Abstract

Properties of spin polarized isospin asymmetric nuclear matter are studied within the framework of the Brueckner–Hartree–Fock formalism. The single-particle potentials of neutrons and protons with spin up and down are determined for several values of the neutron and proton spin polarizations and the asymmetry parameter. It is found an almost linear and symmetric variation of the single-particle potentials as increasing these parameters. An analytic parametrization of the total energy per particle as a function of the asymmetry and spin polarizations is constructed. This parametrization is employed to compute the magnetic susceptibility of nuclear matter for several values of the asymmetry from neutron to symmetric matter. The results show no indication of a ferromagnetic transition at any density for any asymmetry of nuclear matter.

PACS numbers: 26.60.+c, 21.60.Jz, 26.50.+x

I. INTRODUCTION

Pulsars are believed to be rapidly rotating neutron stars endowed with strong magnetic fields [1,2]. Within the dipole magnetic model for pulsars the intensity of the surface (dipole)
magnetic field $B_p$ can be inferred from the measured rotational period $P$ of the pulsar about its rotation axis and from its time derivative $\dot{P} = dP/dt$. For “canonical” pulsars $^1$ this method gives values for $B_p$ in the range $10^{12}$–$10^{13}$ G. Despite the great theoretical effort, there is no general consensus regarding the mechanism to generate such a strong magnetic field in a neutron star. The field could be a fossil remnant from the one of the progenitor star. In fact, assuming magnetic flux conservation during the birth of the neutron star, a magnetic field of $\sim 10^{12}$ G could originate from the collapse of a main sequence star with a typical surface magnetic field of $10$–$10^2$ G. Alternatively, the field could be generated after the formation of the neutron star by some long-lived electric currents flowing in the highly conductive neutron star material.

There are strong theoretical and observational arguments which indicate a decay of the magnetic field during the “life” of a neutron star [3]. The physical processes which are responsible for the magnetic field evolution in neutron stars are not well understood, however, the observational data for pulsar distribution on the $P$-$\dot{P}$ plane can be reproduced by population synthesis codes (see e.g., ref. [4] and references therein quoted) in which the stellar magnetic field evolves according to the following exponential decay law [5]

$$B(t) = B_\infty + (B_0 - B_\infty)e^{-t/\tau_B},$$  \hspace{1cm} (1)

where $\tau_B \sim 10^7$–$10^9$ yr is the field decay time and $B_\infty \sim 10^8$ G is a residue magnetic field. If the residue field is permanent and is not generated by some dynamo mechanism, it could be produced by a spontaneous ferromagnetic transition in the dense stellar core. Several authors have studied the possible existence of a ferromagnetic core in the liquid interior of neutron stars. First models, in which neutron star matter was approximated by pure neutron matter, were proposed just after the discovery of pulsars. Brownell and

\[\text{Brownell and [8]}\]

$^1$Here we use the term “canonical” to distinguish these pulsars from the group of “millisecond” pulsars, for which $B_p \sim 10^8$–$10^9$ G, and from a possible new family of pulsars (the so-called “magnetars”) having $B_p \sim 10^{15}$ G.
Callaway [6] and Rice [7] considered a hard sphere gas model and showed that neutron matter becomes ferromagnetic at $k_F \approx 2.3$ fm$^{-1}$. Silverstein [8] and Østgaard [9] found that the inclusion of long range attraction significantly increased the ferromagnetic transition density (e.g., Østgaard predicted the transition to occur at $k_F \approx 4.1$ fm$^{-1}$ using a simple central potential with hard core only for the singlet spin states). Clark [10] and Pearson and Saunier [11] calculated the magnetic susceptibility for low densities ($k_F \leq 2$ fm$^{-1}$) using more realistic interactions. Pandharipande et al. [12], using the Reid soft-core potential, performed a variational calculation arriving to the conclusion that such a transition was not to be expected for $k_F \leq 5$ fm$^{-1}$. Early calculations of the magnetic susceptibility within the Brueckner theory were performed by Bäckmann and Källman [13] employing the Reid soft-core potential, and results from a correlated basis function calculation were obtained by Jackson et al. [14] with the Reid $v_6$ interaction. A different point of view was followed by Vidaurre et al. [15], who employed neutron-neutron effective interactions of the Skyrme type, finding the ferromagnetic transition at $k_F \approx 1.73 - 1.97$ fm$^{-1}$. Marcos et al. [16] have also studied the spin stability of dense neutron matter within the relativistic Dirac–Hartree–Fock approximation with an effective nucleon-meson Lagrangian, predicting the ferromagnetic transition at several times nuclear matter saturation density. On the contrary, no sign of such a transition has been found by Vidaña et al. [17], who have recently studied the properties of spin polarized neutron matter within the Brueckner–Hartree–Fock approximation employing the realistic Nijmegen II and Reid93 interactions. In connection with the problem of neutrino diffusion in dense matter, Fantoni et al. [18] have recently employed the so-called auxiliary field diffusion Monte Carlo method (AFDMC) using realistic interactions (based upon the Argonne $v_{18}$ two-body potential [19] plus Urbana IX three-body potential [20]), and have found that the magnetic susceptibility of neutron matter shows a strong reduction of about a factor of 3 with respect its Fermi gas value. They pointed out that such a reduction may have strong effects on the mean free path of a neutrino in dense matter and, therefore, it should be taken into account in the studies of supernovae and proto-neutron stars.

In all these studies neutron star matter was approximated by a pure neutron gas. Never-
theless, neutron star matter is also composed of protons, electrons, muons and other exotic constituents [21,22]. The importance of the presence of a small number of protons in neutron matter for the spin stability was pointed out by Kutschera and Wójcik [23]. These authors found that the ferromagnetic state, corresponding to completely polarized protons and weakly polarized neutrons, was energetically preferred over the nonpolarized one. Calculations performed by Bernardos et al. [24] for strongly asymmetric nuclear matter within the relativistic Dirac–Hartree–Fock approximation confirmed also that the presence of an admixture of protons favors the ferromagnetic instability of dense matter.

In this work we study the bulk and single-particle properties of spin polarized isospin asymmetric nuclear matter. Our calculations are based on the Brueckner–Hartree–Fock (BHF) approximation of the well known Brueckner-Bethe-Goldstone (BBG) theory of nuclear matter. To describe the bare nucleon-nucleon interaction we make use of the nucleon-nucleon part of the recent realistic baryon-baryon interaction (model NSC97e) constructed by the Nijmegen group [25]. We study the dependence of the single-particle potentials and the total energy per particle on the asymmetry parameter and neutron and proton spin polarizations. Further, we calculate the magnetic susceptibility for several values of the asymmetry parameter and in particular we explore the possibility of a ferromagnetic transition in the high density region relevant for neutron stars.

The paper is organized in the following way. The formalism of the BHF approximation is briefly reviewed at the beginning of Sec. II, whereas the calculation of the magnetic susceptibility is presented at the end of it. Section III is devoted to the presentation and discussion of the results obtained for the single-particle potentials, the total energy per particle and the magnetic susceptibility. Finally, a short summary and the main conclusions of this work are drawn in Sec. IV.
II. FORMALISM

Spin polarized isospin asymmetric nuclear matter is an infinite nuclear system composed of four different fermionic components: neutrons with spin up and spin down having densities \( \rho_{n\uparrow} \) and \( \rho_{n\downarrow} \) respectively, and protons with spin up and spin down with densities \( \rho_{p\uparrow} \) and \( \rho_{p\downarrow} \). The total densities for neutrons (\( \rho_n \)), protons (\( \rho_p \)), and nucleons (\( \rho \)) are given by:

\[
\rho_n = \rho_{n\uparrow} + \rho_{n\downarrow}, \quad \rho_p = \rho_{p\uparrow} + \rho_{p\downarrow}, \quad \rho = \rho_n + \rho_p. \tag{2}
\]

The isospin asymmetry of the system can be expressed by the asymmetry parameter \( \beta = (\rho_n - \rho_p)/\rho \), while the degree of spin polarization of the system is characterized by the neutron and proton spin polarizations \( S_n \) and \( S_p \), defined as

\[
S_n = \frac{\rho_{n\uparrow} - \rho_{n\downarrow}}{\rho_n}, \quad S_p = \frac{\rho_{p\uparrow} - \rho_{p\downarrow}}{\rho_p}, \tag{3}
\]

Note that the value \( S_n = S_p = 0 \) corresponds to nonpolarized (i.e., \( \rho_{n\uparrow} = \rho_{n\downarrow} \) and \( \rho_{p\uparrow} = \rho_{p\downarrow} \)) matter, whereas \( S_n = \pm 1(S_p = \pm 1) \) means that neutrons (protons) are totally polarized, i.e., all the neutron (proton) spins are aligned along the same direction.

The single component densities are related to the total density and to the isospin and spin asymmetry parameters \( \beta \), \( S_n \), and \( S_p \) via the equations:

\[
\rho_{n\uparrow} = \frac{1 + S_n}{2} \rho_n = \frac{1 + S_n}{2} \frac{1 + \beta}{\rho}, \tag{4}
\]

\[
\rho_{n\downarrow} = \frac{1 - S_n}{2} \rho_n = \frac{1 - S_n}{2} \frac{1 + \beta}{\rho}, \tag{5}
\]

\[
\rho_{p\uparrow} = \frac{1 + S_p}{2} \rho_p = \frac{1 + S_p}{2} \frac{1 - \beta}{\rho}, \tag{6}
\]

\[
\rho_{p\downarrow} = \frac{1 - S_p}{2} \rho_p = \frac{1 - S_p}{2} \frac{1 - \beta}{\rho}. \tag{7}
\]

Finally the Fermi momentum \( k_F^\tau\sigma \) is related to the corresponding partial density by \( k_F^\tau\sigma = (6\pi^2 \rho_{\tau\sigma})^{1/3} \), with \( \tau = n, p \) and \( \sigma = \uparrow, \downarrow \).
Our calculations are based on the BHF approximation of the BBG theory extended to the case in which it is assumed that nuclear matter is arbitrarily asymmetric both in the isospin and spin degrees of freedom (i.e., $\rho_{n\uparrow} \neq \rho_{n\downarrow} \neq \rho_{p\uparrow} \neq \rho_{p\downarrow}$). Therefore, our many-body scheme starts by constructing all the nucleon-nucleon $G$ matrices which describe in an effective way the interaction between two nucleons ($nn, np, pn$ and $pp$) for each one of the spin combinations ($\uparrow\uparrow$, $\uparrow\downarrow$, $\downarrow\uparrow$ and $\downarrow\downarrow$). The $G$ matrices can be obtained by solving the well known Bethe–Goldstone equation

$$\langle \vec{k}_1 \tau_1 \sigma_1; \vec{k}_2 \tau_2 \sigma_2 | G(\omega) | \vec{k}_3 \tau_3 \sigma_3; \vec{k}_4 \tau_4 \sigma_4 \rangle = \langle \vec{k}_1 \tau_1 \sigma_1; \vec{k}_2 \tau_2 \sigma_2 | v | \vec{k}_3 \tau_3 \sigma_3; \vec{k}_4 \tau_4 \sigma_4 \rangle + \sum_{ij} \langle \vec{k}_1 \tau_1 \sigma_1; \vec{k}_2 \tau_2 \sigma_2 | v | \vec{k}_i \tau_i \sigma_i; \vec{k}_j \tau_j \sigma_j \rangle Q_{\tau_i \sigma_i, \tau_j \sigma_j} |G(\omega)| \vec{k}_3 \tau_3 \sigma_3; \vec{k}_4 \tau_4 \sigma_4 \rangle ,$$

(8)

where $\tau_m$ and $\sigma_m$ indicate respectively the isospin and spin projections ($n\uparrow\downarrow, p\uparrow\downarrow$) of the two nucleons in the initial, intermediate and final states, $\vec{k}_m$ are their respective linear momenta, $v$ is the bare nucleon-nucleon interaction, $Q_{\tau_i \sigma_i, \tau_j \sigma_j}$ is the Pauli operator which allows only intermediate states compatible with the Pauli principle, and $\omega$ is the so-called starting energy. Note that the $G$ matrices are obtained from a coupled channel equation. In practice a partial wave decomposition of the Bethe–Goldstone equation is performed, the Pauli operator and the energy denominator is replaced by an angle-averaged one, and the $G$ matrices are solved using relative and center-of-mass coordinates.

The single-particle energy of a nucleon with momentum $k$ and spin projection $\sigma = \uparrow(\downarrow)$ is given by

$$E_{\tau\sigma} = \frac{\hbar^2 k^2}{2m_{\tau}} + U_{\tau\sigma}(k) ,$$

(9)

where the single-particle potential $U_{\tau\sigma}(k)$ represents the mean field “felt” by the nucleon due to its interaction with the other nucleons of the system. In the BHF approximation $U_{\tau\sigma}(k)$ is calculated through the “on-energy shell” $G$ matrix and is given by

$$U_{\tau\sigma}(k) = \sum_{\tau'=n,p} \sum_{\sigma'=\uparrow,\downarrow} U_{\tau\sigma\tau'\sigma'}(k) = \sum_{\tau'=n,p} \sum_{\sigma'=\uparrow,\downarrow} \sum_{k' \leq k_{\tau'\sigma'}} \text{Re}\langle \vec{k'} \tau' \sigma' | G(\omega = E_{\tau\sigma} + E_{\tau'\sigma'}) | \vec{k} \tau \sigma \rangle A ,$$

(10)

6
where $U_{\tau\sigma',\sigma}(k)$ is the contribution to $U_{\tau\sigma}(k)$ due to the Fermi sea of nucleons $\tau'\sigma'$. Note that a sum over the Fermi seas of neutrons and protons with spin up and down is performed and that the matrix elements are properly antisymmetrized. We note also here that the continuous prescription has been adopted for the single particle potential when solving the Bethe–Goldstone equation. As shown by the authors of refs. [26,27], the contribution to the energy per particle from three body clusters are diminished in this prescription.

Once a self-consistent solution of Eqs. (8) and (10) is obtained, the total energy per particle is easily calculated:

$$\frac{E}{A} = \frac{1}{A} \sum_{\tau=n,p} \sum_{\sigma=\uparrow,\downarrow} \sum_{k \leq k_F} \left( \frac{\hbar^2 k^2}{2m_{\tau}} + \frac{1}{2} U_{\tau\sigma}(k) \right) \equiv \frac{T}{A} + \frac{V}{A}. \quad (11)$$

This quantity is a function of $\rho_{n\uparrow}$, $\rho_{n\downarrow}$, $\rho_{p\uparrow}$ and $\rho_{p\downarrow}$ or, equivalently, of $\rho$, $\beta$, and $S_n$ and $S_p$.

The magnetic susceptibility $\chi$ of a system characterizes the response of this system to a magnetic field and gives a measure of the energy required to produce a net spin alignment in the direction of the field. In the case of nuclear matter it is defined by the $2 \times 2$ matrix

$$\frac{1}{\chi} = \begin{pmatrix} 1/\chi_{nn} & 1/\chi_{np} \\ 1/\chi_{pn} & 1/\chi_{pp} \end{pmatrix}, \quad (12)$$

where the matrix elements $1/\chi_{ij}$ are given by

$$\frac{1}{\chi_{ij}} = \frac{\partial H_i}{\partial M_j} \quad (i,j = n,p). \quad (13)$$

Here $M_j$ is the magnetization of the species $j$ per unit volume

$$M_j = \mu_j (\rho_{j\uparrow} - \rho_{j\downarrow}) = \mu_j \rho_j S_j, \quad (14)$$

with $\mu_j$ the magnetic moment of the species $j$, and $H_i$ is the magnetic field induced by the magnetization of the species $i$, which can be obtained from

$$H_i = \rho \frac{\partial (E/A)}{\partial M_i}. \quad (15)$$

Using Eqs. (14) and (15) the matrix elements $1/\chi_{ij}$ can be written as
\[
\frac{1}{\chi_{ij}} = \frac{\rho}{\mu_i \mu_j \rho_j} \left( \frac{\partial^2 (E/A)}{\partial S_i \partial S_j} \right),
\]  
(16)

where the second derivatives can be taken at \( S_i = S_j = 0 \) if the magnetic field is assumed to be small.

It is convenient to study the magnetic susceptibility of the system in terms of the ratio \( \det(1/\chi)/\det(1/\chi_F) \), where \( \chi_F \) is the magnetic susceptibility of the two component free Fermi gas. Writing the total energy per particle as a sum of the kinetic, \( T/A \), and the potential, \( V/A \), energy contributions, the ratio between both determinants can be written as

\[
\frac{\det(1/\chi)}{\det(1/\chi_F)} = \left( 1 + \frac{\partial^2(V/A)}{\partial S_n^2} \right) \left( 1 + \frac{\partial^2(V/A)}{\partial S_p^2} \right) - \left( \frac{\partial^2(V/A)}{\partial S_n \partial S_p} \right) \left( \frac{\partial^2(V/A)}{\partial S_n \partial S_p} \right). 
\]  
(17)

The stability of matter against spin fluctuations is guaranteed if \( \det(1/\chi)/\det(1/\chi_F) > 0 \), indicating a change of sign of the ratio the onset of a ferromagnetic phase in the system.

### III. RESULTS

#### A. Single-particle potentials

In Fig. 1 we show the neutron \( U_{n\uparrow}, U_{n\downarrow} \) (upper panels), and proton \( U_{p\uparrow}, U_{p\downarrow} \) (lower panels) single-particle potentials evaluated at \( \rho = 0.17 \text{ fm}^{-3} \) for several values of \( S_n, S_p \), and \( \beta \). Results for symmetric matter (\( \beta = 0 \)) are plotted on the left and middle panels, whereas results for asymmetric matter with \( \beta = 0.5 \) are reported on the right ones. Solid lines show as a reference the results for nonpolarized matter (\( S_n = S_p = 0 \)), while dotted lines refer to spin polarized matter. On the left panels we show the effects on \( U_{\tau\sigma}(k) \) of a partial polarization (\( S_n = 0.75 \)) of the neutron component in proton- unpolarized (\( S_p = 0 \)) symmetric matter. As it can be seen, the single-particle potentials of both species split off when a partial polarization of the neutron spin is assumed. The single-particle potential for neutrons with spin up (down) becomes less (more) attractive with respect to \( U_n(k) \) for nonpolarized nuclear matter. For the proton single-particle potential, we find an opposite qualitative trend being
$U_{p\uparrow} \ (U_{p\downarrow})$ more (less) attractive than the proton single-particle potential for nonpolarized matter. The additional partial proton polarization (middle panels) produces on $U_{\tau\sigma}(k)$ a reverse global effect with respect to the one produced by polarizing neutrons. This is particularly evident in the case of $U_{p\sigma}(k)$ (compare the results in the lower left and lower central panels). Finally, to obtain the results presented in the right panels of Fig. 1, we further introduce an isospin asymmetry ($\beta = 0.5$) in the system. Notice that the isospin asymmetry generates a splitting of the neutron and proton single particle potentials even in the case of spin-unpolarized matter [28].

The splitting, $U_{\tau\uparrow} - U_{\tau\downarrow}$, in the neutron and proton single-particle potentials can be mainly ascribed to two reasons: (i) the change in the number of pairs which the nucleon under consideration $|k, \tau, \sigma\rangle$ can form with the remaining nucleons $|k' \leq k_F^{\tau'\sigma'}, \tau', \sigma'\rangle$ of the system as nuclear matter is spin polarized, and (ii) the spin dependence (and isospin dependence for asymmetric spin polarized matter) of the nucleon-nucleon $G$ matrices in the spin polarized nuclear medium (see Eq. (8)).

In the general case of spin polarized asymmetric matter, the effect of the neutron and proton spin polarizations and the isospin asymmetry on the single particle potentials can be clarified by considering their partial contributions $U_{\tau\sigma\tau'\sigma'}(k)$ in such a way as to single out explicitly the dependence on their respective phase space:

$$U_{n\uparrow} = U_{n\uparrow} + U_{n\downarrow} + U_{p\uparrow} + U_{p\downarrow} = g_{n\uparrow} n_{\rho\uparrow} + g_{n\downarrow} n_{\rho\downarrow} + g_{n\uparrow} p_{\rho\uparrow} + g_{n\downarrow} p_{\rho\downarrow}, \quad (18)$$

$$U_{n\downarrow} = U_{n\uparrow} + U_{n\downarrow} + U_{p\uparrow} + U_{p\downarrow} = g_{n\uparrow} n_{\rho\downarrow} + g_{n\downarrow} n_{\rho\uparrow} + g_{n\uparrow} p_{\rho\downarrow} + g_{n\downarrow} p_{\rho\uparrow}, \quad (19)$$

$$U_{p\uparrow} = U_{p\uparrow} + U_{p\downarrow} + U_{p\uparrow} + U_{p\downarrow} = g_{p\uparrow} n_{\rho\uparrow} + g_{p\downarrow} n_{\rho\downarrow} + g_{p\uparrow} p_{\rho\uparrow} + g_{p\downarrow} p_{\rho\downarrow}, \quad (20)$$

$$U_{p\downarrow} = U_{p\uparrow} + U_{p\downarrow} + U_{p\uparrow} + U_{p\downarrow} = g_{p\uparrow} n_{\rho\downarrow} + g_{p\downarrow} n_{\rho\uparrow} + g_{p\uparrow} p_{\rho\downarrow} + g_{p\downarrow} p_{\rho\uparrow}, \quad (21)$$

where $g_{\tau\sigma\tau'\sigma'}$ is the average value of the matrix element $\langle \vec{k}\tau\sigma; \vec{k}'\tau'\sigma'|G|\vec{k}\tau\sigma; \vec{k}'\tau'\sigma'\rangle_A$ in the Fermi sphere with radius $k' \leq k_F^{\tau'\sigma'}$ and the density factor $\rho_{\tau'\sigma'}$ in each term arises from the integral over the corresponding Fermi sea.
For small values of the asymmetry parameter \(|\beta| \ll 1\) and the spin polarizations \(|S_n|, |S_p| \ll 1\) one can neglect the dependence on \(\beta, S_n\) and \(S_p\) of the average \(G\) matrices \(g_{\tau\sigma';\sigma'}\) assuming \(g_{\tau\sigma';\sigma'} \sim g_{\tau\sigma';\sigma'}(k, \rho)\) and

\[
\begin{align*}
g_{n\uparrow n\uparrow} & \approx g_{n\downarrow n\downarrow} \approx g_{p\uparrow p\uparrow} \approx g_{p\downarrow p\downarrow} \equiv g_1, \\
g_{n\uparrow n\downarrow} & \approx g_{n\downarrow n\uparrow} \approx g_{p\uparrow p\downarrow} \approx g_{p\downarrow p\uparrow} \equiv g_2, \\
g_{n\uparrow p\uparrow} & \approx g_{n\downarrow p\downarrow} \approx g_{p\uparrow n\uparrow} \approx g_{p\downarrow n\downarrow} \equiv g_3, \\
g_{n\uparrow p\downarrow} & \approx g_{n\downarrow p\uparrow} \approx g_{p\uparrow n\downarrow} \approx g_{p\downarrow n\uparrow} \equiv g_4.
\end{align*}
\]

Clearly, even under these assumptions, in the most general case \(g_1 \neq g_2 \neq g_3 \neq g_4\) because they receive contributions from different spin and isospin channels. Whereas \(g_1\) receives contributions only from the spin and isospin triplet \((S = 1, T = 1)\) channels, \(g_2\) has in addition contributions from the spin singlet ones, \(g_3\) from channels with \(S = 1\), and \(g_4\) from channels with \(S = 0\) and \(T = 0,1\).

Using Eq. (22) the single-particle potentials can then be rewritten in terms of the average \(G\) matrices \(g_{\tau\sigma';\sigma'}\) as:

\[
\begin{align*}
U_{n\uparrow} & \approx \frac{\rho}{4} \left[ (g_1 + g_2 + g_3 + g_4) + (g_1 + g_2 - g_3 - g_4)\beta \\ & + (g_1 - g_2)(1 + \beta)S_n + (g_3 - g_4)(1 - \beta)S_p \right], \\
U_{n\downarrow} & \approx \frac{\rho}{4} \left[ (g_1 + g_2 + g_3 + g_4) - (g_1 + g_2 - g_3 - g_4)\beta \\ & - (g_1 - g_2)(1 + \beta)S_n - (g_3 - g_4)(1 - \beta)S_p \right], \\
U_{p\uparrow} & \approx \frac{\rho}{4} \left[ (g_1 + g_2 + g_3 + g_4) - (g_1 + g_2 - g_3 - g_4)\beta \\ & + (g_3 - g_4)(1 + \beta)S_n + (g_1 - g_2)(1 - \beta)S_p \right], \\
U_{p\downarrow} & \approx \frac{\rho}{4} \left[ (g_1 + g_2 + g_3 + g_4) - (g_1 + g_2 - g_3 - g_4)\beta \\ & - (g_3 - g_4)(1 + \beta)S_n - (g_1 - g_2)(1 - \beta)S_p \right],
\end{align*}
\]

where Eqs. (4), (5) (6) and (7) have been used to write \(\rho_{\tau\sigma}\) in terms of \(\rho, \beta, S_n\) and \(S_p\).

These equations show explicitly the dependence of the single-particle potentials on the spin...
polarizations and isospin asymmetry. This dependence is tested in Fig. 2, where the value at $k = 0$ of the single-particle potentials $U_{n\uparrow}, U_{n\downarrow}, U_{p\uparrow}$ and $U_{p\downarrow}$ at $\rho = 0.17 \text{ fm}^{-3}$ is plotted as a function of neutron (left panels) and proton (right panels) spin polarizations for two values of the asymmetry parameter: $\beta = 0$ (upper panels) and $\beta = 0.5$ (lower panels). In order to make more clear the discussion $S_n(S_p)$ is taken equal to 0 on the right (left) panels. The above equations predict a linear and symmetric variation of the single-particle potentials on $\beta, S_n$ and $S_p$. This prediction is well confirmed from the microscopic results reported on Fig. 2, although deviations from this behavior are found at higher values of the asymmetry and spin polarizations. These deviations have to be associated to the dependence on $\beta, S_n$ and $S_p$ of the average $G$ matrices $g_{\tau\sigma\tau'\sigma'}$ which has been neglected in the present analysis (see Eq. (22)). From the above expressions it can be seen that under charge exchange (i.e., doing the changes $\beta \rightarrow -\beta, S_n \rightarrow S_p, S_p \rightarrow S_n$) the role of neutrons and protons with the same spin projection is exactly interchanged. This can be clearly seen by comparing the left and right top panels of the figure, which correspond to the particular case of symmetric matter. It is also clear that neutrons (protons) with spin up and down interchange their roles when a global flip of the spins is performed (i.e., changing $S_n$ by $-S_n$ and $S_p$ by $-S_p$).

**B. Energy per particle**

The total energy per particle of neutron, asymmetric, and symmetric matter is shown on the left, middle and right panels of Fig. 3, respectively. In the three panels solid lines show results for nonpolarized matter, whereas those for totally polarized matter are reported by dotted lines. Note that in totally polarized asymmetric and symmetric matter, we have distinguished two possible orientations of the neutron and proton spins: that in which neutron and proton spins are aligned along the same direction (i.e., $S_n = S_p = \pm 1$), and that in which neutron and proton spins are orientated along opposite directions (i.e., $S_n = \pm 1, S_p = \mp 1$). As can be seen from the figure, totally polarized matter is always more repulsive than nonpolarized matter in all the density range explored for any value of the
asymmetry parameter. Note also that the case in which neutron and proton spins are parallelly orientated is less repulsive than the case in which they have an antiparallel orientation.

To highlight the effects of the nuclear interaction on the variation of $E/A$ as nuclear matter is polarized, we plot in Fig. 4 the kinetic (upper panels) and potential (lower panels) energy per particle for nonpolarized and totally polarized matter. The kinetic energy contribution of totally polarized neutron matter, asymmetric or symmetric matter is always larger than the corresponding one of nonpolarized matter, simply due to the fermionic nature of nucleons. Also, the potential energy contribution is always more repulsive in the totally polarized case. This can be understood by considering separately the contribution to the potential energy per particle of the spin singlet and spin triplet channels. In Tables I and II we show these contributions for nonpolarized and totally polarized neutron and symmetric matter at densities $\rho = 0.17 \text{ fm}^{-3}$ and $\rho = 0.4 \text{ fm}^{-3}$, respectively. Note, firstly, that an important amount of binding is lost in totally polarized neutron matter and in totally polarized symmetric matter with neutron and proton spins parallelly orientated due to the absence in these cases of the contribution of the spin singlet channels. Note also, in the totally polarized symmetric case, that when the orientation of neutron and proton spins is antiparallel this contribution, although it is present, is much less attractive or even repulsive (see Table II) than the corresponding one of nonpolarized symmetric matter. In addition, in all cases the contribution from spin triplet channels in totally polarized matter is always less attractive (or even repulsive at high density) than the corresponding one to nonpolarized matter. In particular, it is mainly this contribution the one which explains the difference between the energies of totally polarized symmetric matter with a parallel and an antiparallel orientation of neutron and proton spins, being in the antiparallel case less attractive or more repulsive.

An interesting conclusion which can be inferred from our microscopic calculations is that a spontaneous transition to a spin polarized state, i.e., to a so-called ferromagnetic state, of nuclear matter is not to be expected for all the possible isospin asymmetries ranging from symmetric to pure neutron matter. If such a transition would exist, a crossing of the energies of the totally polarized and nonpolarized cases would be observed in neutron, asymmetric
or symmetric matter at some density, indicating that the ground state of the system would
be ferromagnetic from that density on. As can be seen in Fig. 3, there is no sign of such a
crossing and, on the contrary, a spin polarized state becomes less favorable as the density
increases.

It would be interesting to characterize in a simple analytic form the dependence of the
total energy per particle on the asymmetry and spin polarizations. The kinetic energy
contribution is already analytic and well known. It is given by

$$\frac{T}{A}(\rho, \beta, S_n, S_p) = \frac{3}{8} \frac{\hbar^2 k_F^2}{2m} \frac{1}{4} \left[ (1 + \beta)^{5/3}(1 + S_n)^{5/3} + (1 + \beta)^{5/3}(1 - S_n)^{5/3} \
+ (1 - \beta)^{5/3}(1 + S_p)^{5/3} + (1 - \beta)^{5/3}(1 - S_p)^{5/3} \right],$$

being $k_F = (3\pi^2 \rho/2)^{1/3}$ the Fermi momentum of nonpolarized isospin symmetric matter. An
idea of the possible terms appearing in the potential energy contribution can be extracted
from our previous phase space analysis of the single-particle potentials. We start from the
BHF approximation of the potential energy per particle $V/A$ defined through Eq. (11) and
making use of Eqs. (18)–(21) for the single particle potential $U_{\tau\sigma}(k)$, we can write

$$\frac{V}{A} \approx \frac{1}{2A} \sum_{\tau,\sigma} \sum_{\tau',\sigma'} \sum_{k \leq k_F} g_{\tau\sigma\tau'\sigma'}(k, \rho) \rho_{\tau'\sigma'}.$$

Next we use Eq. (22) and perform the average of the quantities $g_i(k, \rho)$ over the correspond-
ing Fermi sphere with radius $k_F$. Finally, we arrive, after some little algebra, to

$$\frac{V}{A} \approx \frac{\rho^2}{8} [\bar{g}_1 + \bar{g}_2 + \bar{g}_3 + \bar{g}_4] + \frac{\rho^2}{4} [\bar{g}_1 + \bar{g}_2 - \bar{g}_3 - \bar{g}_4] [1 + \beta]^2 S_n^2 \
+ \frac{\rho^2}{8} [\bar{g}_1 - \bar{g}_2] (1 - \beta)^2 S_p^2 + \frac{\rho^2}{4} [\bar{g}_3 - \bar{g}_4] (1 - \beta^2) S_n S_p,$$

where $\bar{g}_i$ are just the averages values of the quantities $g_i$. Following this simple analysis we
can finally infer the form of the total energy per particle

$$\frac{E}{A}(\rho, \beta, S_n, S_p) = \frac{T}{A}(\rho, \beta, S_n, S_p) + V_0(\rho) + V_1(\rho)\beta^2 + V_2(\rho)(1 + \beta)^2 S_n^2 \
+ V_2(\rho)(1 - \beta)^2 S_p^2 + V_3(\rho)(1 - \beta^2) S_n S_p.$$

This parametrization is consistent with the spin and isospin structure of the nucleon-
nucleon interaction, in the sense that if we consider a particular configuration, $\beta, S_n, S_p,$ of
the system, a global flip of the spins does not change the energy, whereas a flip only of all neutron or protons spins does. It is also clear that due to charge symmetry, a system with a configuration, $\beta' = -\beta, S_n' = S_p, S_p' = S_n$, will have the same energy as the original one (note that the coefficients of the terms $(1 + \beta)^2 S_n^2$ and $(1 - \beta)^2 S_p^2$ are the same).

The coefficients $V_0(\rho), V_1(\rho), V_2(\rho)$ and $V_3(\rho)$, whose density dependence is shown in Fig. 5, have been determined in the following way

\begin{align*}
V_0(\rho) &= \frac{V}{A}(\rho, \beta = 0, S_n = 0, S_p = 0) , \\
V_1(\rho) &= \frac{V}{A}(\rho, \beta = 1, S_n = 0, S_p = 0) - V_0(\rho) , \\
V_2(\rho) &= \frac{V}{A}(\rho, \beta = 0, S_n = 1, S_p = 0) - V_0(\rho) , \\
V_3(\rho) &= \frac{V}{A}(\rho, \beta = 0, S_n = 1, S_p = 1) - V_0(\rho) - 2V_2(\rho) .
\end{align*}

It is clear, however, that their determination is not unique. Choosing them in this way, we get a parametrization which reproduces with a good quality (see Figs. 6 and 7 and the discussion below) the results of the BHF calculation of the total energy per particle for values of $\beta, S_n$ and $S_p$ around their values for nonpolarized symmetric matter (i.e., $\beta = 0, S_n = 0, S_p = 0$).

Note that for nonpolarized matter this parametrization reduces to

\[ \frac{E}{A}(\rho, \beta, 0, 0) = \frac{T}{A}(\rho, \beta, 0, 0) + V_0(\rho) + V_1(\rho)\beta^2 , \]

which can be identified with the usual parabolic approach of the nuclear matter energy per particle if the kinetic energy contribution is expanded up to order $\beta^2$.

In order to test the quality of this parametrization, we show in Fig. 6 the total energy per particle at $\rho = 0.8$ fm$^{-3}$ as a function of the proton spin polarization for different values of the neutron spin polarization and two values of the asymmetry parameter: $\beta = 0$ (right panel) and $\beta = 0.25$ (left panel). Circles, squares, diamonds and triangles show the results obtained from the BHF calculation, whereas those obtained from the parametrization are reported by
solid lines. As can be seen from the figure, the dependence on the spin polarizations and the asymmetry parameter predicted by Eq. (30) is well confirmed from the microscopic results. It is interesting to note that for a fixed value of $\beta$ and $S_n$ (with $S_n \neq 0$), the minimum of the energy happens for a value of $S_p \neq 0$. However, this is not an indication of a phase transition to a ferromagnetic state, because the real ground state of the system, as we have seen, is that of nonpolarized matter in all the range of densities and isospin asymmetries considered. For completeness, we compare in Fig. 7 the results for the total energy per particle as a function of density for three arbitrarily asymmetric and spin polarized situations: $\beta = 0.25, S_n = 0.3, S_p = 0.4$, $\beta = 0.5, S_n = 0.5, S_p = 0.25$, and $\beta = 0.8, S_n = 0.6, S_p = 0.2$, obtained from the BHF calculation and from the parametrization. As in the previous figure, symbols show results obtained from the BHF calculation, while solid lines refer to those obtained with the parametrization. The quality of the parametrization is quite good, as can be seen in both figures, with deviations from the microscopic calculation of at most $7 - 8\%$ only for the combinations of $\beta, S_n$ and $S_p$ with the largest values. Higher deviations are found, however, at higher values of these parameters, being this an indication that the coefficients need to be refitted when considering matter with $|\beta| \sim 1, |S_n| \sim 1$ and $|S_p| \sim 1$.

C. Magnetic susceptibility

The ratio $\det(1/\chi)/\det(1/\chi_F)$ can be evaluated in a very simple analytic way from Eq. (17) if the parametrization of Eq. (30) is assumed, giving

$$
\frac{\det(1/\chi)}{\det(1/\chi_F)} = 1 + \frac{12mV_2(\rho)}{\hbar^2 k_F^2}[(1 + \beta)^{1/3} + (1 - \beta)^{1/3}] + \frac{36m^2}{\hbar^4 k_F^4}[4V_2^2(\rho) - V_3^2(\rho)](1 - \beta^2)^{1/3},
$$

(36)

with $m$ the average mass of the nucleons. This ratio is shown in Fig. 8 as a function of the density for several values of the asymmetry parameter $\beta$ from neutron to symmetric matter. Note that an important increase of the ratio happens as soon as a small fraction of protons is introduced in the system. This can be understood by looking at Eq. (36) and Fig. 5 from
which it is clear that the ratio \( \det(1/\chi)/\det(1/\chi_F) \) increases as the asymmetry parameter \( \beta \) decreases from 1 to 0. According to the criteria for the appearance of a ferromagnetic instability, such an instability should appear when \( \det(1/\chi)/\det(1/\chi_F) = 0 \). Nevertheless, it can be seen from the figure that \( \det(1/\chi)/\det(1/\chi_F) \) increases monotonously with density, and a decrease is not to be expected even at higher densities for any value of the asymmetry parameter. Therefore, it can be inferred from these results that there is no sign at any density of a possible ferromagnetic phase transition for any asymmetry of nuclear matter.

As we said previously, the coefficients of the parametrization should be refitted when large values of the asymmetry parameter and the spin polarizations are considered. In particular, for the case of pure neutron matter it is more convenient to redefine coefficient \( V_2(\rho) \) in the following way

\[
V_2(\rho) = \frac{1}{4} \left( V_A(\rho, \beta = 1, S_n = 1, S_p = 0) - V_A(\rho, \beta = 1, S_n = 0, S_p = 0) \right), \tag{37}
\]

in order to get a better value of the magnetic susceptibility. We show in the figure by open circles the resulting neutron magnetic susceptibility obtained by refitting coefficient \( V_2(\rho) \) according to Eq. (37). We find a good agreement with the results of ref. [17] where this coefficient is fitted in this way. For comparison, we show also the results for neutron matter obtained recently by Fantoni et al. [18] (filled circles). We note here, that our results confirm the reduction of about a factor 3 of the magnetic susceptibility of neutron matter with respect to its Fermi gas value found by these authors, being the differences ascribed to the method (AFDMC) and the potential (AU6’ two-body + UIX three-body) employed by them. It seems, therefore, that this reduction is largely independent of the two-body modern potential used, and, in addition, that the effects of the three-nucleon interaction on the magnetic susceptibility are not large.

IV. SUMMARY AND CONCLUSIONS

Employing a realistic modern nucleon-nucleon interaction (NSC97e) we have studied properties of spin polarized isospin asymmetric nuclear matter within the Brueckner–
Hartree–Fock approximation. We have determined the single particle potentials of neutrons and protons with spin up and down for several values of the neutron and proton spin polarizations and the asymmetry parameter. We have found that the potentials exhibit an almost linear and symmetric variation as a function of these parameters. Deviations from this behaviour occur at higher values of the asymmetry parameter and spin polarizations. These deviations have to be attributed to the dependence of the nucleon-nucleon $G$ matrices on $\beta, S_n$ and $S_p$.

We have calculated the total energy per particle as a function of the density for totally polarized and nonpolarized neutron, asymmetric and symmetric matter. We have found that in the range of densities explored (up to $7\rho_0$) totally polarized matter is always more repulsive than nonpolarized matter for any asymmetry. This is due to a combination effect of the kinetic and potential energy contributions. We have also found in the totally polarized case that asymmetric and symmetric matter is more repulsive when neutron and proton spins are antiparallelly orientated than when all the spins are aligned along the same direction.

We have constructed an analytic parametrization of the total energy per particle as a function of the asymmetry parameter and spin polarizations. The quality of this parametrization has been tested, finding deviations from the microscopic calculation of at most $7-8\%$. Nevertheless, deviations are higher for higher values of the asymmetry parameter and spin polarization, and the coefficients of the parametrization need to be refitted when large values of the spin and isospin asymmetries are considered.

Employing this parametrization we have determined the magnetic susceptibility of nuclear matter for several values of the asymmetry from neutron to symmetric matter in terms of the ratio $\det(1/\chi)/\det(1/\chi_F)$. We have found that this quantity increases monotonously with density, from which it can be inferred that a phase transition to a ferromagnetic state is not to be expected in nuclear matter at any density for any asymmetry.

Finally, our results confirm the reduction of about a factor 3 of the magnetic susceptibility of neutron matter with respect to its Fermi gas value found recently by Fantoni et al. [18].
ACKNOWLEDGEMENTS

The authors are very grateful to professors A. Fabrocini, A. Polls, A. Ramos and S. Rosati for useful discussions and comments. One of the authors (I.V.) wishes to acknowledge support from the Istituto Nazionale di Fisica Nucleare (Italy).
REFERENCES


TABLE I. Contribution of the spin singlet and triplet channels to the potential energy per particle of neutron and symmetric matter at $\rho = 0.17 \text{ fm}^{-3}$. Second and third columns show results for nonpolarized and totally polarized neutron matter, respectively, whereas those for nonpolarized and totally polarized symmetric matter are reported on the fourth, fifth and sixth ones. Partial waves have been included up to total angular momentum $J = 4$.

<table>
<thead>
<tr>
<th></th>
<th>Neutron matter</th>
<th>Symmetric matter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_n = 0$</td>
<td>$S_n = S_p = 0$</td>
</tr>
<tr>
<td>$S = 0$</td>
<td>$-21.970$</td>
<td>$-14.414$</td>
</tr>
<tr>
<td>$S = 1$</td>
<td>$-0.247$</td>
<td>$9.960$</td>
</tr>
</tbody>
</table>

TABLE II. As in Table I but for $\rho = 0.4 \text{ fm}^{-3}$.

<table>
<thead>
<tr>
<th></th>
<th>Neutron matter</th>
<th>Symmetric matter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_n = 0$</td>
<td>$S_n = S_p = 0$</td>
</tr>
<tr>
<td>$S = 0$</td>
<td>$-37.222$</td>
<td>$-22.619$</td>
</tr>
<tr>
<td>$S = 1$</td>
<td>$10.090$</td>
<td>$48.173$</td>
</tr>
<tr>
<td>Total</td>
<td>$-27.132$</td>
<td>$55.721$</td>
</tr>
</tbody>
</table>
FIGURES

FIG. 1. Neutron ($n^\uparrow,n^\downarrow$) and proton ($p^\uparrow,p^\downarrow$) single-particle potentials as a function of the linear momentum $k$ at $\rho = 0.17$ fm$^{-3}$ for several values of $\beta$, $S_n$ and $S_p$. Left and middle panels show results for symmetric matter, whereas those for asymmetric matter are reported on the right one. Solid lines show as a reference the results for the nonpolarized case, while dotted ones refer to the spin polarized cases.

FIG. 2. Neutron ($n^\uparrow$ and $n^\downarrow$) and proton ($p^\uparrow$ and $p^\downarrow$) single-particle potentials at $k = 0$ and $\rho = 0.17$ fm$^{-3}$ as a function of neutron (left panels) and proton (right panels) spin polarizations. Upper (lower) panels show results for symmetric (asymmetric) matter. $S_n$ ($S_p$) is taken equal to $0$ on the right (left) panels.

FIG. 3. Total energy per particle as a function of the density for nonpolarized (solid lines) and totally polarized (dotted lines) neutron (left panel), asymmetric (middle panel) and symmetric (right panel) matter.

FIG. 4. Kinetic (upper panels) and potential (lower panels) energy contributions to the total energy per particle as a function of the density for nonpolarized polarized (solid lines) and totally polarized (dotted lines) neutron (left panel), asymmetric (middle panel) and symmetric (right panel) matter.

FIG. 5. Density dependence of the coefficients of the parametrization defined in Eq. (30).

FIG. 6. Total energy per particle at $\rho = 0.8$ fm$^{-3}$ as a function of the proton spin polarization for different values of the neutron spin polarization and two values of the asymmetry parameter: $\beta = 0$ (right panel) and $\beta = 0.25$ (right panel). Circles, squares, diamonds and triangles show the results of the BHF calculation, whereas solid lines refer to the parametrization defined in Eq. (30).
FIG. 7. Total energy per particle as a function of the density for three arbitrarily spin polarized asymmetric situations: $\beta = 0.25, S_n = 0.3, S_p = 0.4$, $\beta = 0.5, S_n = 0.5, S_p = 0.25$, and $\beta = 0.8, S_n = 0.6, S_p = 0.2$. As in Fig. 6, symbols show the BHF results, whereas solid lines correspond the parametrization defined in Eq. (30).

FIG. 8. Ratio between the determinants of the matrices $1/\chi$ and $1/\chi_F$ as a function of the density for several values of the asymmetry parameter $\beta$ from neutron to symmetric matter.