Gravitational Microlensing in NUT Space

Sohrab Rahvar\textsuperscript{1,3} and Mohammad Nouri-Zonoz\textsuperscript{2,3}

\textsuperscript{1} Department of Physics, Sharif University of Technology, P.O.Box 11365–9161, Tehran, Iran
\textsuperscript{2} Department of Physics, Tehran University, End of North Karegar St, Tehran 14352, Iran
\textsuperscript{3} Institute for Studies in Theoretical Physics and Mathematics, P.O.Box 19395-5531, Tehran, Iran

18 April 2002

ABSTRACT

We study the theoretical signature of magnetic masses on the light curve of gravitational microlensing effect in NUT space. The light curves for microlensing events are presented and contrasted with those due to lensing produced by normal matter. Associating magnetic masses to MACHOs, we find a constraint on the magnetic mass scales by comparing light curves of microlensing candidates with theoretical light curve of NUT space microlensing. Presence or absence of these feature in observed microlensing events can shed light on the question of the existence of magnetic masses in the Universe.

1 INTRODUCTION

Studying rotational curves of spiral galaxies gives an evidence for existence of dark matter in the galactic halo (Faber & Gallagher 1979; Trimble 1987). Results from 21 cm bands observation also shows that for thousands of spiral galaxies, rotational curve of galaxies remains constant (Persic et al. 1996). Comparing luminescence matter of universe $\Omega_{\text{lum}} = 0.004$ (Fukugita et al. 1995) with baryonic amount of matter $\Omega_B = 0.02h^{-2}$ (obtained from nucleosynthesis models of universe) confirms that the major part of the halo is made of baryonic dark matter (Copi et al. 1995; Burles & Tyler 1998). One of the possible forms of baryonic dark matter in the halo could be MACHO (Massive Astrophysical Compact Halo Object) which are obscure due to their light mass. The pioneer idea of using gravitational
microlensing technique for detection of MACHOs was proposed by Paczyński (1986). Since his proposal, gravitational microlensing theory entered into its observational phase by several groups. In this paper we study the gravitational microlensing in an exotic space-time, called NUT space. The usual gravitational lens effect is based on the bending of light rays passing a point mass $M$ in Schwarzschild space-time. In the paper of Nouri-Zonoz & Lynden-Bell (1997) the gravitational lens effect on light rays passing by a NUT hole has been considered using the fact that all the geodesics of NUT space including the null ones lie on cones. It is shown that compared with the Schwarzschild lens, there is an extra shear due to the gravitomagnetic field which shears the shape of the source. The effect is shown to be small even for big values of the magnetic masses (NUT factor). In this paper we will obtain the gravitational microlensing light curve in NUT space and compare it with observational light curves of few microlensing candidates. The outline of the paper is as follows. In section 2, we give a brief account on the results of gravitational macrolensing by NUT space and then in the third section we discuss the microlensing on light rays by NUT space and in particular we find the magnification in this case. In section 4 we use observational light curves, taken by MACHO collaboration, gravitational microlensing light curve in NUT space. We discuss the constraint on magnetic masses summarize our conclusions in section 5.

2 GRAVITATIONAL MACROLENSING IN NUT SPACE

The metric of NUT space is given (in $t, r, \theta, \phi$ coordinates) by the line element;

$$\text{ds}^2 = f(r) (dt - 2l \cos \theta d\phi)^2 - \frac{1}{f(r)} dr^2 - (r^2 + l^2)(d\theta^2 + \sin \theta d\phi^2)$$ (1)

where $f(r) = 1 - \frac{2(Mr + l^2)}{r^2 + l^2}$ and $l$ is called the magnetic mass or NUT factor and one can think of $Q = 2l$ as the strength of the gravitomagnetic monopole represented by the NUT solution (Lynden-Bell & Nouri-Zonoz 1998). It was shown that all the geodesics of NUT space, including the null ones, lie on a cone whose semi-angle is given by;

$$\sin \chi = \frac{Q}{b[1 + Q^2/b^2]^{1/2}},$$

where $b$ is the impact parameter defined on the cone (Nouri-Zonoz & Lynden-Bell 1997). The geometry of lensing in the case of NUT space could be shown in the following two figures. In Fig.1 the path of a light ray deflected at point $P$ is shown on an open flattened cone and
Figure 1. Open, flattened cone and the light ray (dashed line) which is deflected at $P$ passing the NUT lens. $\nu = 2\pi(1 - \frac{L}{(L^2 + \epsilon^2 Q^2)^{1/2}})$ and $\alpha$ are the deficit and bending angles respectively.

Figure 2. Lens plane and the position of source $S$, image $I$ and observe $O$.

in Fig.2 the positions of the source and image are shown on the lens plane. The relation between the positions of source and image is given by;

$$\frac{r}{r'} = \frac{[4\chi^2 + (\alpha - \beta')^2]^{1/2}}{\beta},$$

(2)

where, $r$ and $r'$ denote the positions of source and image respectively, $\beta' = \beta(1 + \frac{D_d}{D_{ds}})$ with the parameters $\beta$, $D_d$ and $D_{ds}$ as defined in figure 1 and $\alpha = 4Gm/bc^2$ is the bending angle defined on the cone. Using the Jacobian of transformation between image and source positions, the magnification of the image is given by the following relation.
A = \frac{1}{\left(1 - \alpha^2/\beta^2\right)\left(1 - \alpha^2/\beta^2 - 8\chi^2/\beta^2\right)}^{1/2} \tag{3}

It can easily be seen that for $\chi = 0$ one recovers the known result of the Schwarzschild lens. It is shown that for an extended source the orientation of the image is also dependent on the NUT factor through the definition of $\chi$ (Lynden-Bell & Nouri-Zonoz 1997; 1998). Using the above results we study the microlensing by NUT space in the next section.

3 GRAVITATIONAL MICROLENSING IN NUT SPACE

In this section we introduce the basics of gravitational microlensing by a Schwarzschild lens and then study the same effect in NUT space.

3.1 Basics of gravitational microlensing

A considerable gravitational lensing occur when the impact parameter of the light rays is small enough. Since in gravitational microlensing the deflection angle is too small, for present telescopes it is impossible to resolve the two images produced and its effect is only on the magnification of background star. This magnification is given by:

$$A(t) = \frac{u(t)^2 + 2}{u(t)\sqrt{u(t)^2 + 4}}, \tag{4}$$

where $u(t) = \sqrt{u_0^2 + \left(\frac{t - t_0}{t_E}\right)^2}$ is the impact parameter (position of the source in deflector plane normalized by Einstein radius) and in which $t_E$ is the Einstein crossing time (duration of event) defined by $t_E = R_E/v_t$, where $v_t$ is the transverse velocity of deflector with respect to the line of sight. The Einstein radius is given by $R_E^2 = \frac{4GM}{c^2}$, where $M$ is the mass of the deflector and $D = \frac{D_sD_{ls}}{D_{ls}}$. It is seen that the light curve is symmetrical with respect to time and since gravitational lensing effect is independent of the frequency of light, we would expect the same magnification throughout the spectrum. The probability of observing a microlensing event is very low (e.g. toward large and small Magellanic clouds is about $10^{-7}$) and the rate of microlensing events also depends on the galactic models (Rahvar 2002). Comparing the rate of observed microlensing with what have been expected from theoretical galactic halo models reveal that only 20% of halo is made of MACHOs (Spiro & Lasserre 2001; Alcock et al. 2000).
3.2 Gravitational Microlensing in NUT space

In the galactic scales the configurations of gravitational lensing have dynamical behavior and this makes gravitational microlensing light curves very sensitive to the parameters of the space-time under consideration. In what follows, we find the magnification function for gravitational microlensing in NUT metric and compare it with Schwarzschild microlensing. Using equation (3) for the magnification of a point like source in NUT space, it can be written in the following form:

\[ A(u_i) = \left[ \left( 1 - \frac{1}{u_i^2} \right) \left( 1 - \frac{1}{u_i^4} - \frac{8R_i^4}{u_i^4} \right) \right]^{-\frac{1}{2}} \] \hspace{1cm} (5)

where \( R = \frac{R_{\text{NUT}}}{R_E} = c\sqrt{\frac{l^2}{2GM}} \) and \( u_i \) indicates the position of the image in the lens plane (normalized to Einstein radius). In which we define the NUT radius to be;

\[ R_{\text{NUT}}^2 = 2lD. \] \hspace{1cm} (6)

Here we are interested in obtaining the magnification of the background star as a function of the position of the source in the lens plane in the absence of the lens. Using definitions of the Einstein and NUT radii and normalizing all the length scales to Einstein radius, equation (2) can be written in the following form:

\[ u_s^2 = 4R_i^4 + \left( 1 - \frac{1}{u_i^2} - u_i \right)^2, \] \hspace{1cm} (7)

where, \( u_s \) and \( u_i \) are the positions of the source and the image on the lens plane respectively. Hereafter, we omit the index \( s \) of \( u_s \) for convenience. Equation (7) has in principle the following two solutions:

\[ \frac{1}{u_i^{\pm 2}}(u) = 1 + u^2/2 \pm \sqrt{\left( 1 + u^2/2 \right)^2 - (4R_i^4 + 1)} / 4R_i^4 + 1, \] \hspace{1cm} (8)

corresponding to the positions of the two images produced by the lens provided \( u^2 > 2(\sqrt{1 + 4R_i^4} - 1) \). Now using equation (5) the magnification for each of the images can be written in the following form:

\[ A^\pm = \frac{\left( 1 - \frac{8R_i^4}{u_i^{\pm 2} - 1} \right)^{-1/2}}{1 - \frac{1}{u_i^{\pm 2}}}, \] \hspace{1cm} (9)

Substituting equation (8) into equation (9), the total magnification is:

\[ A(u) = |A^-| + |A^+| \] \hspace{1cm} (10)
The magnification of light in NUT space is more than Schwarzschild space with the same impact parameter. 

\[ -\frac{1}{4(1+4R^4)^2} \left( 1 - \frac{8R^4}{(2+u^2+\sqrt{-16R^4+4u^2+u^4})^2} \right) \left( 1 - \frac{8R^4}{4(1+4R^4)^2} \right)^{-1+\frac{8R^4}{4(1+4R^4)^2}} \]

where we use the fact that \( A^+ \) is negative for \( u^2 > 2(\sqrt{1+4R^4} - 1) \). For \( R = 0 \) one can recover Paczyński’s relation (equation 4). One can expand equation (10) in terms of \( R^4 \) to obtain the following simple expression of the magnification:

\[ A(u) = \frac{2 + u^2}{u\sqrt{4 + u^2}} + \frac{8R^4(2 + u^2)}{w^3(4 + u^2)^{3/2}} + \mathcal{O}(R^8) + ... \] (11)

In Fig.(3) the gravitational microlensing light curves in NUT and Schwarzschild spaces are shown. In the next section we use realistic light curves of microlensing candidates toward Large Magellanic Cloud to test their compatibility with theoretical light curves in NUT space.

4 COMPATIBILITY OF MICROLENSING IN NUT SPACE WITH OBSERVATION

In this section we compare the light curves of microlensing candidates, observed by MACHO collaboration, with those obtained (in the previous section) from our study of microlensing in NUT space. Seven light curves of microlensing candidates have been chosen from MACHO database * toward LMC for fitting with theoretical light curves in Schwarzschild and NUT spaces. In order to increase the sensitivity of fitting to the light curves, we express the light curves in terms of magnification rather than magnitude of background star. Fig. 4

* http://www.macho.mcmaster.ca/
Figure 4. Left panel shows the observed light curve of event lmc-8 by MACHO collaboration with the best fitting of gravitational microlensing light curves in NUT (dashed line) and Schwarzschild (solid line) space. The right panel indicated the detail of light curve around the maximum magnification time.

shows one of the light curves of the microlensing candidates (lmc-8) which has been fitted by theoretical microlensing light curves of NUT and Schwarzschild lenses. It is easily seen that the only difference in the fitted light curves is the higher peak in the case of NUT microlensing and for the large impact parameters the two light curves are coincident. In Tables. 1 and 2 the reconstructed parameters and the $\chi^2$ are given for the microlensing light curves in Schwarzschild and NUT space respectively. Table. 2 shows that some of the events like lmc 1b, lmc 5, lmc 6 and lmc 8 have been fitted reasonably well with the NUT space light curve, indeed they are better than the similar results given in Table. 1 for Schwarzschild light curves. For these four events $R$ have been found in the domain of $R \in [0.01, 0.68]$ witch corresponds to the magnetic mass domain $l \in \frac{2GM}{c^2}[10^{-4}, 4.6 \times 10^{-1}]$. Using 0.2 solar mass for the mean mass of MACHOs from microlensing experiments, one can obtain the domain $l \in [10, 2500]cm$.

5 SUMMARY

In this article we have studied the gravitational microlensing in NUT space with the aim of learning more about the magnetic masses and their observability as constitutes of MACHOs. In the first step we introduced the ratio $R = \frac{R_{\text{NUT}}}{R_{\text{E}}}$ which in a sense is the ratio of magnetic mass of the lens to its mass. Then we found the magnification for the microlensing in NUT space in terms of this parameter. In the next step comparing the light curves in NUT space
Table 1. light curve of microlensing candidates towards LMC have been fitted with microlensing in Schwarzschild space, reconstructed parameter with the best $\chi^2$ is obtained.

<table>
<thead>
<tr>
<th>Event</th>
<th>$u_0$</th>
<th>$t_0$</th>
<th>$t_E$</th>
<th>$\chi^2/N_{dof}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>lmc 1a</td>
<td>0.13</td>
<td>56</td>
<td>17</td>
<td>3.72</td>
</tr>
<tr>
<td>lmc 1b</td>
<td>0.12</td>
<td>57</td>
<td>17</td>
<td>1.08</td>
</tr>
<tr>
<td>lmc 4</td>
<td>0.34</td>
<td>646</td>
<td>19</td>
<td>3.3</td>
</tr>
<tr>
<td>lmc 5</td>
<td>0.025</td>
<td>25</td>
<td>27</td>
<td>0.78</td>
</tr>
<tr>
<td>lmc 6</td>
<td>0.44</td>
<td>197</td>
<td>50</td>
<td>0.81</td>
</tr>
<tr>
<td>lmc 7</td>
<td>0.2</td>
<td>463</td>
<td>51</td>
<td>3.14</td>
</tr>
<tr>
<td>lmc 8</td>
<td>0.51</td>
<td>389</td>
<td>34</td>
<td>1.21</td>
</tr>
</tbody>
</table>

Table 2. light curve of microlensing candidates towards LMC have been fitted with NUT space microlensing, reconstructed parameter with the best $\chi^2$ is obtained.

<table>
<thead>
<tr>
<th>Event</th>
<th>$u_0$</th>
<th>$t_0$</th>
<th>$t_E$</th>
<th>$R$</th>
<th>$\chi^2/N_{dof}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>lmc 1a</td>
<td>0.13</td>
<td>56</td>
<td>17</td>
<td>0.0044</td>
<td>3.72</td>
</tr>
<tr>
<td>lmc 1b</td>
<td>0.12</td>
<td>57</td>
<td>16</td>
<td>0.3</td>
<td>1.06</td>
</tr>
<tr>
<td>lmc 4</td>
<td>0.34</td>
<td>646</td>
<td>19</td>
<td>0.031</td>
<td>3.3</td>
</tr>
<tr>
<td>lmc 5</td>
<td>0.027</td>
<td>24</td>
<td>27</td>
<td>0.063</td>
<td>0.93</td>
</tr>
<tr>
<td>lmc 6</td>
<td>0.44</td>
<td>197</td>
<td>50</td>
<td>0.01</td>
<td>0.81</td>
</tr>
<tr>
<td>lmc 7</td>
<td>0.67</td>
<td>463</td>
<td>27</td>
<td>0.81</td>
<td>3.13</td>
</tr>
<tr>
<td>lmc 8</td>
<td>0.89</td>
<td>389</td>
<td>27</td>
<td>0.68</td>
<td>1.21</td>
</tr>
</tbody>
</table>

with the set of observational light curves from MACHO collaboration. We showed that the inclusion of magnetic masses in the form of the NUT metric as the surrounding spacetime of the lens, instead of the usual Schwarzschild lens, will improve the fit. Using the fit for a 0.2 solar mass MACHOs, we found a domain $[10, 2500]$ cm for the magnetic mass scales. In other words the maximum magnetic mass, compatible with observed microlensing, for an object with a mass $\sim M_\odot$ is in the order of $10^3 cm$. Comparing this with the Schwarzschild radius (mass length scale) $\sim 10^5 cm$ we find that magnetic mass constitute of MACHOs (if any) is negligible compared to its mass.
REFERENCES

Alcock C. et al. (MACHO), 2000, APJ 542, 281.
(American Intitute of Physics),(2001) 146.
Rahvar, S.,(astro-ph/0203037)