = \arctan \left( \frac{B^2 - C^2}{A^2 - B^2} \right) \frac{1}{2}.
\]

Note that this expression is different from that for a system with ellipsoidal density contours, \( \rho = \rho(x/a)^2 + (y/b)^2 + (z/c)^2 \), for which

\[
\tau_T = \arcsin \left( \frac{b^2 - c^2}{a^2 - c^2} \right) \frac{1}{2}.
\]

These two formulae agree for near-spherical systems but can differ by 10 degrees or more for moderate triaxialities. For example \( a = 1.2, b = 1, c = 0.8 \) gives \( \tau_T \approx 77^\circ \) and \( f \approx 0.15 \).

In this paper we test the validity of the perturbation theory by calculating orbits in an ellipsoidal triaxial potential. We find that the measured transition angle, \( \tau_T \), which divides precession about the major and minor axes, is usually larger than the theoretical value, \( \tau_T \). Also the boundary dividing the two regions of precession about the major and minor axes is not sharp. The precession axis depends upon the phase of the orbit and \( \tau_T \) can vary by a few degrees. The integration method and results of our simulations are presented in the Section 2. The conclusions are summarised and discussed in Section 3.

Figure 1. The orbital elements.

Figure 2. Precession trajectories for the angular momentum vector of near-circular orbits in a triaxial potential.

This formula is extensively used in the theory of polar rings. It is generally accepted that dissipation will cause a disk of gas to settle into a plane perpendicular to the axis about which it precesses. Even a small triaxiality can lead to a reasonable probability for polar ring formation. For example taking an ellipsoidal density distribution with axis ratios \( a = 1.01, b = 1, c = 0.8 \) gives \( \tau_T \approx 77^\circ \) and \( f \approx 0.15 \).

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2.2 Results

The measured transition angle, \( \tau_T \), is a function of the initial phase, \( \chi \), as illustrated in Figure 3 for one particular choice of axis ratios, \( A = 1.1, B = 1.0 \) and \( C = 0.9 \). The minimum value of \( \tau_T \) occurs at \( \chi = 0 \) and \( \tau_T \) at \( \chi = \pi/2 \) with an approximately sinusoidal variation between the two.
oppositely aligned. Collisions between gas clouds of similar mass may then reduce their velocity relative to the galactic centre to almost zero and they will be accreted into the core. This situation may not be as unlikely as it at first appears because the appropriate range of $i_S$ may span up to 10 degrees even for moderate triaxialities. Also accretion of a gas-rich dwarf spiral or dwarf irregular galaxy is likely to populate a fair region of phase space and a large amount of dissipation is required for this to settle into a disk of gas clouds on circular orbits, as is often assumed for simplicity.

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**References**


**3 CONCLUSIONS**

In this paper we calculate the simulated transition angle for precession about the major or minor axis of a triaxial potential and reach the following two conclusions: (i) $i_S$ is almost always greater than $i_T$ and (ii) $i_S$ is spread out over a few degrees depending upon the initial phase of the orbit.

The first of these results means that theoretical estimates, based on $i_T$, of the expected fraction of accreted gas disk which will settle down to give polar rings will be too high. However the error is not likely to be greater than about 10 percent, much lower than the uncertainty in the observed frequency of polar ring systems.

More interesting is the second result which may provide a mechanism for overcoming the angular momentum barrier which prevents accretion into the core of a galaxy. If material is accreted at an inclination between the measured maximum and minimum values of $i_S$, and if the accreted material is spread out over a range of phases, then precession will occur around both the minor and the major axes. Interactions between the two components can then lead to a large reduction in angular momentum. Indeed, once each component has precessed through 180 degrees then the planes of their orbits again coincide but their angular momenta are
Figure 4.