Fundamental phase-shift detection properties of optical multimode interferometers are analyzed. Limits on perfectly distinguishable phase shifts are derived for general quantum states of a given average energy. In contrast to earlier work, the limits are found to be independent of the number of interfering modes. However, the reported bounds are consistent with the Heisenberg limit. A short discussion on the concept of well-defined relative phase is also included.

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I. INTRODUCTION

It is today well-known that the use of quantum-mechanical states can improve the precision of interferometric measurements. According to the so-called standard quantum limit [1], the precision of optical measurements employing classical states cannot increase faster than $\propto 1/\sqrt{E}$, where $E$ is the energy used in the measurement. However, the use of nonclassical states allows us to reach the Heisenberg limit $\propto 1/E$ [2], which for high energies would give a remarkable improvement in accuracy. Several setups have theoretically been shown to work at the Heisenberg limit [3-6]. The standard quantum limit has also been circumvented experimentally [7-10]. However, the fragile nature of the quantum states has so far prevented these measurements from being carried out with higher energies. Therefore, high intensity classical interferometry reach a much better overall accuracy.

Recently, a bound closely related to the Heisenberg limit was given by Margolus and Levitin [11]. The bound gives the time necessary for a state of a closed system to become orthogonal for a given average energy. This limits the rate of operations in quantum information processing and the resolution in interferometry. In an earlier paper [12], we derived the states that minimize the time needed to freely evolve into another state, whose overlap with the original one was given. This would correspond to minimizing the necessary phase shift for a single-mode state.

The present paper is devoted to the phase-shift detection properties of multimode interferometers. In many interferometric measurements, a single induced phase shift is tracked by monitoring the interference fringes in the output of a two-mode interferometer. However, it is often possible to induce several phase shifts of the same, or smaller, magnitude (possibly with different signs) in the different arms of a multimode interferometer. Here, we investigate whether this fact can be used to improve the accuracy of such measurements. We will assume that the interfering fields have the same optical frequency, so that the total energy is proportional to the number of photons used.

In an earlier investigation [13], it was concluded that the accuracy of the multimode interferometer would improve indefinitely with the number of modes. However, we find that there is no fundamental advantage in using more than two modes and that, in our eyes, the conclusions in Ref. [13] stem from an unfortunate choice of figure of merit. Rather, the accuracy is found to be limited only by the energy used in the measurement and scale according to the Heisenberg limit.

II. PROBLEM FORMULATION

Accuracy can be defined in many different ways. For example, in a recent paper on multimode interferometry [14], the width of the major peak of the phase distribution was taken as a measure of the precision. Here, we will define it as the smallest phase shift required to give a perfectly distinguishable outcome, i.e., the phase-shifted state is required to be orthogonal to the original state.

More precisely, we look for the smallest phase shift that can be detected with certainty using an $M$-mode interferometer and a state whose average energy is given. The smallest phase shift $\phi$ should here be interpreted in the following sense (cf. Figs. 1 and 2). We consider the case where all arms of the interferometer have induced phase shifts that satisfy $\phi_m = \lambda_m \phi$, where $-1 \leq \lambda_m \leq 1$ and $\phi > 0$ is the parameter to be minimized under the condition that it results in a state that is perfectly distinguishable from the original. Since the $N$-photon state $(|0,N\rangle + |N,0\rangle)/\sqrt{2}$ can be made orthogonal in a two-mode interferometer by a relative phase shift of $\pi/N$ [5], and the two-mode interferometer is a special case of the multimode interferometer considered here, we know that the smallest necessary phase shift $\phi$ is smaller than, or equal to, $\pi/2N$.

Minimizing $\phi$ above corresponds to an experimental situation where we have an arbitrary number of induced phase shifts at our disposal (one in each mode). However, we easily identify another experimental situation