A.

Self-induced transparency (SIT) gap solutions

1. INTRODUCTION

The paper is concerned with the problem of self-induced transparency (SIT) in nonlinear optical media. The main focus is on the behavior of light pulses in nonlinear media, such as optical fibers and waveguides, where self-focusing effects can lead to the formation of solitons. The SIT effect is a critical phenomenon in optical communications and has applications in fields such as optical switching, signal processing, and optical data storage.


description of self-focusing and soliton formation in optical fibers and waveguides. The SIT effect is characterized by the propagation of light pulses through nonlinear media, where the pulse intensity causes the refractive index to change, leading to self-focusing and the formation of solitons. The dynamics of these processes are governed by the nonlinear Schrödinger equation (NLSE), which describes the evolution of the optical field in the presence of self-focusing effects.

2. SELF-INDUCED TRANSPARENCY (SIT) GAP SOLUTIONS

The SIT effect can be described by the NLSE, which is given by:

\[ irac{dE}{dt} + \frac{1}{2} \frac{d}{dx} \left( \frac{dE}{dx} \right) + |

Self-induced transparency and...
by the nonlinearity at sufficiently high field intensities. The first type of GS had been predicted [18, 19, 20, 21], and later observed [22], in a Bragg grating possessing Kerr-nonlinearity. A principally different mechanism of GS formation has been theoretically discovered by our group in a periodic array of thin layers of resonant two-level atom [23] separated by half-wavelength nonabsorbing dielectric layers, i.e., a resonant absorbing Bragg reflector (RABR) [13, 14, 15, 16]. As opposed to the 2π-solitons arising in SIT, i.e., resonant field–TLA interaction in uniform media [23, 24], their GS counterparts in a RABR may have an arbitrary pulse area [14, 15]. It must be stressed that, stable, moving or standing, GS solutions have been consistently obtained only in a RABR with thin active TLA layers. By contrast, the case of a periodic structure uniformly doped with active TLA calls either for a solution of the wave (Maxwell) equation without the spatial slowly-varying envelope approximation (SVEA), or for a solution of an infinite set of coupled Bloch equations for all spatial harmonics of the atomic polarization (Fourier components) [13]. Therefore, an attempt [25] to obtain a self-consistent solution for a uniformly-doped periodic structure by imposing the SVEA, or by arbitrarily truncating the infinite hierarchy of equations for the harmonics of atomic population inversion and polarization to its first two orders, is generally unjustified. In fact, it can be shown numerically to fail for many parameter values.

In the simplest case of a uniform (bulk) medium, when the driving field is resonant with the atomic transition, the TLA Bloch equations can be easily integrated and the Maxwell equation then reduces to the sine-Gordon equation

$$\frac{\partial^2 \theta}{\partial \zeta \partial \tau} = -\sin \theta$$

(1)

for the pulse area $\theta = \int_0^\tau \Omega \, dt'$, i.e., the time integral of the Rabi frequency $\Omega$. Equation (1) is written in terms of the dimensionless variables $\tau = (t - n_0 v / c) / \tau_0$ and $\zeta = n_0 z / c \tau_0$, where $\tau_0 = \frac{\mu}{\hbar} \sqrt{\frac{2}{\pi \omega_\lambda g_0}}$ is the cooperative resonant absorption time, $g_0$ being the TLA density (averaged over $z$), $\mu$ the dipole moment of the TLA transition at frequency $\omega_\lambda$ and $n_0$ is the refraction index of the host medium. This sine-Gordon equation is known to have solitary-wave solutions, which propagate without attenuation or distortion with a conserved pulse area of $2\pi$ [23, 24]. These SIT solitons have the form:

$$\Omega(\zeta, \tau) = (\tau_0)^{-1} A_0 \text{sech} \left[ \beta (\zeta - v \tau) \right],$$

(2)

where the pulse width $\beta$ is an arbitrary real parameter uniquely defining the amplitude $A_0 = 2 / \beta$ and group velocity $v = 1 / \beta^2$ of the soliton. In what follows, Eq. (2) will be compared with an SIT GS in a RABR.

B. SIT in RABR: The Model

Let us assume [13, 14, 15, 16] a one-dimensional (1D) periodic modulation of the linear refractive index $n^2(z) = n_0^2[1 + a_1 \cos(2k_0 z)]$. The periodic grating gives rise to a PBG with a central frequency $\omega_c = k_c / n_0$ and gap edges at $\omega_{1,2} = \omega_c \left(1 \pm a_1 / 2 \right)$. The electric field $E$ of a pulse propagating along $z$ can be expressed by means of the dimensionless quantities $\Sigma_{\pm} = \pm 2 \pi \mu n_0^{-1} (\mathcal{E}_F \pm \mathcal{E}_B)$, where $\mathcal{E}_F$ and $\mathcal{E}_B$ denote the slowly varying amplitudes of the forward and backward propagating fields, respectively, as

$$E(z, t) = \hbar (\mu \tau_0)^{-1} \left\{ \Re[\Sigma_+(z, t) e^{-i \omega_c t}] \cos k_c z - \Im[\Sigma_-(z, t) e^{-i \omega_c t}] \sin k_c z \right\}.$$  

(3)

We further assume that very thin TLA layers (much thinner than $1 / k_c$), whose resonance frequency $\omega_\lambda$ is close to the gap center $\omega_c$, are placed at the maxima of the modulated refraction index (Fig. 1). They are located at positions $z_j$ such that

$$e^{i k_c z_j} = 1, \quad e^{i k_c z_{j+1}} = -1,$$

i.e., the TLA density is described by $g = g_0 \lambda / 2 \sum_{j} \delta(z - z_j)$, where $\lambda = 2\pi / k_c$ is the wavelength.

The Bloch equations for the slowly varying polarization envelope $P$ and inversion $w$ in the even numbered layers can be obtained (in the slowly varying envelope approximation) by substituting for the Rabi frequency $\Omega = \tau_0^{-1} (\Sigma_+ \cos k_c z + i \Sigma_- \sin k_c z)$ and applying Eq. (4) at the positions of these layers:

$$\frac{\partial P}{\partial \tau} = -i \delta P + \Sigma_+ w,$$

(5)

$$\frac{\partial w}{\partial \tau} = -\text{Re} \left( \Sigma_+ P^* \right).$$

(6)
Combining Eqs. (5) and (6), one can eliminate the TLA population inversion: \( w = \sqrt{1 - |P|^2} \). The remaining equation, together with the Maxwell equations for \( \Sigma_\pm \) (driven by \( P \)), form a \textit{closed system},

\[
\begin{align*}
\frac{\partial^2 \Sigma_+}{\partial \tau^2} - \frac{\partial^2 \Sigma_+}{\partial \zeta^2} &= \eta^2 \Sigma_+ + 2i(\delta - \eta)P - 2\sqrt{1 - |P|^2} \Sigma_+, \\
\frac{\partial^2 \Sigma_-}{\partial \tau^2} - \frac{\partial^2 \Sigma_-}{\partial \zeta^2} &= -\eta^2 \Sigma_- - 2\delta \frac{\partial P}{\partial \tau}, \\
\frac{\partial P}{\partial \tau} &= -i \delta P - \sqrt{1 - |P|^2} \Sigma_+,
\end{align*}
\]

where \( \tau \equiv t/\tau_0, \zeta \equiv (n_0/c\tau_0) z \) and \( \delta \equiv (\omega_0 - \omega_c) \tau_0 \) are the dimensionless time, coordinate, and detuning, respectively, and \( \eta = l_{\text{abs}}/l_{\text{ret}} = a_1 \omega_0 \tau_0/4 \) is the dimensionless modulation strength, which can be expressed as the ratio of the TLA absorption distance \( l_{\text{abs}} = \tau_0^c/\tau_0 \) to the Bragg reflection distance \( l_{\text{reflection}} = 4\varepsilon/(a_1 \omega_0 n_0) \). We emphasize that the above equations are obtained using the SVEA, which is valid under the assumption that the Bragg reflection does not appreciably change the pulse envelope over a distance of a wavelength, \( l_{\text{ref}} \gg \lambda \), whence \( a_1 \ll 2/\pi \).

To reach a general understanding of the dynamics of the model, one should first consider the spectrum \( \Delta \) produced by the \textit{linearized} version of Eqs. (7)-(9), which describes \textit{weak fields} in the limit of infinitely thin TLA layers. Setting \( \Sigma_+ = A e^{i(k'x - \chi' \tau)}, \Sigma_- = B e^{i(k'x - \chi' \tau)}, w = -1, \) and \( P = C e^{i(k'x - \chi' \tau)} \), we obtain from the linearized equation (9) that \( C \equiv i(\delta - \chi')^{-1}A \). Substituting this into Eqs. (7) and (8), we arrive at the dispersion relation for the wavenumber \( k \) and frequency \( \chi \) in the form

\[
(\chi^2 - k^2 - \eta^2)(\chi - \delta) \times \left\{ (\chi - \delta) \left[ \chi^2 - k^2 + (2 + \eta^2) \right] + 2(\eta - \delta) \right\} = 0.
\]

Different branches of the dispersion relation generated by Eq. (10) are shown in Fig. 2. The roots \( \chi = \pm \sqrt{k^2 + \eta^2} \) (corresponding to the solid lines in Fig. 2) originate from the driven equation (8) and represent the dispersion relation of a Bragg reflector with the gap \( \chi_0 < \eta \) that does not feel the interaction with the active layers. Important roots of Eq. (10) are those of the expression in the curly brackets, shown by the dashed and dash-dotted lines in Fig. 1. These roots correspond to nontrivial spectral features: bright or dark solitons in the indicated (shaded) bands.

The frequencies corresponding to \( k = 0 \) are \( \chi_0 = \eta \) and \( \chi_{0, \pm} = -(\delta - \eta) \pm 2\sqrt{2 + (\eta + \delta)^2}/2 \), while at \( k^2 \to \infty \) the asymptotic expressions for different branches of the dispersion relation are \( \chi = \pm \delta + 2(\eta - \delta) k \). Thus, the linearized spectrum always splits into \textit{two gaps}, separated by an allowed band, except for the special case, \( \eta = \eta_0 \equiv \delta/2 + \sqrt{1 + \delta^2}/4 \), when the upper gap closes down. The upper and lower band edges are those of the periodic structure, shifted by the induced TLA polarization in the limit of a strong reflection. They approach the SH spectral gap for forwards- and backwards-propagating waves [20] in the limit of weak reflection. The allowed middle band corresponds to a \textit{polaritonic excitation} (collective atomic polarization) in the periodic structure.
FIG. 2: The RABR dispersion curves at $\eta = 0.5$ and $\delta = -0.2$. The solid lines show the dispersion branches corresponding to the ‘bare’ (noninteracting) grating, while the dashed and dash-dotted lines stand for the dispersion branches of the grating ‘dressed’ by the active medium. The frequency bands that support the standing dark and bright solitons are shaded.

C. Straddling (quiescent) self-localized pulses

We seek the stationary solutions of Eqs. (7) and (9) corresponding to bright solitons in the form

$$\Sigma_+ = e^{-i\chi t} S(\zeta), \quad P = i e^{-i\chi t} P(\zeta)$$

with real $P$ and $S$. Substituting this into (9), we eliminate $P$ in favor of $S$ and obtain an equation for $S(\zeta)$,

$$\frac{d^2 S}{d\zeta^2} = (\eta^2 - \chi^2) S - 2S \frac{(\eta - \chi) \cdot \text{sign}(\chi - \delta)}{\sqrt{(\chi - \delta)^2 + \delta^2}}.$$  \hspace{1cm} (12)

It then follows [15] that bright solitons can appear in two frequency bands $\chi$, the lower band being $\chi_1 < \chi < \min\{\chi_2, -\eta, \delta\}$, and the upper band being $\max\{\chi_1, \eta, \delta\} < \chi < \chi_2$, where $\chi_{1,2} \equiv 1/2[\delta - \eta \mp \sqrt{(\eta + \delta)^2 + 8}]$ are the boundary frequencies. The lower band exists for all values $\eta > 0$ and $\delta$, while the upper one only exists for the weak-reflectivity case $\delta > \eta - 1/\eta$. On comparing these expressions with the spectrum shown in Fig. 2, we conclude that part of the lower gap is always empty from solitons, while the upper gap is completely filled with stationary solitons in the weak-reflectivity case, and completely empty in the opposite limit.

In an implicit form, the solution of Eq. (12) reads

$$S(\zeta) = 2|\chi - \delta| R(\zeta) \left(1 - R^2(\zeta)\right)^{-1},$$  \hspace{1cm} (13)

with

$$|\zeta| = \sqrt{2} \frac{\chi - \delta}{\chi - \eta} \left[(1 - R^2_{\delta})^{-1/2} \tan^{-1} \frac{R^2_{\delta} - R^2_{\zeta}}{1 - R^2_{\delta} + (2R^2_{\delta} - 1) \ln \left(\frac{R_{\delta} + \sqrt{R_{\delta}^2 - R^2_{\zeta}}}{R_{\delta} - R^2_{\zeta}}\right)}\right],$$  \hspace{1cm} (14)

and $R^2_{\delta} = 1 - |(\chi + \eta)(\chi - \delta)|/2$. This zero-velocity (ZV) gap soliton is always single-humped and its amplitude, found from Eq. (14), is given by

$$S_{\text{max}} = 4R_{\delta}/\sqrt{|\chi + \eta|}.$$  \hspace{1cm} (15)

To calculate the electric field in the antisymmetric $\Sigma_-\text{ mode, we substitute } \Sigma_- = i e^{-i\chi t} A(\zeta) \text{ into Eq. (8) and obtain}$

$$A'' + \left(\chi^2 - \eta^2\right) A = 2P',$$

which can be easily solved by the Fourier transform, once $P(\zeta)$ is known. We note that, depending on the parameters $\eta, \delta$ and $\chi$, the main part of the soliton energy can be carried either by the $\Sigma_+$ or the $\Sigma_-$ mode.
one moving solution, which is the only self-dual solution in the form (1.1) that matches the boundary conditions. Hence, a full family of solutions can be constructed by the method of the extended nonlinear-Stokes (ELS) equation.

is the leading order correction to the self-dual solution. In the limit of the small Reynolds number, the leading order correction is the leading order solution in the form (1.1), which is then the leading order solution in the limit of the small Reynolds number.
E. Light bullets (spatiotemporal solitons) in PCs

The advantageous properties of STG GS can be supplemented by immunity to transverse diffraction, i.e., simultaneous transverse and longitudinal self-localization of light in a PC: multi-dimensional spatio-temporal solitons or “light bullets” (LBs) [28] have been analytically and numerically predicted by our group to exist and be stable, not only in uniform 2D and 3D STG media [29], but also in 2D or 3D periodic structures, wherein STG solutions combining LB and GS properties are demonstrated [17]. Our objective is to consider the propagation of an electromagnetic wave with a frequency close to \( \omega_c \) through a 2D PC doped by thin TLA layers. The forward- and backward-propagating components satisfy equations that are a straightforward generalization of the 1D equations (7) and (8)

\[
\begin{align*}
-\frac{i}{\partial \tau} \Sigma_+ &= A_{\parallel} \sqrt{\text{sech} \Theta_1 \text{sech} \Theta_2} e^{i \left( \zeta \left( \pm \xi \right) + i \eta \zeta \right)}, \\
\Sigma_+ &= \frac{\Sigma_-}{\nu}, \\
P &= \sqrt{\text{sech} \Theta_1 \text{sech} \Theta_2} \left\{ \left( \tanh \Theta_1 + \tanh \Theta_2 \right)^2 + \frac{\hat{\delta} - \eta}{\text{d} \eta} C^2 \left[ \left( \tanh \Theta_1 - \tanh \Theta_2 \right)^2 - 2 \left( \text{sech}^2 \Theta_1 + \text{sech}^2 \Theta_2 \right) \right] \right\}^{1/2} e^{i \left( \zeta \left( \pm \xi \right) + i \eta \zeta \right)}, \\
w &= \left[ 1 - |P|^2 \right]^{1/2},
\end{align*}
\]

with \(\Theta_1(\tau, \zeta) \equiv \beta(\zeta - v \tau) + \Theta_0 + C x\), \(\Theta_2(\tau, \zeta) \equiv \beta(\zeta + v \tau) + \Theta_0 - C x\), the phase \(\nu\) and coefficients \(\Theta_0\) and \(C\) being real constants, while the other parameters are defined as \(A_{\parallel} = 2 \sqrt{\delta / \eta - 1} \), \(\beta = \sqrt{\delta / \eta + 1} \), \(v = -\sqrt{\delta - \eta} / (\delta + \eta) \), \(\kappa = -\sqrt{\delta - \eta} \), and \(\chi \equiv \delta \). The ansatz (21)-(24) satisfies Eqs. (19) and (20) exactly, while Eqs. (5) and (6) are satisfied to order \(\sqrt{\delta / \eta - 1} C^2 \), which requires that \(\sqrt{\delta / \eta - 1} C^2 \ll 1\). The ansatz applies for arbitrary \(\eta\), admitting both weak \((\eta \ll 1)\) and strong \((\eta > 1)\) reflectivities of the Bragg grating, provided that the detuning remains small with respect to the gap frequency. Comparison with numerical simulations of Eqs. (19), (20), (5) and (6), using Eqs. (21)-(24) as an initial configuration, tests this analytical approximation and shows that it is indeed fairly close to a numerically exact solution; in particular,
the shape of the bullet remains within 98% of its originally presumed shape after having propagated a large distance, as is shown in Fig. 4.

Three-dimensional (3D) LB solutions with axial symmetry have also been constructed in an approximate analytical form and successfully tested in direct simulations, following a similar approach [17]. Generally, they are not drastically different from their 2D counterparts described above.

F. Information transmission by SIT GSs and LBs

The efficiency of information transmission is characterized either by channel (information) capacity $C = W \ln(I_s/I_o)$, where $W$ is the bandwidth and $I_s/I_o$ is the ratio of the signal-to-noise intensities, or by the data transmission density $D = N/M$, where $N$ is the number of bits per channel and $M$ is the number of accessible channels. Both $C$ and $D$ can be very high in the case of a SIT GS or LB for the following reasons: (a) The bandwidth $W$ is large, being limited by the PBG width, which can be very large in the optical domain. At the same time, noise is very effectively suppressed by the Bragg reflection and by the absence of diffraction losses in the case of a LB. (b) The maximal transmission density $D$ can be estimated [30] as the ratio of the accessible bandwidth, in our case the PBG width (in excess of $10^{15}$ s$^{-1}$ in the optical domain), to the spontaneous linewidth (10$^9$ s$^{-1}$ for rare-earth ions). Hence, these modes of transmission can be very effective for optical communications.

III. CROSS COUPLING BETWEEN ELECTROMAGNETICALLY-INDUCED AND SELF-INDUCED TRANSPARENCY PULSES

A. EIT in bulk media: Background

Electromagnetically induced transparency (EIT) is based on the phenomenon of coherent population trapping [31, 32], in which the application of two laser fields to a three-level atomic system creates the so-called “dark state”, which is stable against absorption of both fields. Consider a four-level atomic system whose level configuration is depicted in Fig. 5(a). The phase shift and absorption of an optical field $E_i$ are given by the real and imaginary parts of its complex polarizability $\alpha_i$. In the absence of the “control” field $E_b$, the usual EIT spectrum [Fig. 5(b)] for the weak probe field $E_p$ exhibits vanishing phase shift and absorption [Re$(\alpha_p) = \text{Im}(\alpha_p) = 0$] at the two-photon Raman resonance $\omega_2 = \omega_1 + \omega_i$, where $\omega_2$ and $\omega_i$ are the frequencies of the probe and driving fields, respectively, and $\omega_i$ is the frequency of the atomic transition $|i\rangle \leftrightarrow |j\rangle$. An off-resonant control field $E_b$ with the frequency $\omega_b$ such that $|\Delta| = |\omega_b - \omega_2| \gg \gamma_4$, where $\gamma_4$ is the decay rate of the corresponding atomic level, induces an ac Stark shift of level $|3\rangle$ and thereby shifts the EIT spectrum [Fig. 5(b)].

Due to the steepness of the dispersion curve in the vicinity of the Raman resonance, $|\partial_{\omega_a} \text{Re}(\alpha_a)| \gg |\partial_{\omega_a} \text{Im}(\alpha_a)|$. 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig5.png}
\caption{(a) Schematic representation of the atomic system interaction with strong driving field $E_d$ and weak fields $E_a$ and $E_b$ on the transitions $|1\rangle \rightarrow |2\rangle$ and $|2\rangle \rightarrow |4\rangle$, respectively. (b) Absorption and dispersion spectra of the $E_a$ field in the absence (solid line) or presence (dashed line) of the $E_b$ field.}
\end{figure}
this Stark shift leads to a large phase shift along with small absorption of the probe field $E_a$:

$$\phi_a = \text{Re}(a_e)z \simeq -\frac{\alpha_0 \gamma_2 |\Omega_b|^2}{2\Delta_b |\Omega_d|^2} z, \quad \text{Im}(a_a) = -\frac{\gamma_4 \text{Re}(a_a)}{2\Delta_b} \ll \text{Re}(a_a),$$  

(25)

where $\alpha_0$ is the resonant absorption coefficient of the medium at the frequency $\omega_1$, and $\Omega_b = \mu_{ij} E_i / \hbar$ is the Rabi frequency of the corresponding field ($\mu_{ij}$ the dipole matrix element on the respective transition). This is the essence of the so-called giant Kerr cross-phase modulation of a probe field by a control field, introduced first by Schmidt and Imamoğlu [33]. Later Harris and Yamamoto [34] have predicted that a resonant control field $E_b$ with $|\Delta_b| < \gamma_4$ can destroy the coherence between the two ground levels $|1\rangle$ and $|3\rangle$, which leads to a two-photon absorption $\text{Im}(a_{a,b}) = \frac{\alpha_0 \gamma_2 |\Omega_b|^2}{\gamma_4 |\Omega_d|^2}$, that is, the medium absorbs two fields simultaneously, but does not absorb one field alone. This is the essence of a probe-photon switch, conditional on the presence of control photons.

The main limitation of the above schemes [33, 34, 35] stems for the fact that the effective interaction length is limited by the mismatch between the group velocity of the slowly propagating $E_a$ field, $v_g^{(c)} \simeq \frac{2|\Omega_d|^2}{\alpha_0 \gamma_2} \ll c/n_0$ and that of the nearly-free propagating $E_b$ field, $v_g^{(b)} \simeq c/n_0$. For weak (few-photon) pulses, this mismatch ultimately limits the maximal phase shift or absorption of the probe in the presence of the control field.

**B. Simultaneous EIT and SIT in RABR**

In this section we propose a new implementation of the cross-phase modulation, in which the group velocities of both fields can be matched, allowing one to obtain any desired phase shift of the probe field with a weak control field. To this end, we consider the same configuration as in Sec. II, leading to SIT GS and LR solutions: a PC periodically doped by thin layers of atoms at the maxima of its refractive index. However, the multi-level structure of the atoms is now playing a role: it is shown in Fig. 6, along with the polarizations and propagation directions of the fields involved. The states $|1\rangle$, $|3\rangle$ and $|5\rangle$ are the degenerate Zeeman components with $M_F = -1, 0, +1$, respectively, of the atomic ground level having total angular momentum $F = 1$. Similarly, the states $|4\rangle$ and $|6\rangle$ are the degenerate Zeeman components with $M_F = -1, 0$, respectively, of the excited level having angular momentum $F = 1$. Finally, the state $|2\rangle$ corresponds to the single Zeeman component with $M_F = 0$ of another excited level having $F = 0$. Such a level scheme is found, e.g., in alkali atoms, where the ground level is $S_{1/2}, F = 1$ and the two excited levels are $P_{1/2}, F = 1$ and $P_{3/2}, F = 0$. Due to the dipole selection rules, the $\pi$-polarized driving field couples the states with $\Delta M = 0$, the $\sigma_+\sigma_-$-polarized $E_a$ field couples the states with $\Delta M = 1$ and the $\sigma_+\sigma_-$-polarized $E_b$ field couples the states with $\Delta M = -1$.

We assume that initially all the atoms are optically pumped into the states $|1\rangle$ and $|5\rangle$, which then acquire equal populations $1/2$. Hence, the sequence of transitions $|1\rangle \rightarrow |2\rangle \rightarrow |3\rangle \rightarrow |4\rangle$ repeats that of Fig. 5(a), realizing the cross-phase modulation scheme of Sec. IIIA A. The frequency of the $E_a$ field is far from the band gap frequencies of the PC, while the frequency of the $E_b$ field is within the band gap.
As was shown in Sec. II, PC structures doped with the near-resonant TLAs can support standing and slowly moving SIT GSs, whose pulse area (integrated over $z$) can take an arbitrarily small value. In the present setup, the transition $|5\rangle \rightarrow |6\rangle$ realizes that near-resonant TLA, allowing for slow propagation of the $E_b$ field through the PC.

Let us write the propagation equation for the slowly moving SIT soliton in the form

$$\Omega_{\text{SIT}}(z,t) = \Omega_b \text{sech} \left( \frac{v_g^{(b)} - z}{2\beta} \right),$$

where $\Omega_b$ is the peak Rabi frequency, $v_g^{(b)}$ is the group velocity and $\beta = (2\alpha_b)^{-1}$, with $\alpha_b$ being the absorption coefficient of the active medium at the carrier frequency $\omega_b$ of the soliton. The temporal width of the pulse is given by $\tau_b = 2\beta/v_g^{(b)}$. The area of the $E_b$ pulse

$$\theta_b = \int_{-\infty}^{\infty} \Omega_{\text{SIT}}dt = \frac{\Omega_b}{v_g^{(b)}\alpha_b}\pi,$$

is then inversely proportional to the group velocity of the pulse. Hence, the SIT condition (Sec. IIIIA) $\theta_b = 2\pi$ imposes a unique relation between the Rabi frequency of the SIT soliton and its group velocity:

$$v_g^{(b)} = \frac{\Omega_b}{2\alpha_b}.$$  

The absorption-free propagation of the SIT soliton is limited to $z < v_g^{(b)}/\gamma_6$, where $\gamma_6$ is the decay rate of the upper atomic state $|6\rangle$.

Our aim is to match the group velocities of the $E_a$ field subject to EIT and the $E_b$ field having the form of a slow SIT gap soliton: $v_g^{(a)} = v_g^{(b)}$. This requires that $|\Omega_a|^2 = \Omega_b\alpha_b\gamma_6/4\alpha_b$, i.e., an appropriate choice of the driving field Rabi frequency $\Omega_a$, for a given Rabi frequency $\Omega_b$ of the control field. Such velocity matching of the two copropagating weak fields would maximize their interaction.

One possibility to launch the required slow SIT soliton is to irradiate the PC by a laser beam at a small angle $\psi$ relative to the periodicity direction $z$, $\psi \simeq Dn_0v_g^{(b)}/(Lc) \ll 1$, where $D$ and $L$ are, respectively, the transverse and longitudinal dimensions of the structure. This choice of $\psi$ ensures that the $z$-component of the beam, which forms the SIT soliton and propagates in the PC with the group velocity $v_g^{(b)}$ over the distance $L$, will traverse the structure during the same time as the transverse component of that beam, which covers the distance $D$ with the velocity $(c/n_0)\sin\psi$.

We have checked that for the parameter values corresponding to dopant atoms (or ions) with the mean density $N = 10^{13}$ cm$^{-3}$ (surface density of $4 \times 10^6$ cm$^{-2}$ in the thin layers), $\Delta_b = 30\gamma_4$, $|\Omega_b| \simeq 10^5$ rad/s and $|\Omega_a| \simeq 4 \times 10^5$ rad/s, we obtain $\pi$ phase shift of the $E_a$ field over a distance $z \simeq 4$ cm, while the absorption probability remains less than $10\%$.

One possible difficulty of our scheme is that, with the parameters listed above, the temporal width of the $E_b$ field is $\tau_b \sim 10^{-6}$ s, and the interaction time is of the order of $10^{-3}$ s, while the lifetime of the SIT soliton is of the order...
of the decay time of the excited atomic state $\gamma_6^{-1} \sim 10^{-7}$ s. One can cope with this problem by employing the atomic level scheme shown in Fig. 7, which allows one to launch Raman solitons. We irradiate the system with an additional strong cw field $E_B$, which couples the $F = 1$ excited state with the $F = 2$ metastable ground states. The fields $E_R$ and $E_B$ are largely detuned from the fast decaying excited states $|4\rangle$ and $|6\rangle$ by an amount $\Delta \gg \gamma_{4,6}$. Then, upon adiabatically eliminating the states $|4\rangle$ and $|6\rangle$, we obtain that the Rabi frequency $\Omega_B$ of the control field in Eq. (26) is simply replaced by $\Omega_B \Omega_R$. The lifetime of the SIT soliton is given now by the lifetime of the $F = 2$ ground states, which can be very large, reaching in some instances a fraction of a second! In addition, such a setup allows one to launch slow Raman GSs [36], and thus circumvent the difficulty of launching a standing (ZV) or slowly moving GSs, which must otherwise overcome the high reflectivity of the PC boundaries.

IV. CONCLUSIONS

In this paper we have focused on properties of solitons in a doped PC or RABR, combining a periodic refractive-index superlattice (Bragg reflector) in 1D or 2D and a periodic set of thin active layers (consisting of TLAs resonantly interacting with the field). We have demonstrated that the system supports a vast family of bright GSs, whose properties differ substantially from their counterparts in periodic structures with either cubic or quadratic off-resonant nonlinearities. Depending on the initial conditions, these can be either standing (ZV) or slowly moving stable solitons that exhibit SIT irrespective of their photon number (pulse energy) for an appropriate group velocity. A multidimensional version of this model corresponds to a periodic set of thin active layers placed at the maxima of a 2D- or 3D-periodic refractive index. It has been found to support stable propagation of spatiotemporal solitons in the form of 2D- and 3D-localized LSs.

The best prospect of realizing a PC which is adequate for observing the GSs and LSs is to use thin layers of rare-earth ions [37] embedded in a spatially-periodic semiconductor structure [38]. The TLAs in the layers should be rare-earth-ions with the density of $10^{15} - 10^{16}$ cm$^{-3}$, and large transition dipole moments. The parameter $\eta$ can vary from 0 to 100 and the detuning is $\sim 10^{12} - 10^{13}$ s$^{-1}$. Cryogenic conditions in such structures can strongly extend the dephasing time $T_2$ and thus the soliton's or LS's lifetime, well into the $\mu$sec range [37], which would greatly facilitate the experiment.

In a 2D PC, LSs can be envisaged to be localized on the time and transverse-length scales, respectively, $\sim 10^{-13}$ s and 1 $\mu$m. The incident pulse has uniform transverse intensity and the transverse diffraction is strong enough. One needs $d^2/\lambda_0 \lambda_0 < 1$, where $\lambda_0$ and $d$ are the resonant absorption length, carrier wavelength, and the pulse diameter, respectively. For $l_{\text{abs}} \sim 10^{-3}$ m and $\lambda_0 \sim 10^{-4}$ m, one thus requires $d < 10^{-4}$ m, which implies that the transverse size of the PC must be a few $\mu$m.

We have considered here (Sec. IIIIB) the cross-coupling of optical beams in a PC or RABR. We have pointed out, for the first time, the advantageous features of the cross coupling between EIT and SIT pulses, which is capable of producing extremely strong correlations between the two pulses. With doping parameters as above, and driving and control fields with Rabi frequencies of the order of $10^8$ rad/s, we can obtain a phase shift of $\pi$ for the weak probe pulse over a distance of few cm. This is much larger than any corresponding phase shift (for similar control fields) in other media.

We strongly believe that the highly promising payoff expected from the construction of suitable structures justifies the experimental challenge they pose. If, and when the schemes proposed above are experimentally realized, they may prove to be useful for producing ultra-sensitive nonlinear phase shifters or logical photon switches for both classical and quantum information processing or communication, owing to the unique advantages of the doped PCs over conventional EIT schemes [33, 34, 35, 39] or high-Q cavities [40, 41]:

Acknowledgments

This work was supported by the EU (ATESIT) Network, the US-Israel BSF and the Feinberg Fellowship (D.P.).


