Brane cosmology driven by the rolling tachyon

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(April 11, 2002)

Brane cosmology driven by the tachyon rolling down to its ground state is investigated. We adopt an effective field theoretical description for the tachyon and Randall-Sundrum type brane world scenario. After formulating basic equations, we show that the standard cosmology with a usual scalar field can mimic the low energy behavior of the system near the tachyon ground state. We also investigate qualitative behavior of the system beyond the low energy regime for positive, negative and vanishing 4-dimensional effective cosmological constant \( \Lambda_4 = \kappa_5^4 V(T_0)^2/12 - |\Lambda_5|/2 \), where \( \kappa_5 \) and \( \Lambda_5 \) are 5-dimensional gravitational coupling constant and (negative) cosmological constant, respectively, and \( V(T_0) \) is the (positive) tension of the brane in the tachyon ground state. In particular, for \( \Lambda_4 < 0 \) the tachyon never settles down to its potential minimum and the universe eventually hits a big-crunch singularity.

I. INTRODUCTION

Pioneered by Sen [1], the study of non-BPS objects such as non-BPS branes, brane-antibrane configurations or spacelike branes [2] has been attracting physical interests in string theory [3]. These objects are expected to be important for our understanding non-perturbative dualities beyond the BPS level. Moreover, they may play roles in cosmology. Sen showed that classical decay of unstable D-brane in string theories produces pressure-less gas with non-zero energy density [4]. Gibbons took into account coupling to gravitational field by adding an Einstein-Hilbert term to the effective action of the tachyon on a brane, and initiated a study of “tachyon cosmology”, cosmology driven by the tachyon rolling down to its ground state [5]. Fairbairn and Tytgat considered possibility of inflation driven by the rolling tachyon [6].

Another subject in which branes play important roles is the brane-world scenario. Actually, in this scenario a brane in a higher dimensional bulk spacetime is supposed to be our universe itself. Randall and Sundrum [7] showed that, in a 5-dimensional AdS background, 4-dimensional Newton’s law of gravity can be reproduced on the world-volume of a 3-brane, despite the existence of the infinite fifth dimension. There are also cosmological solutions in this scenario, in which the standard cosmology is restored at low energy, provided that a parameter representing the mass of a bulk black-hole is small enough [8–13].

Considering that the tachyon is a degree of freedom on a brane, it is perhaps interesting to consider the brane-world scenario to take into account gravity. In this case, the Einstein-Hilbert term is introduced in the bulk action rather than in the brane action. Hence, purpose of this paper is to initiate the brane-world version of the tachyon cosmology. We adopt an effective field theoretical description for the tachyon and Randall-Sundrum type brane world scenario. After formulating basic equations, we show that the standard cosmology with a usual scalar field can mimic the low energy behavior of the system near the tachyon ground state. An obvious consequence of this result is that if the tachyon potential has a minimum at finite distance from the maximum and if it oscillates about the minimum then the tachyon will behave like pressure-less gas. We also investigate qualitative behavior of the system beyond the low energy regime for positive, negative and vanishing 4-dimensional effective cosmological constant.

The rest of this paper is organized as follows. In Sec. II we briefly review the effective field theoretical description for the rolling tachyon. In Sec. III we consider the Randall-Sundrum type brane cosmology driven by the rolling tachyon and investigate the low energy behavior of the system near the tachyon ground state. In Sec. IV we investigate a simple double-well potential and a run-away potential to see qualitative behavior of the system beyond the vicinity of the tachyon ground state. Sec. V is devoted to a summary of this paper.

II. ROLLING TACHYON

Let us consider an n-brane in D-dimensional spacetime. The imbedding of the world volume of the brane can be described by the parametric equation

\[ x^M = Z^M(y), \]  

(1)
where \( \{ x^M \} \) is a coordinate system in the \( D \)-dimensional bulk and \( y \) denotes a set of \( (n+1) \) parameters \( \{ y^\mu \} \). The parameters \( y^\mu \) will play a role of \( (n+1) \)-dimensional coordinates on the world-volume of the \( n \)-brane. Throughout this paper we shall adopt the effective field theoretical description of non-BPS branes proposed in refs. [14–16], which includes a tachyon. The effective action for the brane with the tachyon field \( T \) is

\[
S_{\text{brane}} = - \int d^{n+1}y \sqrt{|\det \tilde{q}|} V(T),
\]

where \( T(V) \) is a tachyon potential,

\[
\dot{q}_{\mu\nu} = q_{\mu\nu} + \partial_\mu T \partial_\nu T,
\]

\( q_{\mu\nu} \) is the induced metric defined by

\[
q_{\mu\nu} = g_{MN} \frac{\partial Z^M}{\partial y^\mu} \frac{\partial Z^N}{\partial y^\nu},
\]

and \( g_{MN} \) is the bulk metric.

Note that the choice of the tachyon potential \( V(T) \) is very important. Different potentials give different dynamics of the tachyon and the brane universe. Hence, if we could, we would like to use potentials predicted by rigorous calculations based on boundary string field theory or conformal field theory for configurations of our interest. As we shall see, we would like to consider very simple forms of the tachyon potential. In Sec. IV we shall consider a simple double-well potential

\[
V(T) = (T^2 - 1)^2 + V_0,
\]

and a simple run-away potential

\[
V(T) = \frac{1}{\cosh(T)} + V_0
\]

to investigate qualitative behavior of the system.

The surface stress energy tensor \( S^{\mu\nu} \) is given by

\[
S^{\mu\nu} = \frac{2}{\sqrt{|q|}} \delta S_{\text{brane}} / \delta q_{\mu\nu} = - \sqrt{\frac{|\det \tilde{q}|}{|\det q|}} V(T)(\tilde{q}^{-1})^{\mu\nu}.
\]

The equation of motion for the tachyon is

\[
\frac{1}{\sqrt{|\det q|}} \partial_\mu \left[ \sqrt{|\det \tilde{q}|} V(T)(\tilde{q}^{-1})^{\mu\nu} \partial_\nu T \right] - V'(T) = 0.
\]

For a homogeneous isotropic brane, we can assume the following form of the induced metric \( q_{\mu\nu} \) and the tachyon field \( T \) without loss of generality.

\[
q_{\mu\nu} dy^\mu \, dy^\nu = -dt^2 + a(t)^2 \Omega_{ij}^K dy^i \, dy^j,
\]

\[
T = T(t),
\]

where \( \Omega_{ij}^K \) is the metric of the \( n \)-dimensional constant curvature space with the curvature constant \( K \):

\[
\Omega_{ij}^K dy^i \, dy^j = \frac{d\rho^2}{1 - K \rho^2} + \rho^2 d\Omega_{n-1}^2.
\]

Here, \( d\Omega_{n-1}^2 \) is the metric of the \( (n-1) \)-dimensional unit sphere. Positive, zero and negative values of \( K \) correspond to \( S^n \), \( R^n \) and \( H^n \), respectively. With the above form of \( q_{\mu\nu} \) and \( T \), the equation of motion and the surface stress energy tensor are reduced to

\[
\frac{V\ddot{T}}{1 - T^2} + \frac{\dot{a}}{a} V\dot{T} + V' = 0,
\]

\[
S_{\nu}^{\mu} = \begin{pmatrix}
-\rho & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & \ddots & 0 \\
0 & 0 & 0 & p
\end{pmatrix},
\]

where \( \rho \) is the surface density.
where a dot denotes derivative with respect to $t$, and

$$\rho = \frac{V}{\sqrt{1 - \dot{T}^2}},$$

$$p = -V \sqrt{1 - \dot{T}^2}. \hspace{1cm} (11)$$

The equation of motion is formally equivalent to the conservation equation $\nabla_\mu S^\mu_\nu = 0$, or

$$\dot{\rho} + n\frac{\dot{a}}{a}(\rho + p) = 0. \hspace{1cm} (12)$$

### III. BRANE COSMOLOGY

Now we consider brane cosmology driven by the rolling tachyon. We consider Randall-Sundrum brane world scenario on a 3-brane ($n = 3$) in a 5-dimensional bulk spacetime ($D = 5$) [7]. We assume that the bulk is invariant under the $Z_2$ reflection along the brane and described by 5-dimensional Einstein pure gravity with a negative cosmological constant and that the brane motion is described by Israel’s junction condition [17]. With these assumptions, the bulk geometry is AdS-Schwarzschild spacetime [18] and the evolution of the brane is governed by [19,20]

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa_5^4}{36} \rho^2 - \frac{K}{a^2} + \frac{\mu}{a^4} - \frac{1}{l^2}, \hspace{1cm} (13)$$

where $\kappa_5$ is the 5-dimensional gravitational coupling constant, $l = \sqrt{6/|\Lambda_5|}$ is the length scale of the bulk cosmological constant $\Lambda_5$, and $\mu$ is the mass parameter of the bulk black hole. This equation looks very different from the standard Friedmann equation in the sense that the first term in the right hand side is proportional to $\rho^2$. Nonetheless, the standard cosmology is restored at low energy if brane tension is properly introduced [8–13]. The term $\mu/a^4$ is due to Weyl tensor in the bulk [21] and can be understood as dark radiation [11]. Thus, our basic equations for brane cosmology driven by the rolling tachyon are the equation of motion (9) and the generalized Friedmann equation (13) with $\rho$ given by (11).

Let us analyze behavior of the system near the tachyon ground state $T = T_0$ and show that the standard cosmology with a usual scalar field can mimic the low energy behavior of the system. For this purpose, we assume that

$$[V(T) - V(T_0)]/V(T_0) \sim l[V'(T)/V(T) - T^2 \sim lT \sim O(\epsilon)], \hspace{1cm} (14)$$

where $\epsilon$ is a dimensionless small parameter. The actual value of $T_0$ can be either finite or infinite. With this assumption, the generalized Friedmann equation (13) and the tachyon equation (9) are reduced to

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3} \left[\frac{1}{2} \dot{\phi}^2 + V_{eff}(\phi)\right] - \frac{K}{a^2} + \frac{\Lambda_4}{3} + \frac{\mu}{a^4} + O(\epsilon^2), \hspace{1cm} (15)$$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + V'_{eff}(\phi) = O(\epsilon^2), \hspace{1cm} (16)$$

where $\phi = \sqrt{V(T_0)}T$, $V_{eff}(\phi) = V(T) - V(T_0)$, $G_N = \kappa_5^4 V(T_0)/48\pi$ and $\Lambda_4 = \kappa_5^4 V(T_0)^2/12 - 3l^{-2}$. These equations are the same as the corresponding equations in the standard cosmology driven by a usual scalar field $\phi$ with the potential $V_{eff}(\phi)$, the cosmological constant $\Lambda_4$ and the dark radiation $\mu/a^4$, up to corrections of order $O(\epsilon^2)$. Consistency of the above reduced equations with the assumption (14) requires that

$$Kl^2a^{-2} \sim l^2\Lambda_4 \sim \mu l^2a^{-4} \sim O(\epsilon). \hspace{1cm} (17)$$

In particular, we need to impose a fine-tuning between the tachyon vacuum energy $V(T_0)$ and the bulk cosmological constant $\Lambda_5$ so that the 4-dimensional effective cosmological constant $\Lambda_4$ is small compared to $\Lambda_5$.

It is easy to introduce other matter fields on the brane, following refs. [8–13]. In this case, the generalized Friedmann equation (13) becomes

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3} \left[\frac{1}{2} \dot{\phi}^2 + V_{eff}(\phi) + \rho_{\text{matter}}\right] - \frac{K}{a^2} + \frac{\Lambda_4}{3} + \frac{\mu}{a^4} + O(\epsilon^2), \hspace{1cm} (18)$$
where $\rho_{\text{matter}}$ is energy density of other matter fields on the brane, and the equation of motion of the tachyon is unchanged. Here, we have assumed that $\rho_{\text{matter}}/V(T_0) = O(\epsilon)$. Therefore, the standard cosmology can mimic the low-energy behavior of the brane cosmology driven by the tachyon.

It is well-known in the standard cosmology that a scalar field oscillating about a potential minimum behaves like pressure-less gas. Hence, if the tachyon potential in the effective field theory has a minimum at finite distance from the maximum and if it oscillates about the minimum then the tachyon will behave like pressure-less gas. Note that calculations in boundary string field theory suggest that the minimum of the tachyon potential is a finite distance away from the maximum [22,23]. It is probably worth while mentioning Sen’s result that decay of unstable D-branes in string theory produces pressure-less gas with non-zero energy density but that the minima must be at infinity in the effective field theory description [4]. There is an interesting accidental coincidence (production of pressure-less gas) between Sen’s result in conformal field theory and the above conclusion based on the effective field theory and the brane world.

IV. SIMPLE EXAMPLES

To see behavior of the system beyond the low energy regime satisfying (14) and (17), we need to analyze the generalized Friedmann equation (13) and the tachyon equation (9) directly. In the following, for simplicity we consider a spatially flat brane ($K = 0$) in the pure AdS bulk ($\mu = 0$). By introducing dimensionless quantities $\tau = t/l$, $x = T/l$, $y = \partial_x x$ and $z = \partial_x a/a$, our basic equations in this case are written as

$$
\begin{align*}
\partial_\tau x &= y, \\
\partial_\tau y &= -(1 - y^2) \left[ 3yz + \frac{v'(x)}{v(x)} \right],
\end{align*}
$$

where $v(x) = \kappa_5^2 lV(T)/6$, $v'(x) = \partial_x v(x)$, and $z$ is given by

$$
z^2 = \frac{v(x)^2}{1 - y^2} - 1.
$$

There are the expanding branch ($z > 0$) and the contracting branch ($z < 0$). Hence, the projection to the $xy$-plane is two-fold. Hereafter, we shall consider the following three cases separately: (i) $v(x_0) > 1$ ($\Lambda_4 > 0$), where $x_0$ is a potential minimum; (ii) $v(x_0) < 1$ ($\Lambda_4 < 0$); (iii) $v(x_0) = 1$ ($\Lambda_4 = 0$). In the case (i), expanding and contracting branches are disconnected. In the case (ii) the two branches are connected in the $xyz$-space through the intersection of the surfaces $v(x)^2 = 1 - y^2$ and $z = 0$. In the critical case (iii), two branches are just touching at a point $(x,y,z) = (x_0,0,0)$ in the $xyz$-space. (See ref. [24] for discussion about the standard scalar field cosmology with negative potentials.)

Hereafter, unless otherwise stated, we shall consider the expanding branch. The behavior of the system can be easily understood by plotting the vector field $(\partial_\tau x, \partial_\tau y)$ in the $xy$-plane.

First, let us consider the critical case (iii). For a simple double-well potential $v(x) = (x^2 - 1)^2 + 1$, figure 1 shows a plot near the minimum $x = 1$. This plot shows that the tachyon field oscillates about the minimum. Since the conservation equation (12) implies that expansion of the brane universe dilutes the energy density $\rho$, the amplitude of the oscillation decays. In order to see a global picture of the vector field, it is convenient to introduce a normalized vector field $(N\partial_\tau x, N\partial_\tau y)$, where $N = 1/\sqrt{(\partial_\tau x)^2 + (\partial_\tau y)^2}$. Figure 2 shows the global picture of the normalized vector field. It is easy to see how the tachyon decays into the minimum with oscillation. Hence, for the critical case (iii), we may generally expect that the tachyon rolls down to the minimum of the potential, that it oscillates about the minimum with decaying amplitude and that the universe approaches to Minkowski spacetime ($H = z/l = 0$).

Next, let us consider the case (i). Figure 3 and figure 4 show plots for the potential $v(x) = (x^2 - 1)^2 + 1.5$. In this case, rolling down of the tachyon is slower than the case (iii) due to the larger “cosmic friction term” $3yz$ in (19). For the same reason, decay of the oscillation about the minimum in the case (i) is faster than the case (iii). In the case (i) the universe approaches to de Sitter spacetime ($H = z/l > 0$).

Finally, let us consider the case (ii). Figure 5 and figure 6 show the expanding branch and the contracting branch, respectively, for the potential $v(x) = (x^2 - 1)^2 + 0.9$. These two branches are connected in the $xyz$-space through a throat. In each figure, the throat is the boundary between regions with and without arrows. Hence, even if the universe was initially in the expanding branch, the system approaches to the throat and goes into the contracting branch. Eventually, the universe hits a big-crunch singularity. Note that the tachyon never settles down to the potential minimum in the case (ii).
The general qualitative behavior explained here is consistent with the result of ref. [24] in which the standard scalar field cosmology was analyzed, provided that $V_0$ in ref. [24] is replaced by the 4-dimensional effective cosmological constant $\Lambda_4 = \kappa_5^4 V(T_0)^2 / 12 - |\Lambda_5|/2$.

![Diagram](image1.png)

**FIG. 1.** A plot of the vector field $(\partial_x, \partial_y)$ for the double-well potential $v(x) = (x^2 - 1)^2 + 1$. The diamond at $(x, y) = (1, 0)$ represents an attractor corresponding to Minkowski spacetime with the tachyon in its ground state.

![Diagram](image2.png)

**FIG. 2.** The global picture of the normalized vector field for the double-well potential $v(x) = (x^2 - 1)^2 + 1$. The diamond at $(x, y) = (1, 0)$ represents an attractor corresponding to Minkowski spacetime with the tachyon in its ground state.
FIG. 3. A plot of the vector field $(\partial_x, \partial_y)$ for the double-well potential $v(x) = (x^2 - 1)^2 + 1.5$. The diamond at $(x, y) = (1, 0)$ represents an attractor corresponding to de Sitter spacetime with the tachyon in its ground state.

FIG. 4. The global picture of the normalized vector field for the double-well potential $v(x) = (x^2 - 1)^2 + 1.5$. The diamond at $(x, y) = (1, 0)$ represents an attractor corresponding to de Sitter spacetime with the tachyon in its ground state.
FIG. 5. The global picture of the normalized vector field for the double-well potential \( v(x) = (x^2 - 1)^2 + 0.9 \) in the expanding branch. The region without arrows is not allowed. The boundary between the allowed and disallowed regions corresponds to the throat through which the system evolves to the contracting branch shown in figure 6.

FIG. 6. The global picture of the normalized vector field for the double-well potential \( v(x) = (x^2 - 1)^2 + 0.9 \) in the contracting branch. The region without arrows is not allowed. The boundary between the allowed and disallowed regions corresponds to the throat through which the system evolves from the expanding branch shown in figure 5 to the contracting branch shown here.

The above examples of double-well potentials have minima at finite \( T \). The finiteness is consistent with the result in boundary string field theory that the minimum of the tachyon potential is a finite distance away from the maximum [22,23]. However, this is not, at least apparently, consistent with the result in conformal field theory that the tachyon evolves to the minimum without oscillation [25,4]. Sen suggested that the minima must be at infinity in the effective field theory description [4]. Hence, it may be relevant to consider not a double-well potential but a run-away potential.

As a simple example, let us consider a ran-away potential \( v(x) = 1/\cosh(x) + 1 \). Figure 7 shows a plot of the vector \( (\partial_x, \partial_y) \). In order to see a global behavior of the system, figure 8 plots the normalized vector field \( (N\partial_x, \)
\( N \partial_t y \), where \( N = 1/\sqrt{(\partial_t x)^2 + (\partial_t y)^2} \). From these two figures, it is easy to see how rolling down of the tachyon slows down. Hence, we may expect inflation on the brane by the rolling tachyon, depending on the form of the potential. Actually, the effective potential \( V_{\text{eff}}(\phi) \) corresponding to the run-away potential \( v(x) = 1/\cosh(x) + 1 \) is

\[
V_{\text{eff}}(\phi) = 6\kappa_4^{-2}l^{-2}/\cosh(\kappa_4 \phi/\sqrt{6}) \sim \exp(-\kappa_4 \phi/\sqrt{6}) \quad (\kappa_4 \phi \gg 1),
\]

where \( \kappa_4^2 = \kappa_3^2/l \), and it is known that in the standard cosmology the so called power-law inflation occurs for this potential \( V_{\text{eff}}(\phi) \) [26–28].

**FIG. 7.** A plot of the vector field \((\partial_t x, \partial_t y)\) for the run-away potential \( v(x) = 1/\cosh(x) + 1 \). The dotted line represents points satisfying \( \partial_t y = 0 \).

**FIG. 8.** The global picture of the normalized vector field for the run-away potential \( v(x) = 1/\cosh(x) + 1 \). The dotted line represents points satisfying \( \partial_t y = 0 \).
V. SUMMARY

We have investigated brane cosmology driven by the tachyon rolling down to its ground state. We have adopted the effective action (2) for the tachyon and Randall-Sundrum type brane world scenario. We have shown that the standard cosmology with a usual scalar field can mimic the low energy behavior of the system near the tachyon ground state. In particular, if the tachyon potential in the effective field theory has a minimum at finite distance from the maximum and if it oscillates about the minimum then the tachyon will behave like pressure-less gas. There is an interesting accidental coincidence (production of pressure-less gas) between Sen’s result in conformal field theory [25,4] and the above conclusion based on the effective field theory and the brane world.

We have also analyzed qualitative behavior of the system beyond the low energy regime for a spatially flat brane in pure AdS bulk. As an example we have considered a double-well potential. For the double-well potential, qualitative behavior of the system is classified by the sign of \( \Lambda_4 = \kappa_4^5 V(T_0)^2/12 - |\Lambda_5|/2 \): (i) \( \Lambda_4 > 0 \); (ii) \( \Lambda_4 < 0 \); (iii) \( \Lambda_4 = 0 \). In the case (i) the tachyon rolls down the potential hill and starts oscillating about a minimum. The amplitude of the oscillation decays due to cosmic expansion and the universe approaches to de Sitter spacetime. The case (iii) is similar to the case (i), but the universe approaches to Minkowski spacetime. In the case (ii), even if the universe was initially expanding, it starts contracting and eventually hits a big-crunch singularity. In this case the tachyon never settles down to the potential minimum. Finally, we considered a run-away potential, for which the ground state is at infinity. For the run-away potential with \( \Lambda_4 = 0 \), rolling down of the tachyon slows down and power-law inflation can occur.

ACKNOWLEDGMENTS

The author would like to thank Lev Kofman, Shiraz Minwalla, Ricardo Schiappa, Andrew Strominger and Lisa Randall for useful discussions and/or comments. He would be grateful to Werner Israel for continuing encouragement. This work is supported by JSPS Postdoctoral Fellowship for Research Abroad.