Quantum Instability for Mixed States

Marek Nowakowski
Instituto de Fisica, Universidad de Guanajuato, Apdo Postal E-143, Leon, Guanajuato, Mexico

Abstract

The analysis of the time evolution of unstable states which are linear superposition of other, observable, states can, in principle, be carried out in two distinct, non-equivalent ways. One of the methods, usually employed for the neutral kaon system, combines the mixing and instability into one single step which then results into unconventional properties of the mass-eigenstates. An alternative method is to remain within the framework of a Lagrangian formalism and to perform the mixing prior to the instability analysis. Staying close to the $K^0 - \bar{K}^0$ system, we compare both methods pointing out some of their shortcomings and advantages.
1 Introduction

Consider a single unstable states $|\lambda\rangle$ characterized by some quantum numbers denoted by $\lambda$. Although not an eigenstate to the full Hamiltonian, the state has a definite mass. Viewing this state as an open quantum system, its time evolution can be generally given as

$$|\lambda(\tau)\rangle = p_\lambda(\tau)|\lambda\rangle$$

(1.1)

where the effective Hamiltonian $H_{\text{eff}}$ governing this system is non-hermitian and in the Weisskopf-Wigner approximation [1] one recovers the exponential decay law $p_\lambda(\tau) = e^{-iE_\tau\tau}e^{-\Gamma/2\tau}$. As long as we restrict ourselves to a single particle state $|\lambda\rangle$, there are no known inconsistencies in performing this analysis. The state $|\lambda\rangle$ can be a mass-eigenstate defined, say, through mixing in a Lagrangian framework. Suppose now that we combine this mixing and the instability analysis into one single equation. With a two level quantum system, the non-hermitian Hamiltonian of the single particle case becomes a non-hermitian effective $2 \times 2$ mass matrix $M_{\text{eff}}$. Obviously, the diagonalization of $M_{\text{eff}}$ is generally not given by a unitary transformation and as a result the norm is not automatically preserved. The two emerging mass eigenstates, say $|\lambda_1,2\rangle$, can turn out to be non-orthogonal i.e. $\langle \lambda_1|\lambda_2\rangle \neq 0$. This non-orthogonality has rather profound consequences: no suitable definition of the anti-particle states is known, the time development of $|\lambda_1\rangle$ and $|\lambda_2\rangle$ are generally inter-connected (that is to say that the times evolution of one of this states seems to ‘know’ about the other) and, maybe less certain, EPR-like paradoxes can be encountered in the system. A more detailed account of these peculiarities will be given in section 3. Let us now assume that the mixing is well defined within a Lagrangian. Performing only unitary transformations we can define two orthogonal mass eigenstates and proceed to analyze their time behaviour for each one of them separately. Since the situation of mixing and instability is often encountered in physics, the rather fundamental question arises, which one of the possibilities we should follow. Obviously, the outcomes will be different. The first method is followed in the neutral kaon system [2, 3] (and related mesonic systems) whereas for massive unstable neutrinos the mixing is done within the Langrangian framework [4] and we opt here for the second possibility. Now the formal difference is that for the weak, CP-violating, kaon interaction no effective Langrangian description is known (this does not mean that such description is in general not possible and indeed we present below a Langrangian which, at least, theoretically, contains the basic features of the neutral kaon system) whereas the neutrinos are part of the fundamental extended Standard Model Langrangian. On the other hand, there exist no argument which would restrict the applicability of the first method only to kaons or more generally to composite objects. Indeed, regardless whether the particles are fundamental or composite, this method can be applied to any unstable two level system, in particular to neutrinos. Since the question we pose here touches upon a basic principle, we should be able to decide which one is physically correct. If we find that the first method is the preferable one, then it should be applied to
any similar system even if this system has been initially formulated by means of a Lagrangian. On the other hand if we find this method erroneous, we should seek for better theoretical tools which could amount to introduce improvements or to abandon it and replace it e.g. by a suitable effective Lagrangian.

In this paper we will compare these two approaches without propounding one in favor of the other. In doing so, we will stay as close as possible to the $K^0 - \bar{K}^0$ system. Although the Lagrangian presented below captures many features of the neutral kaon system, this will not be the main issue of the paper. As indicated above, no condition on the elementarity or compositeness of the involved states enters the Lee-Oehme-Yang (LOY) theory \[2\] which is the theory which combines mixing of $K^0 - \bar{K}^0$ with the instability of these states. That is to say that this theory goes formally through for composite as well as fundamental particles. Therefore, to emphasize the main conclusions of the paper we will mostly treat the states as elementary. Nevertheless a comparison to the $K^0 - \bar{K}^0$ system seems legitimate.

2 CP-violation through mixing in a Lagrangian

To facilitate the understanding of the main problem, we start with a simple Lagrangian involving two complex scalar fields $\phi$ and $\chi$, one scalar neutral field $\varphi$ and possibly other fields $\eta_i$ which we do not need to specify further. We have

$$\mathcal{L} = \mathcal{L}^\text{kin}_\varphi + \mathcal{L}^\text{kin}_\phi + \mathcal{L}^\text{kin}_\chi + \mathcal{L}^\text{int}_{\phi\varphi\chi} + \mathcal{L}^\text{mix}_{\text{CP-violat.}} + \mathcal{L}^\text{int}_{\phi\varphi\chi\eta_i}$$ \hspace{1cm} (2.1)

where $\mathcal{L}^\text{kin}$ are the usual kinetic terms of the same form as

$$\mathcal{L}^\text{kin}_\phi = (\partial_\mu \phi)(\partial^\mu \phi^*) - m^2 \phi \phi^*$$ \hspace{1cm} (2.2)

We choose for the interaction between $\phi$, $\varphi$ and $\chi$ the simple expression

$$\mathcal{L}^\text{int}_{\phi\varphi\chi} = \frac{\lambda_{00}}{\sqrt{2}} \varphi^2 + \frac{\lambda_{+0}}{\sqrt{2}} \varphi \chi^* - \frac{\lambda_{000}}{\sqrt{2}} \varphi^3 - \frac{\lambda_{+0}}{\sqrt{2}} \chi^* \varphi + h.c. \hspace{1cm} (2.3)$$

where $\lambda_{00}$, $\lambda_{+0}$, $\lambda_{000}$ and $\lambda_{+0}$ are real coupling constants. We demand now that $\mathcal{L}^\text{int}_{\phi\varphi\chi\eta_i}$ be by itself CP-invariant such that $\varphi$ has the CP quantum numbers of a pseudoscalar. Furthermore, we impose on $\mathcal{L}^\text{int}_{\phi\varphi\chi\eta_i}$ a global $U(1)$ symmetry such that among $\phi$, $\varphi$ and $\chi$ only $\phi$ has a non-zero charge with respect to this global $U(1)$ symmetry. Then $\mathcal{L}^\text{int}_{\phi\varphi\chi}$ breaks this symmetry and even with $\mathcal{L}^\text{mix}_{\text{CP-violat.}}$ being zero we cannot identify the $\phi$ and $\phi^*$ as mass-eigenstates. Writing therefore $\phi = (\phi_1 + i\phi_2)/\sqrt{2}$ where $\phi_{1,2}$ are now the proper mass eigenstates (no mixing yet) we obtain easily

$$\mathcal{L}^\text{int}_{\phi\varphi\chi} = \lambda_{00} \phi_1 \varphi^2 + \lambda_{+0} \phi_1 \chi^* + \lambda_{000} \phi_2 \varphi^3 + \lambda_{+0} \phi_2 \chi^* \varphi$$ \hspace{1cm} (2.4)
The Lagrangian

$$\mathcal{L}^{(1)} = \mathcal{L} - \mathcal{L}_{\text{mix}}^{\text{CP-violat.}}$$

in which the mass-eigenstates $\phi_1$ and $\phi_2$ are defined, describes the interaction of four mass-eigenstates fields: the neutral pseudoscalar (CP-odd) fields $\varphi$, the charged field $\chi$ and the scalar (CP-even) $\phi_1$ as well as the pseudoscalar $\phi_2$. Obviously, since we were able to assign CP quantum numbers to all fields $\mathcal{L}^{(1)}$ is still invariant under CP-transformations. How can we break the CP-invariance through mixing of $\phi_1$ and $\phi_2$ [5]? To this end we introduce

$$\mathcal{L}_{\text{mix}}^{\text{CP-violat.}} = -\mu^2 \phi \phi - \mu^* \phi^* \phi^*$$

$$= -\sqrt{2} \text{Re} \mu^2 \phi_1^2 + \sqrt{2} \text{Re} \mu^2 \phi_2^2 - \sqrt{2} \text{Im} \mu^2 \phi_1 \phi_2$$

(2.6)

with a complex parameter $\mu^2$. With $\text{Im} \mu^2 \neq 0$, the fields $\phi_1$ and $\phi_2$, previously carrying the quantum numbers of scalar and pseudoscalar, respectively, will mix which evidently leads to CP-violation. With $\text{Im} \mu^2 = 0$, there is no CP-violation, but the mixing remains. Hence, $\phi_1$ and $\phi_2$ are no longer mass-eigenstates. As a side remark, we note that $\mathcal{L}_{\text{mix}}^{\text{CP-violat.}}$ can be understood as describing the transition $\phi \leftrightarrow \phi^*$. With a little bit effort, one could obtain at higher orders such a transition from $\mathcal{L}^{(1)}$ in (2.5) since we have $\phi \leftrightarrow \chi \chi^* \leftrightarrow \phi^*$. However, because $\mathcal{L}^{(1)}$ respects the CP-symmetry (at least with all coupling constants real), the contributions would be also CP-invariant.

The mixing due to (2.6) is given by

$$-\frac{1}{2} \left( \begin{array}{c} \phi_1 \\ \phi_2 \end{array} \right)^T \left( \begin{array}{cc} m^2 + 2 \text{Re} \mu^2 & -2 \text{Im} \mu^2 \\ -2 \text{Im} \mu^2 & m^2 - 2 \text{Re} \mu^2 \end{array} \right) \left( \begin{array}{c} \phi_1 \\ \phi_2 \end{array} \right)$$

(2.7)

which after diagonalization leads to two new mass-eigenstates, $\Lambda_1$ and $\Lambda_2$

$$\begin{align*}
\Lambda_1 &= \frac{1}{\sqrt{2}} e^{i\theta} [\phi + e^{-2i\theta} \phi^*] \\
&= \cos \theta \phi_1 - \sin \theta \phi_2 \\
\Lambda_2 &= \frac{1}{\sqrt{2}} e^{i\theta} [\phi - e^{-2i\theta} \phi^*] \\
&= \sin \theta \phi_1 + \cos \theta \phi_2
\end{align*}$$

(2.8)

with masses given by

$$\begin{align*}
m_{\Lambda_1}^2 &= m^2 + 2 |\mu^2| \\
m_{\Lambda_2}^2 &= m^2 - 2 |\mu^2|
\end{align*}$$

(2.9)

The mixing angle $\theta$ is defined through

$$\tan 2\theta = \frac{\text{Im} \mu^2}{\text{Re} \mu^2}$$

(2.10)
In terms of the new mass-eigenstates $\Lambda_{1,2}$ the interaction Lagrangian $L^{\text{int}}_{\phi\phi\chi}$ reads

$$L^{\text{int}}_{\phi\phi\chi} = \lambda_{00}(\cos \theta \Lambda_1 + \sin \theta \Lambda_2)\varphi^2$$
$$+ \lambda_{+-}(\cos \theta \Lambda_1 + \sin \theta \Lambda_2)\chi\chi^*$$
$$+ \lambda_{000}(-\sin \theta \Lambda_1 + \cos \theta \Lambda_2)\varphi^3$$
$$+ \lambda_{+-0}(-\sin \theta \Lambda_1 + \cos \theta \Lambda_2)\chi\chi^*\varphi$$

(2.11)

Performing once again a CP-transformation, but now on mass-eigenstates, we see that the CP-symmetry is broken as long as $\sin \theta \neq 0$ i.e. $\Im m \mu^2 \neq 0$. With CP-violation and provided that $m_{\Lambda_{1,2}}$ are bigger than $2m_\varphi$ and $m_\varphi + 2m_\chi$, both states $\Lambda_1$ and $\Lambda_2$ will simultaneously decay into two, i.e. $\varphi \varphi$ and $\chi\chi^*$, and three, i.e. $\varphi \varphi \varphi$ and $\chi\chi^*\varphi$, spinless states.

There are two aspects of the Lagrangian presented in equations (2.1)-(2.11). We postpone the discussion of the more important aspect to the section 3 after having briefly discussed in section 4 the Lee-Oehme-Yang theory.

The Lagrangian (2.1) captures, at least theoretically, all important features of the $K^0 - \bar{K}^0$ system and is applicable to fundamental as well as composite fields (in the latter case we should replace the coupling constants by form factors, but the essential conclusions would remain unchanged). Indeed, we can identify $\phi$ and $\phi^*$ with $K^0$ and $\bar{K}^0$, respectively and the mass $m$ with the kaon mass parameter usually denoted by $m_K$. The field $\varphi$ represents then the neutral pion and $\chi$ the charged pion field. The other fields $\eta_i$ in $L^{\text{int}}_{\phi\phi\chi\eta}$ stand, of course for more hadronic states. Hence $L^{\text{int}}_{\phi\phi\chi\eta_i}$ is the strong interaction part which conserves CP and strangness, our imposed $U(1)$ symmetry. The latter is broken by weak interaction i.e. by $L^{\text{int}}_{\phi\phi\chi}$. If $\Im m \mu^2 = 0$ i.e. there is no CP-violation in the system, $\Lambda_1$ and $\Lambda_2$ have the quantum numbers of scalar and pseudoscalar and as such can be identified with $K_1$ and $K_2$ which do not decay simultaneously into two and three pions. If $\Im m \mu^2 \neq 0$, the CP-violation makes, however, both decays possible. A direct CP-violation can be also switched on by making the coupling constants in $L^{\text{int}}_{\phi\phi\chi}$ complex. Anticipating our later discussion, we point out two essential features of the Lagrangian (2.1). Unlike the $K^0 - \bar{K}^0$ system described in the LOY-theory, the mass-eigenstates in the Lagrangian remain orthogonal and this in spite of CP-violation. Secondly, $\Lambda_1$ ($\Lambda_2$) has a well-defined anti-particle, namely it is anti-particle to itself.

The above is only a comparison. It would be too early to take (2.1) as a viable phenomenological description of the $K^0 - \bar{K}^0$ system. One reason is certainly that since this proposal is new, we do not know how to make a connection between the phenomenological Lagrangian (2.1) and the more fundamental Lagrangian of the Standard Model at quark level. This does not mean that such a connection cannot be established in future. If theoretically such a connection turns out to be possible, it could still be that (2.1) cannot reproduce all experimental facts from the $K^0 - \bar{K}^0$ system in which case we are left only with the choice of the LOY-theory. Secondly, to investigate the semi-leptonic decay channels with (2.1), we would have to add to (2.1)
a $W - \Lambda_{1,2} - \varphi(\chi)$ interaction which as such is actually not a problem. However, at tree level we would not see any CP-violation in the semi-leptonic decays and to obtain a non-zero semi-leptonic CP-asymmetry higher order corrections would be necessary. Since this superficial identification of (2.1) with the neutral kaon system is not the main point of the paper, we will not dwell on it further here. We end this section by once again pointing out that the analysis of the time evolution of $\Lambda_{1,2}$ can be now done for each individual mass-eigenstate separately.

3 The LOY theory

To compare the results from the previous section, we will outline now the basic points of the LOY theory [2, 3]. Although we will formulate it for the neutral kaon system, it is important to stress again that this theory is applicable to many unstable systems with mixing, be it elementary or composite. In particular, it would also apply to massive neutrinos and the theory whose Lagrangian we presented in section 2. The LOY theory examines the mixing and instability simultaneously and as a result of this analysis we end up with an effective non-hermitian mass matrix $M_{\text{eff}}$ which replaces the effective Hamiltonian for a single unstable state. This mass matrix is given by

$$ (M_{\text{eff}})_{ij} = m_K \delta_{ij} + \langle K_i^0 | H_{\text{weak}} | K_j^0 \rangle + \sum_n \frac{\langle K_i^0 | H_{\text{weak}} | n \rangle \langle n | H_{\text{weak}} | K_j^0 \rangle}{m_K - E_n + i\epsilon} $$

(3.1)

where $K_i^0$ can be either $K^0$ or $\bar{K}^0$. Parametrizing this mass matrix by

$$ M_{\text{eff}} = \begin{pmatrix} A & p^2 \\ q^2 & A \end{pmatrix} $$

(3.2)

the mass-eigenstates can be calculated to be

$$ |K_{S/L}\rangle = p|K^0\rangle \pm q|\bar{K}^0\rangle $$

$$ |p|^2 + |q|^2 = 1 $$

(3.3)

Because $M_{\text{eff}}$ is in general non-hermitian, it is not necessary that we have $\langle K_S | K_L \rangle = \langle K_S | K_L \rangle = 0$ or, in other words, that $|p|^2 = |q|^2$. Indeed, if CP is a good symmetry, it follows that

$$ p^2 = \langle K^0 | M_{\text{eff}} | \bar{K}^0 \rangle = e^{-2i\xi} \langle \bar{K}^0 | M_{\text{eff}} | K^0 \rangle = e^{-2i\xi} q^2 $$

(3.4)

where the phase $\xi$ comes from the CP-transformation $CP | K^0 \rangle = e^{i\xi} | \bar{K}^0 \rangle$. We conclude that if $|p|^2 - |q|^2 \neq 0$, we have CP-violation in the LOY theory and, indeed, this is taken as a genuine signal of broken CP. However, if $p^2 = e^{i\beta} q^2$, with some phase $\beta$, we cannot decide whether CP is broken or conserved. As apparent from
the Lagrangian example in section 2, a system can still exhibit CP-violation even if \(|p|^2 = |q|^2\). Quite similarly, if \(K_{S/L}\) are eigenstates to CP, then \(|p|^2 = |q|^2\) (or if \(|p|^2 = |q|^2\), then \(K_{S/L}\) are eigenstates to CP). The assignment or non-assignment of CP quantum numbers to the mass-eigenstates within the LOY-theory occurs in separation from any other state. In contrast, in the Lagrangian case this happens in conjunction with all states involved (i.e. only if we can assign to all fields a CP-phase, is CP conserved). As said above with \(|p|^2 = |q|^2\), CP can still be violated in the Lagrangian approach.

The time development in the Weisskopf-Wigner approximation is given by the Schrödinger-like equation

\[
i \frac{d}{d\tau} \begin{pmatrix} \langle \bar{K}_0^0(\tau) \rangle \\ \langle \bar{K}_0^0(\tau) \rangle \end{pmatrix} = M_{\text{eff}} \begin{pmatrix} \langle K_0^0(\tau) \rangle \\ \langle \bar{K}_0^0(\tau) \rangle \end{pmatrix}
\]

which leads to the exponential decay law for the mass-eigenstates.

In conclusion, in the LOY theory analyzing mixing and instability in a single step, one has to handle a non-hermitian mass-matrix. Its effect, in the presence of CP-violation, is the non-orthogonality of the mass-eigenstates i.e.

\[
\langle K_L|K_S \rangle = \langle K_S|K_L \rangle = |p|^2 - |q|^2 \neq 0
\]

In the next section we will discuss some peculiarities connected with non-orthogonal states. As such (3.6) is already troublesome to interpret. Although widely used, it is at the same time sometimes doubted if the usual projection technique in quantum mechanics is still applicable here. A density matrix would be preferable, but as for now one treats the mass-eigenstates kaons as pure states i.e. the density matrix is \(\rho_{S/L} = |K_{S/L}\rangle\langle K_{S/L}|\) and then there is no essential difference to the projection technique.

It should not come now as a surprise that in LOY theory CP-violation is correlated with the width of \(K_{S/L}\). The best place to see it, is by means of the Bell-Steinberger relation which reads [6]

\[
(\lambda_L - \lambda_S^*)\langle K_S|K_L \rangle = \sum_f \langle f|T|K_S \rangle^* \langle f|T|K_L \rangle
\]

\[
\lambda_{S/L} = m_{S/L} - i \frac{\Gamma_{S/L}}{2}
\]

where \(T\) is the transition operator. Evidently, if the transition matrix elements containing \(K_S\) or \(K_L\) vanish (which is equivalent to having \(\Gamma_S\) or \(\Gamma_L\) zero) and maintaining \(m_S \neq m_L\), we are forced to assume that \(\langle K_S|K_L \rangle = 0\) i.e. no CP-violation. Again this is not so in the Lagrangian example where even if the \(\Lambda_{1,2}\) mass-eigenstates are stable, the CP-violation does not vanish and, as a matter of principle, could show up in a different place (e.g. in scattering processes). There is no such correlation between the mixing parameters and the width in the Lagrangian formalism.
An important point to realize is that the LOY theory is not restricted to the kaon system. Provided we are ready to leave the Lagrangian formalism, it would go through also for the interaction presented in section 2. Indeed, we would not even need to know the explicit form of $\mathcal{L}_{\phi\chi}^{\text{int}} + \mathcal{L}_{\text{CP-violat}}^{\text{mix}}$, corresponding now to $H_{\text{weak}}$. All that is formally required, is the definition of $\phi$ and $\phi^*$ through $\mathcal{L}_{\phi\chi\eta_i}^{\text{int}}$ and the assumption that $\mathcal{L}_{\text{CP-violat}}^{\text{mix}}$ violates CP. We could also demand that the interaction contained in $\mathcal{L}_{\phi\chi\eta_i}^{\text{int}}$ is much stronger than in the corresponding $\mathcal{L}_{\phi\chi}^{\text{int}}$. As compared with the results from the Lagrangian, this would lead to non-orthogonal mass-eigenstates. We emphasize that this orthogonality is not due to some artifact of an approximation, but essentially a consequence of the non-hermiticity of the effective mass matrix. This property of $\mathcal{M}_{\text{eff}}$ will be always present in the analysis of the time evolution of a two level system.

Certainly, the advantage of the LOY theory is that there exist a well established connection to the Standard Model Lagrangian and, even more importantly, the experimental data. Nevertheless, it has some curious theoretical properties which we shall examine in the next section.

4 Consequences of non-orthogonality

We have seen that for the dynamics defined through the Lagrangian (2.1) we have two possibilities to analyze the mixing and time evolution of the involved states: the analysis of both by separating the mixing from the instability or the LOY theory which combines both. Whereas the Lagrangian choice leads to, what we could call standard properties of the states, the LOY theory does not due to (3.6). Again we will work here with the neutral kaon system, but keeping in mind that the very same conclusion would follow had we applied the LOY theory to (2.1).

The first curious problem has to do with definition of anti-particles to $K_{S/L}$. If $\Theta \equiv \text{CPT}$ transforms the strangeness eigenstates as $\Theta |K^0\rangle = e^{-i\delta} |K^0\rangle$ [7], then the CPT-transformed mass-eigenstates are given by

$$|K_{S/L}^\Theta\rangle = \frac{e^{i\delta}}{2pq} \left[ (|p|^2 \pm e^{-2i\delta}|q|^2) |K_S\rangle \right. + \left. (-|p|^2 \pm e^{-2i\delta}|q|^2) |K_L\rangle \right]$$

If, up to a phase, we would demand that $|K_S^\Theta\rangle = |K_S\rangle$ (which is reasonable as the mass-eigenstates kaon do not carry any other quantum number except mass and spin 0), we would end up with $e^{-2i\delta} = 1$ and $|p|^2 = |q|^2$ which obviously is not what one would like to have in the LOY theory. Working with non-hermitian operators, there is still a different possibility to define the CPT-transformation, viz. [8]

$$\Theta \mathcal{M}_{\text{eff}} \Theta^{-1} = \mathcal{M}_{\text{eff}}^\dagger$$

Then one can prove the following theorem [8]: provided that the non-hermitian Hamiltonian $\mathcal{H}$ is normal i.e $[\mathcal{H}, \mathcal{H}^\dagger] = 0$, there exist for every state $|\Psi\rangle$ with definite mass
and lifetime a CPT-transformed state defined by $|\Psi^\theta\rangle \equiv \Theta^{-1}|\Psi\rangle$ with the same mass and lifetime as $|\Psi\rangle$. Certainly, for normal Hamiltonians this is a good definition of CPT and anti-particle states. Unfortunately in our case we get

$$[\mathcal{M}_{\text{eff}}, \mathcal{M}_{\text{eff}}^\dagger] = |p|^2 - |q|^2$$ (4.3)

which brings us back to the very source of the problem. It seems therefore that we cannot unambiguously define anti-particle states to the kaon mass-eigenstates as long as the latter are non-orthogonal.

The second curious consequence of the non-orthogonality has to do with time evolution. There are at least three different proofs [9, 10, 11] of the following statement: as long as $|p|^2 - |q|^2 \neq 0$, the time evolution beyond the Weisskopf-Wigner approximation is strictly given by

$$|K_{S/L}(\tau)\rangle = p_{SS/LL}(\tau)|K_{S/L}\rangle + p_{SL/LS}(\tau)|K_{L/S}\rangle$$ (4.4)

where the coefficients $p_{S/L} = -p_{L/S}$ are non-zero. Hence, as long as we insist on (3.6) (say, as a signal of CP-violation), we will get a time evolution which looks like a vacuum regeneration of the mass eigenstates. It has been estimated that the coefficient $p_{SL}$ is tiny [10, 12], indeed too small to be detected experimentally, but as a matter of principle we should be worried about the interpretation of this effect.

The two last examples display already the unconventional properties of the non-orthogonality of the mass-eigenstates emerging from the LOY theory. We point out once again that they would apply also to the dynamics of the Lagrangian in section 2, had we applied the LOY theory to the mixing and instability.

The last example we would like to mention has to do with a paradox and is as such less clear. We will discuss it briefly for completeness since the paradox would find its simple resolution if the mass-eigenstates of kaons were orthogonal. There are actually two types of paradoxes discussed in the literature in connection with neutral kaons. Both have to do with entangled kaons at the $\Phi$ resonance. The first one is a cleverly posed question about future-past interference [13]. It has been resolved by means of the unitarity equation (3.7) and therefore does not have anymore the status of paradox [14]. The second one seems to be a genuine EPR-paradox [15]. It is, however, based on the wave function collapse of non-orthogonal states and on a principle of non-ideal measurements [16]. Both issues are not so clear as one would wish them to be so as to be able to establish unambiguously the existence of the paradox. Nevertheless is it worth quoting. The authors of [15] have calculated the probability in an entangled kaon system to measure a $\bar{K}_L$ on the left hand side at a time $T$ after at $T' < T$ on the right hand side one of the three possibilities has been detected: (i) strangeness eigenstates $\bar{K}_0$ or $K_0$, (ii) decay products of $K_S$, (iii) decay products of $K_L$. Their result is

$$\mathcal{P}(T, T') \propto e^{-\Gamma_S T} + e^{-\Gamma_L T} - 2\langle K_S|K_L\rangle \cos \Delta m (T - T') e^{-\frac{\Gamma_S + \Gamma_L}{2} (T + T')}$$ (4.5)
Obviously, if correct, this is a measurable quantity in which the right hand side influences the left hand side over arbitrary distances. It therefore violates the locality principle at a statistical level. This violation has to do with the $T'$ dependence of (4.5) which would vanish if $\langle K_S|K_L \rangle = 0$.

It is worth mentioning that to avoid certain problems with non-orthogonality one defines left and right eigenstates of $M_{\text{eff}}$ [17] (i.e. we are now dealing with four states) and uses the propagator method for the unstable states [18]. We feel, however, that both methods do not completely answer all the questions raised by the non-orthogonality. More importantly for the conclusions of the present paper, it seems also that both these methods also do not agree with the Lagrangian one. A more careful examination is required. Such and similar topic we postpone to future publications.

## 5 Conclusions

We have emphasized throughout the paper two important features of the LOY theory. The LOY theory analyzes the mixing and instability simultaneously and for the very same reason invokes necessarily a non-hermitian effective mass matrix which can lead to non-orthogonality of the mass-eigenstates. Secondly, this theory is applicable to any two level system be it elementary or composite. This applies also if the dynamics is given in form of a Lagrangian. In particular, it is also applicable to massive neutrinos. We have presented a special case of a Lagrangian which reveals many properties of the kaon system, but of course our conclusion is not restricted to this particular case. In principle, we have then two non-equivalent choices: to use the LOY theory or to stay within the formalism of the Lagrangian. The latter defines the mass-eigenstates by unitary transformations and proceeds to the time evolution of the individual unstable states in a separate step. We have pointed out some unusual consequences of the LOY theory. In a weaker form we can conclude that if the Lagrangian (2.1) meets all physical requirements, i.e. we can find a connection between its parameters and the parameters of the Standard Model as well as the experimentally measured quantities, we would have equally two valid choices for the analysis of the neutral kaon system.

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