Nuclear matter with off-shell propagation

PACS. 21.20.+x Nuclear Matter

Key words. Nuclear matter - interaction term - thermodynamic properties - hadronization. In this Letter we present a new method for the calculation of off-shell propagation of nuclear matter. This method, based on the off-shell approximation, does not require the imposition of the hadronization condition, which is a common approximation in nuclear matter calculations. The method is demonstrated in the context of the Skyrme model, where it is shown to provide results that are in excellent agreement with those obtained using other methods. 

January 12, 2002
ergy is small. We restrict ourselves to normal nuclear matter for all densities studied here. It allows us to compare with BHF calculations which are performed exclusively in the normal phase of nuclear matter.

The results here presented are obtained using a separable parameterization of the Paris potential [18] for $S$, $P$, $D$ and $F$ partial waves, for symmetric nuclear matter. We use rank 3 and rank 4 parameterization for the $^1S_0$ and $^3S_1 - ^3D_1$ partial waves. In the $^3P_0$ partial wave we use Morgan I interaction, in order to avoid unphysical resonances far off-shell. In the numerical iteration the full spectral function is discretized. For momenta close to the Fermi momentum the spectral function is separated into a background part and a quasiparticle peak approximated by a delta function. The numerical treatment of the energy integrations for the spectral functions is done using convolution algorithms [10].

The $T$-matrix approximation resumes ladder diagrams with dressed particle-particle and hole-hole propagators

\[
<\mathbf{p}|T(\mathbf{P}, \Omega)|\mathbf{p}'> = V(\mathbf{p}, \mathbf{p}') + \int \frac{d\omega_1}{2\pi} \frac{d\omega_2}{2\pi} \int \frac{d^3 q}{(2\pi)^3} V(\mathbf{p}, \mathbf{q}) \frac{1 - f(\omega_1) - f(\omega_2)}{\Omega - \omega_1 - \omega_2 + i\epsilon}
\]

where

\[
A(\mathbf{p}, \omega_1)A(\mathbf{p}, \omega_2) < \mathbf{p}|T(\mathbf{P}, \Omega)|\mathbf{p} >
\]

\[
\text{Im}\Sigma(\mathbf{p}, \omega) = \int \frac{d\omega_1}{2\pi} \int \frac{d^3 k}{(2\pi)^3} A(k, \omega_1)
\]

\[
< (\mathbf{p} - \mathbf{k}) | 2i\text{Im}\Sigma(\mathbf{p} + \mathbf{k}, \omega + \omega_1) | (\mathbf{p} - \mathbf{k}) > > A
\]

\[
(f(\omega_1) + b(\omega + \omega_1))
\]

where \(A(\mathbf{p}, \omega) = \frac{\text{Im}\Sigma(\mathbf{p}, \omega)}{(\omega - p^2/2m - \text{Re}\Sigma(\mathbf{p}, \omega))^2 + \text{Im}\Sigma(\mathbf{p}, \omega)}\)

is the self-consistent spectral function of the nucleon and \(b(\omega)\) is the Bose distribution. The real part of the self-energy is related to \(\text{Im}\Sigma\) by a dispersion relation

\[
\text{Re}\Sigma(\mathbf{p}, \omega) = \Sigma_{HF}(\mathbf{p}) + \mathcal{P} \int \frac{d\omega_1}{\pi} \frac{\text{Im}\Sigma(\mathbf{p}, \omega_1)}{\omega - \omega_1}
\]

with \(\Sigma_{HF}(\mathbf{p})\) the Hartree-Fock self-energy. Eqs. 2, 3, 5 and 4 are to be solved iteratively and at each iteration the chemical potential \(\mu\) is adjusted to fulfill the condition on the density \(\rho\)

\[
\int \frac{d\omega}{2\pi} \int \frac{d^3 p}{(2\pi)^3} A(p, \omega)f(\omega) = \rho
\]

The spectral functions obtained in the self-consistent solution consist of a quasiparticle peak and a broad background (Fig. 1). As function of momentum the position of the peak in the spectral function follows approximately the quasiparticle dispersion relation

\[
\omega_p = \frac{p^2}{m} + \text{Re}\Sigma(p, \omega_p)
\]

The quasiparticle peak is very sharp for momenta close to the Fermi momentum. It is a manifestation of the quasiparticle nature of excitations close to the Fermi surface. Indeed we find that the single-particle width \(-2\text{Im}\Sigma(p, \omega_p)\) is proportional to \((p - p_F)^2\) close to the Fermi momentum. The background of the spectral functions extend far from the quasiparticle peak. The part of the spectral function below the Fermi energy leads to nonzero occupancy for momenta above \(p_F\) and gives a large, negative contribution to the binding energy for all momenta.

The nucleon momentum distribution

\[
n(\mathbf{p}) = \int \frac{d\omega}{2\pi} A(\mathbf{p}, \omega)f(\omega)
\]

is very different different from the Fermi-Dirac distribution. Momentum states below the Fermi momentum are depleted and a tail in the distribution \(n(\mathbf{p})\) for large momenta appears. The T-matrix approximation leads to a Fermi liquid behavior in the normal phase, with a jump in the fermion density of \(Z_{\rho, \mathbf{p}} = \left(1 - \frac{\text{Re}\Sigma(p, \omega_p)}{\text{Im}\Sigma(p, \omega_p)}\right)^{-1} \simeq .74\) at the Fermi momentum. In the calculation the Fermi momentum is fixed by the constraint (6) on the total density. For a conserving approximation the Fermi momentum should be the same as the Fermi momentum of a free
fermion gas [19, 11]. This thermodynamical consistency relation is verified to a good accuracy by our calculations for the range of densities studied.

The energy per particle, in the case of only two-body interactions, can be obtained from the single-particle spectral function

\[ E/N = \frac{1}{\rho} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\pi} \omega A(p, \omega) f(\omega). \quad (9) \]

The binding energy per nucleon as function of the Fermi momentum is presented in Fig. 2 for the self-consistent T-matrix approximation and compared to results from G-matrix calculations using the continuous choice of the auxiliary potential. The results of the T-matrix approach lie above the BHF binding energy for densities close to the phenomenological saturation point. Since we know that further hole line corrections do not modify the continuous BHF results drastically, we get an assessment of the accuracy of the T-matrix approach. The higher the density the larger the discrepancy becomes. Correspondingly the saturation point in the T-matrix approach is shifted to lower densities \( \rho = 1.4\rho_0 \) instead of \( 2.4\rho_0 \) and lower binding energies (the binding energy is reduced by 4MeV at \( \rho_0 \)). Very similar results are found for the equation of state of pure neutron matter [17]. We note the the Hugenholtz-Van Hove condition (11) is very well satisfied.

The origin of the of the binding energy in the T-matrix approximation can be understood writing Eq. (9) as

\[ E/N = \frac{1}{\rho} \int \frac{d^3p}{(2\pi)^3}n(p)\frac{1}{2} \left( \frac{p^2}{2m} + \omega_p \right). \quad (10) \]

In Fig. 3 we compare the removal energy \( \omega_p \) to the quasiparticle energy \( \omega_p \). Due to a large contribution of the background strength of the spectral function lying below the quasiparticle peak the removal energy is much below the quasiparticle energy. On the other hand, the single-particle energy \( \omega_p \) in the T-matrix approximation is generally above the one obtained from G-matrix calculations. It can be seen by comparing the Fermi energies in the two approximations in Fig. 2. These differences explain why the Fermi energy in the conserving T-matrix approximation is equal to the binding energy at the local saturation point, following the Hugenholtz-Van Hove relation (11). The binding energy is determined by \( \omega_p \) and \( E_F = \omega_p \), for the T-matrix calculation, whereas both quantities are determined by \( \omega_p \) in the G-matrix scheme.

This paper presents the first results self-consistent T-matrix calculation of saturation properties of nuclear matter with a realistic potential. The binding energy obtained is smaller than the BHF result with the continuous auxiliary potential. We note that a very similar shift in binding energy is observed in a BHF calculation, when including the rearrangement terms contribution to the binding energy [15]. In Ref. [15] by considering rearrangement term corrections to the single particle energies and to the binding energy an improvement of the fulfillment Hugenholtz-
Van Hove relation is found. The same can be observed in the $T$-matrix approach, we destroy a bit the binding energies and improve considerably the single particle energies from the BHF approach to get the relation (1) right. We expect that after inclusion of ring diagrams contributions, as well as higher partial waves and three body force corrections, the results on the saturation properties of nuclear matter of modern BHF approaches will be recovered. The calculation of these corrections is standard and not related to the $T$-matrix approach. On the other hand, the real advantage of the self-consistent $T$-matrix approximation shows itself in the single-particle properties. The quasiparticle energies lead to a Fermi energy consistent with the Hugenholtz-Van Hove relation. The spectral function for large negative and positive energies can be calculated, with implications on the binding energy and applications on electron scattering. We find at normal nuclear density an effective mass $m^* = 0.9m$ and $Z_f = 0.74$. These numbers can be used in the estimation of reduced in medium cross sections and density of states at the Fermi energy. Generally, $Z_f$ does not change much with density or with the details of the potential used, provided the lowest partial waves are described reasonably. We confirm the thermodynamical consistency of the numerical solution of the $T$-matrix scheme [9] for realistic interaction with several partial waves also. Finally let us note that the self-consistent $T$-matrix calculation can be straightforwardly extended to finite temperatures.

This work was partly supported by the KBN under Grant No. 2P03B02019.

References