Physics potential and present status of neutrino factories

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I briefly review the recent status of research on physics potential of neutrino factories with emphasis on measurements of the CP phase.

1 Introduction

The observation of atmospheric neutrinos (See, e.g., Ref.1) and solar neutrinos (See, e.g., Ref.2) gives the information on the mass squared differences and the mixings, which can be written in the three flavor framework of neutrino oscillations as $(|\Delta m^2_{32}|, \theta_{23})$ and $(\Delta m^2_{21}, \theta_{12})$, where I have adopted the standard parametrization$^3$ for the $3 \times 3$ MNSP$^5,6$ matrix. On the other hand, the CHOOZ result$^4$ tells us that $|\theta_{13}|$ has to be small ($\sin^2 2\theta_{13} < 0.1$). So the MNSP matrix looks like

$$U_{\text{MNSP}} \simeq \begin{pmatrix} c_\odot & s_\odot & \epsilon \\ -s_\odot/\sqrt{2} & c_\odot/\sqrt{2} & 1/\sqrt{2} \\ s_\odot/\sqrt{2} & -c_\odot/\sqrt{2} & 1/\sqrt{2} \end{pmatrix},$$

where I have used $\theta_{23} \simeq \pi/4$, $\sin^2 2\theta_{12} \equiv \sin^2 2\theta_\odot \simeq 0.8$ and $|\epsilon| \ll 1$.

The next thing to do is to determine $\theta_{13}$, the sign of $\Delta m^2_{32}$ and the CP phase $\delta$. During the past few years a lot of research have been done on the possibilities of future long baseline experiments. One is a super-beam experiment and the other one is a neutrino factory. The former is super intense conventional neutrino beam which is obtained from pion decays while the latter is from muon decays in a storage ring. The background fraction $f_B$ in the case of super-beams$^7$ is of order $10^{-2}$, while in the case of a neutrino factory$^8$ it is of order $10^{-5}$. The advantage of a neutrino factory is such low background fraction and neutrino factories are expected to enable us to determine $\theta_{13}$ and the sign of $\Delta m^2_{32}$ ($\delta$) for $\sin^2 2\theta_{13} \gtrsim 10^{-5}$ ($10^{-3}$), respectively.

In this talk I will mainly discuss measurements of the CP phase $\delta$ at neutrino factories and will try to clarify the reason why some group obtains different results for the optimized muon energy and baseline.
2 Measurements of the CP phase at neutrino factories

Measurement of the CP phase in neutrino oscillations is difficult not only because CP violating contribution in the oscillation probability is in general small but also because there is matter effect. Namely, the dependence of the probabilities for $\nu$ and $\bar{\nu}$ on $\delta$ and $A$ is given by $P(\nu_\mu \rightarrow \nu_e) = f(E, L; \theta_{ij}, \Delta m_{ij}^2, \delta; A)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = f(E, L; \theta_{ij}, \Delta m_{ij}^2, -\delta; -A)$, where $f$ is a certain function and $A \equiv \sqrt{2}G_F N_e$ stands for the matter effect. The quantity obtained from experiments on $\bar{\nu}$ is not $f(\cdots, -\delta; A)$, so that direct comparison between $f(\cdots, \delta; A)$ and $f(\cdots, -\delta; A)$ is impossible in a strict sense. The situation here is different from the $K^0 - \bar{K}^0$ system where the quantity $N(K_L \rightarrow 2\pi)/N(K_L \rightarrow 3\pi)$ immediately gives us evidence for CP violation. Hence I have to compromise and adopt a kind of indirect measurement of CP violation, i.e., I deduce the values of $\delta$ etc. by comparing the energy spectra of the data and of the theoretical prediction with neutrino oscillations assuming the three flavor mixing. In determining $\delta$, there are other oscillation parameters as well as the density of the Earth whose values are not exactly known, so that I have to take into account correlations of errors of these parameters. Thus I introduce the following quantity to see the significance of the case with nonvanishing $\delta$:

$$\Delta \chi^2 \equiv \min_{\theta_{ij}, \Delta m_{ij}^2, A} \sum_j \left\{ \frac{[N_j(\nu_e \rightarrow \nu_\mu) - \tilde{N}_j(\nu_e \rightarrow \nu_\mu)]^2}{\sigma_j^2} + \frac{[N_j(\bar{\nu}_e \rightarrow \bar{\nu}_\mu) - \tilde{N}_j(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)]^2}{\sigma_j^2} + \frac{[N_j(\nu_\mu \rightarrow \nu_\mu) - \tilde{N}_j(\nu_\mu \rightarrow \nu_\mu)]^2}{\sigma_j^2} + \frac{[N_j(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu) - \tilde{N}_j(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu)]^2}{\sigma_j^2} \right\}, \quad (1)$$

where $N_j(\nu_\alpha \rightarrow \nu_\beta) \equiv N_j(\nu_\alpha \rightarrow \nu_\beta; \theta_{kl}, \Delta m_{kl}^2, \delta, A)$, $\tilde{N}_j(\nu_\alpha \rightarrow \nu_\beta) \equiv \tilde{N}_j(\nu_\alpha \rightarrow \nu_\beta; \theta_{kl}, \Delta m_{kl}^2, \delta = 0, A)$ stand for the numbers of events of the data and of the theoretical prediction with a vanishing CP phase, respectively, and $\sigma_j^2$ stands for the error which is given by the sum of the statistical and systematic errors. At neutrino factories appearance and disappearance channels for $\nu$ and $\bar{\nu}$ are observed, and I have included the numbers of events of all the channels in

\[^{a}\text{Correlations of errors at neutrino factories were studied in Ref.}^{9,10,11,12,13}.\]
to gain statistics. In the present analysis, $N_j(\nu_\alpha \rightarrow \nu_\beta)$ is substituted by theoretical prediction with a CP phase $\delta$ and $\Delta \chi^2$ obviously vanishes if $\delta = 0$. The quantity $\Delta \chi^2$ reflects the strength of the correlation of the parameters, i.e., if $\Delta \chi^2$ turns out to be very small for a certain value of $A$ then the correlation between $\delta$ and $A$ would be very strong and in that case there would be no way to show $\delta \neq 0$. To reject a hypothesis “$\delta = 0$” at the $3\sigma$ confidence level I demand

$$\Delta \chi^2 \geq \Delta \chi^2(3\sigma CL)$$

where the right hand side stands for the value of $\chi^2$ which gives the probability 99.7% in the $\chi^2$ distribution with a certain degrees freedom, and $\Delta \chi^2(3\sigma CL) = 20.1$ for 6 degrees freedom. From (2) I get the condition for the detector size to reject a hypothesis “$\delta = 0$” at $3\sigma CL$.

On the other hand, Koike et al.\textsuperscript{11} claim that another quantity

$$\Delta \tilde{\chi}^2 \equiv \min_{\delta, A} \sum_j \frac{1}{\sigma_j^2} \left( \frac{[N_j(\nu_e \rightarrow \nu_\mu) - \bar{N}_j(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)]^2}{N_j(\nu_e \rightarrow \nu_\mu) - \bar{N}_j(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)} \right)^2$$

should be used since this particular combination improves the correlation between $\delta$ and $A$. In my opinion, however, both analyses with $\Delta \chi^2$ and $\Delta \tilde{\chi}^2$ are based on indirect measurements of CP violation anyway and there is no reason why one has to discard other information on $\delta$. In fact it turns out that (3) is a combination in which the correlation between $\delta$ and $A$ improves for low energy and worsens at high energy.

It should be pointed out here that indirect measurements of CP violation are also considered in the $B^0 - \bar{B}^0$ system.\textsuperscript{5} To measure the phase $\phi_1$, direct CP violating process $B(\bar{B}) \rightarrow J/\psi K_s$ is used\textsuperscript{14,15}, while those to measure $\phi_2$ and $\phi_3$ are $B \rightarrow 2\pi\textsuperscript{16}$ and $B \rightarrow DK\textsuperscript{17}$, which are not necessarily CP–odd processes. Their strategy is to start with the three flavor framework, to use the most effective process, which may or may not be CP–odd, to determine the CP phases and to check unitarity or consistency of the three flavor hypothesis.

The optimized muon energy $E_\mu$ and baseline $L$ for the measurement of CP the phase at a neutrino factory have been investigated by several groups by taking into consideration the correlations of $\delta$ and all other parameters and the results are summarized in Table 1. The results in Ref.\textsuperscript{13} are given in Fig. 2, which shows that the more uncertainty I have in the density, the shorter baseline I have to choose because the correlation between $\delta$ and $A$ improves for low energy and worsens at high energy.

\textsuperscript{b}This is the reason why the quantity in (1) is denoted as $\Delta \chi^2$ instead of absolute $\chi^2$. $\Delta \chi^2$ represents deviation from the best fit point rather than the goodness of fit.

\textsuperscript{c}The term ”indirect” in the B system is different from that in neutrino oscillations.
Figure 1: Unitarity triangle in the B meson system. The phase $\phi_1$ is measured by a direct CP violating process $B(\bar{B}) \to J/\psi K_s$, whereas the most promising way to determine $\phi_2$ and $\phi_3$ is through $B \to 2\pi$ and $B \to DK$, which are not direct CP violating processes.
Figure 2: The contour plot of equi-number of data size required to reject a hypothesis \( \bar{\delta} = 0 \) at 3\( \sigma \) with the background fraction \( f_B = 10^{-5} \) or \( 10^{-3} \) and the uncertainty of the matter effect \( \Delta A = 5\% \), 10\% or 20\%. 

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\begin{align*}
\text{f}_B &= 10^{-5} \quad \Delta A = 5\% \\
\text{f}_B &= 10^{-3} \quad \Delta A = 5\% \\
\text{f}_B &= 10^{-5} \quad \Delta A = 10\% \\
\text{f}_B &= 10^{-3} \quad \Delta A = 10\% \\
\text{f}_B &= 10^{-5} \quad \Delta A = 20\% \\
\text{f}_B &= 10^{-3} \quad \Delta A = 20\% \\
\end{align*}
\]
becomes stronger for larger baseline and muon energy. The works $^{12,13,18}$, which basically used $\Delta \chi^2$ in (1), agree with each other to certain extent (there are some differences on the reference values for the oscillation parameters) while the result by Koike et al.$^{11}$ is quite different from others. This discrepancy is due to the fact that they adopted $\Delta \tilde{\chi}^2$ in (3) as was mentioned above. There are slight differences between the results by the group $^{12,18}$ and those by the other $^{13}$ and this appears to come from different statistical treatments. In the future it should be studied what makes a difference to get the optimum set $(E_\mu, L)$. The detector size required to reject a hypothesis "$\delta=0$" as a function of $\theta_{13}$ is given in Figure 2 (taken from Ref.$^{19}$). Figure 2 shows the sensitivity of neutrino factories to the CP phase.

### Table 1: Comparison of different works

| Ref. | correlations of $\theta_{ij}, m_{ij}^2$ | $|\Delta A/A|$ | $f_B$ | $\Delta m^2_{21}/10^{-5}$eV$^2$ | $E_{th}$/GeV | Optimized $E_\mu$/GeV | Optimized $L$/km |
|------|----------------------------------------|-----------------|-------|-------------------------------|---------------|------------------------|-----------------|
| KOS$^{11}$ | included | 10% | 0 | 5 | 1 | $\lesssim 6$ | 600 – 800 |
| | | | | | | 10 | $\lesssim 50$ | 500 – 2000 |
| FHL$^{12}$ | included | 0 | 0 | 10 | 4 | 30 – 50 | 2800 – 4500 |
| PY$^{13}$ | included | 5% | $10^{-9}$ | 3.2 | 0.1 | $\sim 50$ | $\sim 3000$ |
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| H$^{18}$ | included | 10% | 0 | 10 | 0.1 | $\geq 20$ | $\geq 2000$ |
| | | | | | | | | $\sim 25$ | $\sim 1500$ |

Some people have questioned whether the sensitivity to the CP phase at a neutrino factory increases infinitely as the muon energy increases, and Lipari$^{20}$ concluded that the sensitivity is lost at high energy. In the work$^{13}$ the behaviors of $\Delta \chi^2$ in (1) was studied analytically for high muon energy and it was shown after the correlations between $\delta$ and any other oscillation parameter or

3 High energy behaviors of $\Delta \chi^2$

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Figure 3: Data size (kt-yr) required to reject a hypothesis of $\delta = 0$ at 3$\sigma$ when the true value is $\delta = \pi/2$, in the case of a neutrino factory with $10^{21}$ useful muon decays per year and a background fraction $f_B = 10^{-3}$. 

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A is taken into account that

\[ \Delta \chi^2 \propto \left( \frac{J}{\sin \delta} \right)^2 \frac{1}{E_{\mu}} \left( \sin \delta + \text{const} \frac{\Delta m^2_{32} L}{E_{\mu} \cos \delta} \right)^2 \]  

for large \( E_{\mu} \), where \( J \equiv \frac{(c_{13}/8)}{8} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} \sin \delta \) stands for the Jarlskog parameter. The behavior (4) is the same as that for \( \Delta \tilde{\chi}^2 \) in (3) and it is qualitatively consistent with the claim by Lipari. It is remarkable that (4) is different from a naively expected behavior

\[ \Delta \chi_{\text{naive}}^2 \propto E_{\mu} (\cos \delta - 1)^2. \]

This is because the correlation between \( \delta \) and other parameters is taken into account. Koike et al. criticize the analysis with \( \Delta \chi^2 \) by saying that this quantity looks mainly at the CP conserving part at high muon energy (10GeV \( \lesssim E_{\mu} \lesssim 50 \)GeV). The behavior (4) indicates, however, that CP violating part becomes dominant after the correlation between \( \delta \) and other parameters is taken into consideration and the criticism by Koike et al. does not apply.

### 4 Parameter degeneracy

The discussions in the previous sections have been focused on rejection of a hypothesis \( \delta = 0 \). Once \( \delta \) is found to be nonvanishing, it becomes important to determine the precise value of \( \delta \). It has been pointed out that various kinds of parameter degeneracy exist. Burguet-Castell et al. found degeneracy in (\( \delta, \theta_{13} \)), Minakata and Nunokawa found the one in the sign of \( \Delta m^2_{32} \), and Barger et al. found the one in the sign of \( \pi/4 - \theta_{23} \). To understand the four-fold ambiguity it is instructive to draw a trajectory in the \( P(\nu_{\mu} \to \nu_e) - P(\bar{\nu}_{\mu} \to \bar{\nu}_e) \) plane (See Figure 4 taken from Ref.24). From Figure 4 it is obvious that there exists four-fold ambiguity for a given set of the oscillation parameters. Also the position of the ellipse has degeneracy in interchanging \( \theta_{23} \leftrightarrow \pi/2 - \theta_{23} \), so that in general eight-fold degeneracy is expected. It was proposed to do experiments at two baselines to remove this degeneracy. In reality one has to evaluate numbers of events for \( \nu_{\mu} \to \nu_e \) and \( \bar{\nu}_{\mu} \to \bar{\nu}_e \) (Notice that Figure 4 deals with the probabilities only), and the correlations of errors have to be taken into consideration as well, so determination of the oscillation parameters is even more difficult than what Figure 4 indicates.

### 5 Summary

People in the field reached consensus that neutrino factories can measure the CP phase \( \delta \) with the detector size larger than \( 10^{21} \mu \cdot 100 \text{kt-yr} \) for \( \sin^2 2\theta_{13} > 10^{-3} \)
unless $|\delta|$ is small. The detailed study on the optimized muon energy and baseline still needs to be done.

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References

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