Entropy in
Black Hole Pair Production

DAVID GARFINKLE$^{1,3}$
STEVEN B. GIDDINGS$^{2,1}$
ANDREW STROMINGER$^{1,2}$

$^1$Institute for Theoretical Physics
and
$^2$Department of Physics
University of California
Santa Barbara, CA 93106

$^3$Department of Physics
Oakland University
Rochester, MI 48309

Abstract

Pair production of Reissner-Nordström black holes in a magnetic field can be described by a euclidean instanton. It is shown that the instanton amplitude contains an explicit factor of $e^{A/4}$, where $A$ is the area of the event horizon. This is consistent with the hypothesis that $e^{A/4}$ measures the number of black hole states.

† Email addresses: giddings@denali.physics.ucsb.edu, steve@voodoo.bitnet.
1. Introduction

The elegant laws of black hole thermodynamics[1,2] have yet to find a microscopic explanation in an underlying statistical mechanics of black hole states. Particularly interesting is the interpretation of the Bekenstein-Hawking entropy. In most cases this entropy is given by the simple formula

\[ S_{bh} = A/4, \quad (1.1) \]

where \( A \) is the area of the black hole horizon in Planck units. Up to an additive constant, this formula can be derived by insertion of the semiclassical black hole mass-temperature relation[3] into the thermodynamic formula \( 1/T = \partial S/\partial E \) followed by integration. Additional assumptions are required to fix the constant part of the entropy. A widely utilized, but mysterious, procedure is to fix the constant by relating it to the black hole instanton in the Euclidean path integral[4].

The relation (1.1) acquires additional meaning in light of Bekenstein’s conjectured generalized second law[1], which states that the sum of the usual entropy plus \( S_{bh} \) always increases. Although there is no complete proof of this conjecture, evidence is provided by the many ingenious gedanken attempts[2] to violate the generalized second law which have been foiled by the subtle dynamics of quantum mechanical black holes.

If the traditional connection between thermodynamics and statistical mechanics were to extend to black holes, then the number of quantum states of the black hole would be finite and given by

\[ N = e^{S_{bh}}. \quad (1.2) \]

These microstates might be either “internal states” inside the black hole or “horizon states” somehow associated with degrees of freedom of (or near) the black hole horizon, or both.

The issue of whether (1.2) can be taken literally has bearing on the vexing question of what happens to information cast into a black hole\(^1\). If one assumes that (1.2) counts all the black hole states, and that information is preserved, then one is forced to conclude that information escapes from a black hole at a rapid rate (proportional to the rate of area decrease) during the Hawking process. We do not think this is likely because it seems to requires a breakdown of semiclassical methods for arbitrarily large black holes and at arbitrarily weak curvatures, although this point is certainly the subject of heated debates! On the other hand one might try to account for the decrease in (1.2) during black hole

\(^1\) For recent reviews see [5-8].
evaporation by assuming that information is truly lost in the black hole interior, perhaps being eaten by the singularity. A problem with this is that - for large neutral black holes - the spacelike slice on which the quantum Hilbert space is defined can be extended through the interior of the black hole in a manner which avoids the singularity and all strong-curvature regions. Dynamics on such a slice is weakly coupled, and it is therefore hard to see how information could be lost\(^2\).

An alternate interpretation of (1.2) is that it counts only the horizon states. One is then not pushed into the conclusion that information either rapidly escapes or is eaten at weak coupling. Indeed if one assumes that there are \(e^{1/4}\) states per Planck area of the horizon, one precisely recovers (1.2). Certainly a derivation of this strange factor would be of great interest! Of course even if such a derivation were found, it would still remain to understand why - if (1.2) counts only horizon states - the generalized second law appears to be valid.

Yet a third microphysical explanation of (1.1) is suggested by recent work\(^\[9\], in which the entropy of the free scalar field vacuum outside a ball of surface area \(A\) was computed by tracing over states inside the ball. The result was found to leading order in \(A\) to be

\[
S_{\text{bh}} = A\Lambda^2, \tag{1.3}
\]

where \(\Lambda\) is an ultraviolet cutoff. (1.3) has a microphysical explanation by construction, but it is not in terms of states at or inside the surface of the ball. Rather the entropy arises from correlations between the quantum state inside and outside the ball. It is tempting to try to relate this observation to (1.1), but this would require explaining why \(\Lambda^2\) is precisely 1/4 in Planck units. Furthermore, such an interpretation of (1.1) would not appear to readily explain the validity of the generalized second law. Certainly no such law is valid in the free field example of [9].

For these reasons it is clearly of interest to seek a deeper understanding of the meaning of the black hole entropy. One promising avenue of exploration is the phenomenon of pair production of charged black holes. In Schwinger production of charged particles in a background field, the total production rate grows as the number of particle species produced. If this is extrapolated to black hole production in a background field\(^[10,11]\) then one would likewise expect the rate to be proportional to the number of independent black

\(^2\) Although perhaps (1.2) makes sense only with respect to a specific slicing of spacetime which differs from the one described here.
hole states produced. In this paper we show that the factor \((1.2)\) indeed multiplies the pair production amplitude, consistent with its interpretation as somehow counting black hole microstates. While the nature of these supposed states is still very mysterious, we do hope that our result will constrain future interpretations.

The desired factor \((1.2)\) is isolated from the rest of the pair production amplitude by consideration of the family of stable solutions discussed in [12] corresponding to gravitationally corrected ‘t Hooft-Polyakov monopoles of charge \(q\). For \(q M_{\text{GUT}} \ll M_{\text{Planck}}\), these closely resemble the ‘t Hooft-Polyakov solutions. For \(q M_{\text{GUT}} > M_{\text{Planck}}\), the monopole drops inside an event horizon and the solutions are identical to extremal Reissner-Nordstrom monopole black holes. Pair production of these monopoles can be analyzed using instanton methods. For fixed magnetic field \(B\), consider a one-parameter family of instantons labeled by \(M_{\text{GUT}}\). For \(q M_{\text{GUT}} \ll M_{\text{Planck}}\), the instanton resembles the one described by Affleck, Alvarez, and Manton [13,14] as an ‘t Hooft-Polyakov monopole in a circular orbit in euclidean space. For \(q M_{\text{GUT}} > M_{\text{Planck}}\), the instanton is precisely the one found in [11] describing Reissner-Nordstrom monopole pair production. At the critical value of \(M_{\text{GUT}}\) near \(M_{\text{Planck}}/q\), where the monopole drops inside a horizon, one finds that the action discontinuously changes by precisely \(-S_{\text{bh}}\).

Of course even our well-funded gedanken experimentalist can not observe this threshold because coupling constants such as \(M_{\text{GUT}}\) cannot be varied in the laboratory. Fortunately it will be seen from a precise description of the production process that the same threshold can be observed by varying the magnetic field \(B\) while keeping \(M_{\text{GUT}}\) fixed. Our gedanken experimentalist who discovers that the production rate suddenly jumps up at precisely the critical \(B\) field which produces monopoles with horizons, will likely conclude that he has crossed a threshold for production of \(e^{S_{\text{bh}}}\) new states. This assigns a new, physical significance to the relation \((1.2)\).

In section two we briefly review ref. [11] and present an exact formula for the pair production rate. It is however somewhat difficult to extract from this the contribution of the entropy because structure dependent Coulomb terms give contributions of similar magnitude. To circumvent this difficulty, section three compares this amplitude to the pair production of a GUT monopole (with parameters tuned so that its surface is barely outside the would-be horizon) and thereby extracts the entropy factor. Finally, in section four we perform the same comparison in the two-dimensional reduced theory that arises in the weak-field limit. Although this yields exactly the same result, it provides a simplified description of the process. Section five closes with discussion. The appendix contains a
derivation of the exact action of the black hole pair-production instanton, which is valid even for black holes of size (or charge) comparable to $1/B$. This extends the leading-order-in-$B$ expression given in [11].

2. Reissner-Nordstrøm pair production

The amplitude for production of magnetically charged black holes in a magnetic field can be calculated in the semiclassical approximation by finding an analogue of the Schwinger instanton in gravity coupled to electromagnetism, with euclidean action

$$S = \frac{1}{16\pi} \int d^4x \sqrt{g} [-R + F_{\mu\nu} F^{\mu\nu}] - \frac{1}{8\pi} \int d^3x \sqrt{h}K$$  

(2.1)

where we have included the surface term written in terms of the extrinsic curvature $K$ and boundary metric $h$. First consider the solution corresponding to the background field. Because of the magnetic energy this solution is not flat, but rather for a magnetic field in the $z$ direction is given by the euclidean Melvin universe[15],

$$ds^2 = (1 + \frac{1}{4}B^2\rho^2)^2(dt^2 + dz^2 + d\rho^2) + \frac{\rho^2}{(1 + \frac{1}{4}B^2\rho^2)^2}d\phi^2$$

$$F = \frac{B\rho d\rho \wedge d\phi}{(1 + \frac{1}{4}B^2\rho^2)^2}$$

(2.2)

where $-\infty < t, z < \infty$, $0 < \rho < \infty$, and $0 < \phi < 2\pi$. This solution corresponds to a flux tube with total flux

$$\Phi = \int F = \frac{4\pi}{B}$$

(2.3)

through a transverse hypersurface.

The instanton describes circular motion of an extremal Reissner-Nordstrøm black hole in the euclidean continuation of the Melvin universe. It is given by the Ernst solution[16],

$$ds^2 = \frac{\Lambda^2}{A^2(x-y)^2}[-G(y)dt^2 - G^{-1}(y)dy^2 + G^{-1}(x)dx^2] + \frac{G(x)}{\Lambda^2A^2(x-y)^2}dz^2$$

$$F = dz \wedge dE$$

(2.4)

Here

$$G(x) = 1 - x^2(1 + qAx)^2$$

$$E = \frac{2}{\Lambda B}(1 + \frac{1}{2}qBx)$$

(2.5)

(2.6)