A Modification of the Saturation Model: DGLAP Evolution

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Abstract

We propose to modify the saturation model of Golec-Biernat and Wüsthoff by including DGLAP evolution. We find considerable improvement for the total deep inelastic cross section, in particular in the large $Q^2$ region. The successful description of DIS diffraction is preserved.
The saturation model \cite{1, 2, 3} has provided a successful description of HERA deep inelastic scattering data, in particular for the transition from the perturbative region to the nonperturbative photoproduction region. This includes both the total $\gamma^*p$ cross section and the DIS diffractive cross section. Whereas the formulae are particularly appealing through their simplicity, they also have an attractive theoretical background, namely the idea of saturation. Despite its success, the model suffers from shortcomings which should be cured. In particular, the model does not include logarithmic scaling violations, i.e. at larger values of $Q^2$ it does not exactly match with QCD evolution (DGLAP). This becomes clearly visible in the energy dependence of $\sigma_{tot}^{\gamma^*p}$ in the region $Q^2 > 20$ GeV$^2$ where the model predictions are below the data. One expects that QCD evolution should enhance the cross section in this region.

It is the purpose of this letter to propose a modification of the saturation model. We attempt to preserve the success of the model in the low-$Q^2$ and in the transition region, while incorporating DGLAP evolution in the large-$Q^2$ domain. Since the energy dependence in the large-$Q^2$ region is mainly due to the behaviour of the dipole cross section at small dipole sizes $r$, our changes will affect mostly the small-$r$ region. At the same time, particular attention will be given to DIS diffraction for which the saturation model correctly describes the energy dependence. Since the inclusive diffractive cross section mostly depends upon the large-$r$ behaviour of the dipole cross section, we attempt to leave the dipole cross section unchanged in this region. A recent attempt \cite{4} along the same lines indicates that, in fact, diffraction provides a highly nontrivial restriction on possible modifications of the saturation model.

1 The Model

Before we describe the modifications of the saturation model, we briefly review the main features of its original formulation. Within the dipole formulation of the $\gamma^*p$ scattering,

$$\sigma_{T,L}^{\gamma^*p}(x, Q^2) = \int d^2 r \int dz \psi_{T,L}(Q, r, z) \hat{\sigma}(x, r) \psi_{T,L}(Q, r, z),$$

(1)

where $T, L$ denotes the virtual photon polarisation, the dipole cross section was proposed to have the form

$$\hat{\sigma}(x, r) = \sigma_0 \left\{ 1 - \exp \left( -\frac{r^2}{4R_0^2(x)} \right) \right\},$$

(2)

where $R_0(x)$ is the saturation scale which decreases when $x \to 0$,

$$R_0^2(x) = \frac{1}{\text{GeV}^2} \left( \frac{x}{x_0} \right)^{\lambda}.$$  

(3)

In order to be able to study the formal photoproduction limit, the Bjorken variable $x = x_B$ was modified to be

$$x = x_B \left( 1 + \frac{4m_q^2}{Q^2} \right) = \frac{Q^2 + 4m_q^2}{W^2},$$

(4)

where $m_q$ is an effective quark mass, and $W^2$ denotes the $\gamma^*p$ center-of-mass energy squared. The parameters of the model, $\sigma_0 = 23$ mb, $\lambda = 0.29$ and $x_0 = 3 \cdot 10^{-4}$ (for the assumed quark mass $m_q = 140$ MeV) were found from a fit to small-$x$ data \cite{1}. For alternative forms of the dipole cross section parameterisation see \cite{5}.
As it is well known [6], in the small-$r$ region the dipole cross section is related to the gluon density

$$\hat{\sigma}(x, r) \simeq \frac{\pi^2}{3} r^2 \alpha_s x g(x, \mu^2).$$

(5)

where the scale $\mu^2$ for small $r$ behaves as $C/r^2$. Equation (5) is valid in the double logarithmic approximation in which the constant $C$ is not determined. In the saturation model, eq. (2), we find for small $r \ll 2R_0(x)$

$$\hat{\sigma}(x, r) \simeq \frac{\sigma_0 r^2}{4 R_0^2(x)},$$

(6)
i.e., the gluon density is modelled as:

$$x g(x, \mu^2) = \frac{3}{4\pi^2 \alpha_s} \frac{\sigma_0}{R_0^2(x)}.$$

(7)

For fixed $\alpha_s$ this gluon density is clearly scale independent, which contradicts the QCD DGLAP evolution. Thus, in order to correctly take into account the scale dependence as given by the DGLAP evolution equations we have to modify the small-$r$ behaviour of the dipole cross section by incorporating the properly evolved gluon density. At the same time, we wish to preserve the idea of saturation, which reflects unitarity, and to keep unaltered the large-$r$ behaviour of the dipole cross section which determines the diffractive cross section.

Therefore, we propose the following modification of the model (2)

$$\hat{\sigma}(x, r) = \sigma_0 \left\{ 1 - \exp \left( -\frac{\pi^2}{3} \frac{r^2 \alpha_s(\mu^2) x g(x, \mu^2)}{\sigma_0} \right) \right\},$$

(8)

where the scale $\mu^2$ is assumed to have the form

$$\mu^2 = \frac{C}{r^2} + \mu_0^2.$$

(9)

The parameters $C$ and $\mu_0^2$ will be determined from a fit to DIS data. In a first approximation, $g(x, \mu^2)$ is evolved with the leading order DGLAP evolution equation for the gluon density. In the spirit of the small-$x$ limit, we neglect quarks in the evolution equations. We assume the following gluon density at the initial scale $Q_0^2 = 1$ GeV$^2$:

$$x g(x, Q_0^2) = A_g x^{-\lambda_g} (1 - x)^{5.6},$$

(10)

where $A_g$ and $\lambda_g$ are parameters to be determined from a fit to data. The exponent 5.6, determining the large $x$ behaviour, is motivated by one of the versions of the MRST parameterisation [7] of the gluon density.

For small $r$, the exponential in (8) can be expanded in powers of its argument, and the relation (5), with the running $\alpha_s = \alpha_s(\mu^2)$, is found. In contrast to the original dipole cross section, the rise in $1/x$ now has become $r$-dependent. When inserting $\hat{\sigma}$ into (1) and convoluting with the photon wave function, the integrand peaks near $r \sim 2/Q$ for large $Q^2$, and the argument of the gluon density turns into $\mu^2 \approx Q^2$. Consequently, with increasing $Q^2$, DGLAP evolution will strengthen the rise in $1/x$, whereas in the original saturation model the power of $1/x$ had been constant. For sufficiently large $r$, the scale $\mu^2$ is frozen at the value $\mu_0^2$. This prevents the effective scale of the gluon density from becoming unreasonably small. The saturation value of the dipole cross section is $\hat{\sigma}(x, r) \approx \sigma_0$, as in the original model (2). The transition from the small to the large $r$ region depends on $x$, but in detail it will be different from the original model. This will be discussed in detail in the section presenting numerical results.
2 Momentum space formulation

Although in this letter we will restrict ourselves to the total (and later on to the diffractive) cross section, it is instructive to rephrase these features in momentum space. In a future step, we intend to study the effects of saturation in more exclusive final states, and the translation of our dipole cross section into momentum space may serve as a first step into this direction. For this purpose let us start with the $k_T$-factorization formula [8] for the $\gamma^*p$ cross section, e.g. for the transversely polarised photon [9],

$$\sigma_{T}^{\gamma p} = \frac{\alpha_{em}}{\pi} \sum_f e_f^2 \int_{l^2} d^2 l \alpha_s f(x, l^2) \int d^2 k \int_0^1 dz \left[ z^2 + (1-z)^2 \right] \left\{ \frac{k}{k^2 + Q'^2} - \frac{k+1}{(k+1)^2 + Q'^2} \right\}^2,$$

(11)

where $f(x, l^2)$ is the gluon amplitude describing an interaction of the $q\bar{q}$ pair with the proton, $l$ is the transverse momentum of the gluon coupled to the quark pair and $Q'^2 = z(1-z)Q^2$. Using the relation

$$\frac{k}{k^2 + Q'^2} = i \frac{Q}{2\pi} \int d^2 r \, e^{i k \cdot r} \frac{r}{r} K_1(Q r),$$

(12)

the following formula is found

$$\sigma_{T}^{\gamma p} = \frac{3\alpha_{em}}{2\pi^2} \sum_f e_f^2 \int_0^1 dz \left[ z^2 + (1-z)^2 \right] \frac{Q}{2\pi} \int d^2 r \int d^2 r' \, e^{i k \cdot (r-r')} D(r, r')$$

$$\times \int \frac{d^2 k}{(2\pi)^2} e^{i k \cdot (r-r')} D(r, r')$$

(13)

where

$$D(r, r') = \frac{2\pi}{3} \int \frac{d^2 l}{l^2} \alpha_s f(x, l^2) \left( 1 - e^{i l \cdot r} \right) \left( 1 - e^{-i l \cdot r'} \right).$$

(14)

If the argument of the strong coupling $\alpha_s$ and the variable $x$ in the gluon amplitude in $D(r, r')$ do not depend on the quark transverse momenta $k$, the integration over $k$ in eq. (13) gives the delta function $\delta^2(r-r')$ which reflects the conservation of a dipole transverse size vector $r$ during the collision. In this case the dipole formula (1) is obtained with the following identification of the dipole cross section

$$\hat{\sigma}(x, r) = \frac{2\pi}{3} \int \frac{d^2 l}{l^2} \alpha_s f(x, l^2) \left( 1 - e^{i l \cdot r} \right) \left( 1 - e^{-i l \cdot r} \right).$$

(15)

Going beyond the leading $\log(1/x)$ approximation in which eq. (11) was derived, e.g. by taking into account the exact gluon kinematics [10] or considering a quark virtuality $k^2 + Q'^2$ as an argument of the running coupling $\alpha_s$, we find that $r$ is no longer conserved during the scattering process, and the simple relation (15) ceases to exist. As a result, the $k_T$-factorization formula (11) can no longer be written in the form (1), and the simple dipole picture fails. We want to avoid this situation, thus we assume that the argument of $\alpha_s$ is given by the gluon momentum $l^2$ and $x = x_Bj$. Since the integration in (15) includes also small momenta, the modelling of the infrared behaviour of $\alpha_s$ cannot be avoided. However, we will hide this fact by analysing the combined quantity $\alpha_s f(x, l^2)$.

From the requirement that in the double logarithmic limit (DLL) formula (11) should be consistent with the DLL of the DGLAP formalism one can derive a relation between the gluon
amplitude \( f(x, l^2) \) and the conventional gluon distribution \( x g(x, Q^2) \). Starting from eq. (11), using the relation \( F_T = Q^2/(4\pi^2\alpha_{em}) \sigma_T^{\gamma p} \) and imposing the strong ordering condition: \( l^2 \ll k^2 \ll Q^2 \), one arrives at

\[
\frac{\partial F_T(x, Q^2)}{\partial \log Q^2} = \frac{1}{3\pi} \sum_f e_f^2 \int_0^{Q^2} \frac{d^2l}{\pi l^2} \alpha_s f(x, l^2).
\]

By comparison with an analogous formula obtained in the DLL of the DGLAP evolution equations, one finds the following relation at large \( Q^2 \)

\[
\alpha_s(Q^2) x g(x, Q^2) = \int_0^{Q^2} \frac{d^2l}{\pi l^2} \alpha_s f(x, l^2).
\]

In the model (8) we go beyond the \( k_T \)-factorization formula (11) where \( f(x, l^2) \) represent a two gluon amplitude. In the region of small \( l^2 \), the relation (17) between \( f(x, l^2) \) and the gluon density no longer holds, and \( f(x, l^2) \) is defined through relation (15) where for the l.h.s we use our model (8). In general, provided the dipole cross section has a finite limit: \( \lim_{r \to \infty} \hat{\sigma}(x, r) = \hat{\sigma}_\infty(x) \), the equation (15) can be inverted with the help of the following relation:

\[
\frac{\alpha_s f(x, l^2)}{l^4} = \frac{3}{4\pi} \int \frac{d^2r}{(2\pi)^2} \exp{\{i \cdot r\}} \{ \hat{\sigma}_\infty(x) - \hat{\sigma}(x, r) \} = \frac{3}{8\pi^2} \int_0^{\infty} dr \, J_0(lr) \{ \hat{\sigma}_\infty(x) - \hat{\sigma}(x, r) \}.
\]

(18)

In the original dipole model we find [2]:

\[
\alpha_s f(x, l^2) = \frac{3\sigma_0}{4\pi^2} R_0^2(x) l^4 \exp{-R_0^2(x) l^2}.
\]

(19)

For the modified dipole cross section, this inversion has to be done numerically. DGLAP evolution will affect mainly the large-\( l \) behaviour while at small \( l \) our modification should be less severe. The most interesting question to be addressed below concerns the transition region: to what extent does our modification affect the region of moderate momenta, i.e. could one ‘see’ saturation in diffractive final states?

3 Numerical results

Let us now turn to numerical results. We performed global fits to the DIS data with \( x < 0.01 \) in the range of \( Q^2 \) between 0.1 and 500 GeV\(^2\). For H1 and ZEUS HERA experiments the new 1996-1997 data sets were used [11, 12, 13]. In addition to the HERA data also the data of the E665 experiment [14] were used. The statistical and systematic errors were added in quadrature. The number of degrees of freedom, \( N_{df} \), was around 330.

The new data sets are considerably more precise (with much smaller statistical and systematic errors) than the ones used in the original analysis [1]. As a preparatory step, we applied the original model (2), using the parameter values of the original fit, to the new data and obtained rather high value of \( \chi^2/N_{df} \approx 3 \) (for the old data, the corresponding value was \( \chi^2/N_{df} = 1.18 \)). Next, we let the new data to determine their own values of the parameters of the original model, \( \sigma_0, \lambda \) and \( x_0 \) in eq. (2). This led to an improvement of the fit, \( \chi^2/N_{df} \approx 2.2 \). Nevertheless, this
<table>
<thead>
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<th>$m_q$ (MeV)</th>
<th>$\sigma_0$ (mb)</th>
<th>$A_g$</th>
<th>$\lambda_g$</th>
<th>$C$</th>
<th>$\mu_s^2$</th>
<th>$\chi^2/N_{df}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fit 1</strong></td>
<td>140</td>
<td>23.0</td>
<td>1.20</td>
<td>0.28</td>
<td>0.26</td>
<td>0.52</td>
</tr>
<tr>
<td><strong>Fit 2</strong></td>
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<td>23.8</td>
<td>13.71</td>
<td>-0.41</td>
<td>11.10</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 1: The parameters of the fits to the ZEUS, H1 and E665 data with $x < 0.01$ (333 points). The H1 data was rescaled by a factor of 1.05. The numbers in bold are fixed during the fits.

relatively poor agreement indicates that the original model is doing not so well with the new data, especially for large values of $Q^2$. As a first step for the improvement, we modify the dipole cross section at small values of $r$ by including QCD DGLAP evolution, as given in eq. (8).

In the modified saturation model, there are five parameters to be determined: $\sigma_0$, $C$, $\mu_s^2$, $A_g$ and $\lambda_g$ from eqs. (8, 9, 10). We use the leading order DGLAP evolution equation for the gluon density, and we put $\Lambda_{QCD} = 200$ MeV in $\alpha_s$ and set the number of active flavours $N_f = 3$. Thus, although the evolution equation for the gluon is decoupled from the quarks, their presence is encoded in the assumed value of $N_f$.

We performed first the fit leaving all five parameters free and assumed the value of the light quark mass $m_q = 140$ MeV, as in the original formulation [1]. A good quality fit was obtained with $\chi^2/N_{df} = 1.05$. The found value of the dipole cross section $\sigma_0 = 27.4$ mb, however, was higher than the saturation model value, $\sigma_0 = 23$ mb. Also, the corresponding value of the photoproduction cross section ($\sigma^{\gamma p} = 204$ mb) was significantly higher than the measured value $174 \pm 1(st) \pm 13(sys)$ mb at $W = 209$ GeV [15]. Thus we decided to decrease $\sigma_0$ and fixed it to the saturation model value 23 mb. This value is also advantageous for the description of the inclusive DIS diffractive cross section which is more sensitive to large dipole sizes, i.e., to the saturation region, than the total $\gamma^* p$ cross section [2]. The resulting parameters in such fit are presented in Table 1 (Fit 1). The description is slightly worse that that described above, but both the photoproduction cross section ($\sigma^{\gamma p} = 189$ mb) and the diffractive cross section are properly described. This is because we modified only the small dipole size part of the dipole cross section (2), without affecting the saturation part. The found gluon density gives 39% of the total proton momentum carried by gluons resolved at the initial scale $Q_0^2 = 1$ GeV$^2$.

The results of Fit 1 are compared to the data on $F_2$ in Fig. 1 for $Q^2 < 1$ GeV$^2$ and in Fig. 2 for large $Q^2$ points. In all presented plots, the solid lines refer to the results obtained with the DGLAP improved model (8) and the dashed lines correspond to the saturation model (2) with the original parameters from [1]. We see that the DGLAP evolution significantly improves agreement with the data at large $Q^2$ while at small $Q^2$ the results are practically the same. This effect is summarised in Fig. 3 where the effective slopes $\lambda(Q^2)$, obtained from the parameterisation of $F_2$ at small $x$: $F_2 \sim x^{-\lambda(Q^2)}$, are plotted. Thus, the DGLAP modification of the dipole cross section for small $r$ is crucial for much better agreement with the data. The same effective slopes characterise the energy dependence of the $\gamma^* p$ cross section: $\sigma^{\gamma^* p} \sim (W^2)^{\lambda(Q^2)}$. The change from a soft dependence at small $Q^2$ to a hard one for large $Q^2$ is shown in Fig. 4. In Fig. 5 we show another aspect of the transition of $F_2$ to low $Q^2$ values, namely the emergence of the behaviour: $F_2 \sim Q^2$ approached in the limit $Q^2 \to 0$ and $y = W^2/s$ fixed. The class of the saturation models described here nicely reproduce this behaviour, see recent Ref. [16] for more details on this transition.

In a second step of our investigations we relax our requirement of staying in the low-$Q^2$ region as close as possible to the original model. In particular, we allow the quarks in the $q\bar{q}$
dipole to become massless. Thus we set $m_q = 0$ in the wave function $\Psi_{T,L}$ and in the kinematic relation (4). In the original model, the quark mass was introduced as an effective parameter for modelling the large-$r$ behaviour of the photon wave function. The non-zero quark mass allows us to study the photoproduction limit of our model after the modification (4) of the Bjorken-$x$ in the dipole cross section, therefore, setting this parameter to zero eliminates this possibility. It allows, however, for a better description of the current data. We also fix the minimal value $\mu_0^2$ of the scale $\mu^2$ in eq. (8) to 1 GeV$^2$ in the fits in order to avoid negative gluon density below the input scale $Q_0^2 = 1$ GeV$^2$ for the gluon evolution. Since the parameters $\sigma_0$ and $\lambda_g$ are strongly correlated, we have performed a systematic search of the best $\chi^2$ on the grid of fixed $(\sigma_0, \lambda_g)$. In each case, the remaining parameters, $A_g$ and $c$, were fitted. In this way we found two local minima for $\chi^2$, shown in Fig. 6, for positive value of $\lambda_g$ in eq. (10) leading to strongly rising gluon density, and for negative value of $\lambda_g$, corresponding to valence-like initial gluon. The latter scenario gives considerably better description (with $\chi^2/N_{df} = 0.97$) than the first one (with $\chi^2/N_{df} = 1.13$). In the final analysis, after a quantitative estimation of the position of the best fit in the parameter space using the grid method, we allow $\sigma_0$ and $\lambda_g$ to be fitted together with $A_g$ and $c$. The values of these parameters for the best fit are given in Table 1 (Fit 2). The corresponding value of the gluon momentum at the input scale equals 84%. The effective slope $\lambda(Q^2)$ from the Fit 2 parameterisation is shown as the dotted line in Fig. 3. As expected, slight differences between the two fit scenarios only appear for small values of $Q^2$, below 1 GeV$^2$.

It is interesting to compare the results of the Fit 1 and Fit 2 since they lead to a different picture of the dynamics of the $\gamma^*p$ interaction. In Fit 1, the initial gluon density, $xg(x, Q_0^2)$, quickly rises with $1/x$, whereas in Fit 2 it even decreases with rising $1/x$. Therefore, in Fit 1 the rise of the cross section with the energy is mainly due to the intrinsic properties of the initial gluon density, with only slight corrections being due to the evolution effects at high $Q^2$ values, and considerable damping effects resulting from saturation at low $Q^2$. In Fit 2, the evolution effects are very strong (note the value of the parameter $C$, which is much higher then in Fit 1). The small-$x$ rise of the cross section is due solely to the DGLAP evolution effects with some corrections coming from saturation.

Further insight into the physical picture lying behind the fits can be gained by a closer look at $r$-dependence of the dipole cross section $\hat{\sigma}(x, r)$, and a momentum dependence of the related gluon amplitude, $f(x, l^2)$. In the first row of Fig. 7 we show the dipole cross section for the two fits. As we have already discussed, for Fit 1 the DGLAP modification (solid lines) affects mostly the region of moderately small $r$ ($< 1$ GeV$^{-1}$). In Fit 2 both the small and large $r$ regions are affected. In particular, the structure close to the saturation region is different from that in the saturation model (2) (dashed lines). The differences between the models are particular visible if we turn to momentum space and compute the gluon amplitude $\alpha_s f(x, l^2)$ for different values of $x$, using relation (18). The results are shown in the second row of Fig. 7, where the full lines denote the gluon amplitude from the DGLAP improved model and the dashed lines correspond to the saturation model (2). The small-$r$ region of the dipole cross section corresponds to the large $l^2$ region in the gluon amplitude. The dipole cross section from Fit 1 is translated into a gluon amplitude with a double-bump structure. Notice that the second bump results from the DGLAP modification of the small-$r$ part of the dipole cross section. In Fit 2, however, the second bump disappears and the gluon amplitude is similar but significantly broader than the one corresponding to the saturation model. Although in Fig. 7 the various gluon amplitudes are clearly distinct, after convolution with the impact factors and turning to the $\gamma^*p$ cross sections,

1The valence-like gluon density preferred in the massless fit, as described in the text, becomes negative below the input scale due to backward evolution. In this case the dipole cross section (8) does not saturate at large $r$. 

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these differences are becoming much less visible. It is natural to expect, that some of these differences should become visible in more exclusive final states. As a first example, one might think of DIS diffraction. The inclusive diffractive process, however, is sensitive to the region of small $l^2$ or large $r$ and, therefore, has only limited value in distinguishing between the two fit solutions. On the other hand, other physics processes like jet, charm and bottom production should be more sensitive to the behaviour of the unintegrated gluon density at large gluon momenta $l^2$.

Looking at Fig. 7 notice that starting from certain values of $l^2$, the gluon amplitudes become negative. In order to understand this let us differentiate the relation (17) with respect to the large scale $\mu^2$,

$$\alpha_s f(x, \mu^2) \simeq \frac{\partial}{\partial \ln \mu^2} \left\{ \alpha_s(\mu^2) x g(x, \mu^2) \right\} = \alpha_s(\mu^2) \left\{ \frac{\partial x g(x, \mu^2)}{\partial \ln \mu^2} - \frac{x g(x, \mu^2)}{\ln(\mu^2/\Lambda^2)} \right\}. \quad (20)$$

The quantity in the curly brackets in the last equality can become negative, which is shown in the bottom row of Fig. 7 by plotting the r.h.s of the above equation (dashed lines). In the shown range of $l^2$, eq. (20) is especially well satisfied for the parameters from Fit 2. For Fit 1 the equality is reached for much larger (not shown) values of $l^2 = \mu^2$.

In ref. [1] the critical line in the $(x, Q^2)$ plane was defined which marks the transition to the saturation region where a new behaviour of the structure function, $F_2 \sim Q^2$, emerges. Near this line, the characteristic size of the $q\bar{q}$ dipole, $\bar{r} \simeq 2/Q$, equals the saturation radius $R_0(x)$, see section 1. In this case the argument of the exponent in eq. (2) equals one,

$$Q^2 R_0^2(x) = 1 \quad (21)$$

and $\hat{\sigma}(x, \bar{r}) \sim \sigma_0$. We adopt the same criterion for the critical line in the DGLAP improved saturation model (8). Thus, we have the following condition

$$\frac{4\pi^2}{3 \sigma_0 Q^2} \alpha_s(\mu^2) x g(x, \mu^2) = 1, \quad (22)$$

where $\mu^2 = C Q^2/4 + \mu_0^2$. Eq. (22) is an implicit equation for the critical line $Q^2 = Q^2_0(x)$, shown in Fig. 8 for the two fits. As expected, for FIT 1 the found critical line is not different from that defined in the original saturation model (dashed line) and the transition region stays around 1 GeV$^2$ in the HERA kinematics (lower band). For FIT 2 the critical line is situated at lower values of $Q^2$ (around 0.5 GeV$^2$). It is interesting to note that both fits predict that in the THERA kinematic range (upper band) the saturation region lies at $Q^2 \approx 2$ GeV$^2$, which puts the perturbative QCD description of saturation effects on more solid ground.

### 4 Diffraction

One of the main advantages of dipole models is their straightforward description of diffractive processes. The generalised optical theorem applied in the framework of the dipole picture allows to express the cross section for diffractive $q\bar{q}$ production in which proton remains intact as

$$\frac{d\sigma_{\text{dif}}}{dt} \bigg|_{t=0} = \int d^2 r \int dz \psi_{T,L}^*(Q, r, z) \sigma^2(x, r) \psi_{T,L}(Q, r, z), \quad (23)$$
where \( t = \Delta^2 \), and \( \Delta \) is the four-momentum transferred into the diffractive system from the proton. In addition to the contributions of the \( q\bar{q} \) states it is important to include the contributions of the \( q\bar{q}g \) final states [17].

In the phenomenological analysis [2], the \( q\bar{q}g \) diffractive amplitude was computed in the two-gluon exchange approximation with an additional assumption of strong ordering of transverse momenta of the \( q\bar{q} \) pair and the gluon. This allows to treat the \( q\bar{q}g \) system as a color octet dipole \((8\bar{8})\) in the transverse coordinate representation. Compared to the triplet dipole, the coupling of two \( t \)-channel gluons in the singlet state to the octet dipole carries the relative weight \( C_A/C_F = 2N_c^2/(N_c^2 - 1) \). Thus, in order to take into account the repeated exchange of a two gluon system, the equation (8) for the triplet dipole cross section is modified for the octet dipole as

\[
\hat{\sigma}_{gg}(x,r) = \sigma_0 \left\{ 1 - \exp \left( -\frac{C_A}{C_F} \cdot \frac{\pi^2 r^2 \alpha_s(\mu^2)xg(x,\mu^2)}{3\sigma_0} \right) \right\}.
\]

The above modification is done in the spirit of multiple pomeron exchange, i.e. the term proportional to \((r^2)^n\), resulting from the expansion of the exponent in (24), would correspond to \( n \) exchanged pomerons with an appropriate colour factor \((C_A/C_F)^n\). In addition, compared to the diffractive \( q\bar{q} \) production, the cross section formula for a diffractive \( q\bar{q}g \) system contains an overall factor \((N_c^2 - 1)/N_c\).

One of the most important results of the original saturation model was that, at fixed \( Q^2 \), the ratio of the inclusive diffractive cross section and the total \( \gamma^*p \) cross section is nearly constant in agreement with data. This prediction is not changed in the DGLAP improved saturation model since we modified only the short distance part of the dipole cross section. Even in the case of Fit 2, the constant ratio is preserved, as shown in Fig. 9, in contrast to the attempts in [4]. The theoretical curves in these figures are computed using the Fit 2 results, and the experimental data are taken from ZEUS [18]. The results for the Fit 1 computation differ only slightly from the Fit 2 one.

The diffractive data shown in Fig. 9 were obtained without the experimental identification of the forward going proton. Therefore, as described in [18], this data have a substantial contribution of the proton dissociation process which was estimated as 31\pm15\%. To take into account this contribution, the prediction of our model shown in Fig. 9 were multiplied by a normalisation factor of \( 1/(1 - 0.31) = 1.45 \). The agreement of the height of the predicted cross section with the data is satisfactory only within the relatively large error of the estimated proton dissociation factor.

We also compare our predictions with the recent diffractive data in which the forward going proton was identified in the ZEUS Leading Proton Spectrometer (LPS) [19]. The diffractive slope \( B_D = 7 \text{ GeV}^{-2} \) was assumed in agreement with the measurement. As shown in Fig. 10 the predictions of our model (without any double dissociation renormalisation factor) are in good agreement with the LPS data. This agreement gives further support to the dynamical picture of the \( \gamma^*p \) interactions developed in this and previous papers on the saturation model.

5 Conclusions

In this letter we have proposed a modification of the saturation model which takes into account the QCD DGLAP evolution of the gluon distribution. Fitting the parameters of our model we found a solution that describes the new HERA data on \( F_2 \) significantly better than the original
saturation model, especially in the region of larger $Q^2$. The agreement with the DIS diffractive HERA data is also kept.

Somewhat surprisingly, we found another set of parameters which lead to even better data description departing from the original saturation model. For this description, we set the effective quark mass of the original model equal to zero, and in our comparison with the HERA data we have disregarded photoproduction data points. We found indications that this solution represents a slightly different physical picture: the initial gluon density no longer rises at small $x$, and QCD-evolution plays a much more significant role than in our first solution. The fact that the effective quark mass of the original model has its strongest influence in the limit $Q^2 \rightarrow 0$ suggests that the large-$r$ behavior of the photon wave function requires further considerations.

We have found it useful to discuss the various versions of the saturation model not only in the $r$-space but also in momentum space since the latter provides more direct connection with exclusive final states. As a future step, it will be instructive to trace saturation effects in less inclusive cross sections.

We consider the modification of the saturation model presented in this paper as a first step of a more systematic program. The success of the original model indicates that this simple ansatz contains elements of the correct dynamics. Next, we have to analyse this model within QCD and to find the necessary corrections. With precise HERA data on various reactions becoming available, all modifications have to be tested by careful comparisons.

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References

Figure 1: $F_2$ as a function of $x$ for fixed low $Q^2$ values. The comparison with the low $Q^2$ data from ZEUS. The solid lines: the model with the DGLAP evolution (8) (FIT 1) and the dotted lines: the saturation model (2).
Figure 2: H1 and ZEUS data on $F_2$ as a function of $x$ for fixed values of $Q^2 > 1$ GeV$^2$ and the saturation model curves. The solid lines: the model with the DGLAP evolution (8) (FIT 1) and the dotted lines: the saturation model (2).
Figure 3: The effective slope $\lambda(Q^2)$ from the parameterization $F_2 \sim x^{-\lambda(Q^2)}$ as a function of $Q^2$. The model with the DGLAP evolution (8): the solid line (FIT 1) and the dotted line (FIT 2). The saturation model (2): the dashed line. The open circles: ZEUS analysis and the full circles: H1 data [20].
Figure 4: The $\gamma^* p$ cross section as a function of energy $W^2$ at various $Q^2$. The solid lines: the model with the DGLAP evolution (8) (FIT 1) and the dotted line: the saturation model (2), shown for $x < 0.01$. 

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Figure 5: $F_2(x, Q^2)$ as a function of $Q^2$ for fixed $y = Q^2/(sx)$. The solid lines: the model with DGLAP evolution (8) (FIT 1) and the dashed lines: the saturation model (2). The curves are plotted for $x < 0.01$. Full circles: ZEUS data and open circles: H1 data.
Figure 6: The lines of constant values of $\chi^2$ in the space of $(\sigma_0, \lambda_g)$ in the massless case, $m_q = 0$. The two local minima are indicated by the black dots.
Figure 7: The dipole cross section $\hat{\sigma}(x,r)$ (upper row) for $x = 10^{-2}...10^{-6}$ (from right to left) and the gluon amplitude $\alpha_s f(x,l^2)$ at $x = 10^{-2}...10^{-4}$ (from bottom to top) for the two fits. The solid lines correspond to the DGLAP improved model while the dashed lines describe the saturation model (2). The dotted lines in the bottom row show the r.h.s of eq. (20).
Figure 8: The position of the critical line in the \((x, Q^2)\) plane in the DGLAP improved model (solid lines) and the original saturation model (dshed line). The bands indicate acceptance regions for the colliders HERA (lower) and future THERA (upper).
Figure 9: The ratio of $\sigma_{\text{diff}}/\sigma_{\text{tot}}$ versus the $\gamma^*p$ energy $W$. The data is from ZEUS and the solid lines correspond to the results of the DGLAP improved model (FIT 1).
Figure 10: The diffractive structure function as a function of $x_{IP}$. The data is from the LPS measurement of ZEUS and the solid lines are the DGLAP saturation model prediction (FIT 1).