Electron Mass Operator in a Strong Magnetic Field

The electron mass operator in a strong magnetic field is considered. The contribution of higher Landau levels to the electron mass is calculated. The contribution of higher Landau levels is shown to be essential in the strong magnetic field. The electron mass operator is expressed in terms of the effective magnetic field. The effective magnetic field is obtained from the electron mass operator in the strong field limit. The effective magnetic field is shown to be equal to the external magnetic field in the strong field limit.

\[ M = 1 - \frac{1}{(\alpha/2\pi)^2} \log |B/m_0| \]

\[ \alpha = \frac{\mu}{m_0} \]

Where \( \mu \) is the electron mass without a field, \( m_0 \) is the electron mass in a vacuum, \( B \) is the magnetic field, and \( \alpha \) is the fine structure constant.

Asymptotic properties of the ODE diagrams and effective magnetic fields B_eff, B_m are also discussed. The electron mass operator is expressed in terms of the effective magnetic field. The effective magnetic field is obtained from the electron mass operator in the strong field limit. The effective magnetic field is shown to be equal to the external magnetic field in the strong field limit.

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electrons occupying the lowest Landau level. In this case, in the gauge where $\Gamma_\mu \simeq \gamma_\mu$ the exact photon propagator can be presented in the form [5]

$$G^{(\gamma)}(q) = -i D(q_\perp^2, q_\parallel^2) \hat{\Lambda}_{\mu\nu},$$

(4)

$$D(q_\perp^2, q_\parallel^2) = \frac{1}{q^2 - \mathcal{P}(q_\perp^2, q_\parallel^2)},$$

(5)

Here $q_\parallel^2 = q_\mu \lambda_{\mu\nu} q_\nu$, $q_\perp^2 = q_\mu \tilde{\lambda}_{\mu\nu} q_\nu$, $q_\parallel^2 = q_\mu \Omega q_\mu$, $\lambda_{\mu\nu} = v_{\mu\nu}$, $\hat{\lambda}_{\mu\nu} = \tilde{v}_{\mu\nu}$, $\Omega$ is the dimensionless tensor of external magnetic field, $\lambda_{\alpha\beta} = F_{\alpha\beta}/\sqrt{F_{\mu\nu}^2}$, $\hat{\lambda}_{\alpha\beta} = \frac{1}{\sqrt{F_{\alpha\beta} F_{\mu\nu}}}$ is the dual tensor. The function $\mathcal{P}(q_\perp^2, q_\parallel^2)$ is the eigenvalue of the photon polarization operator $\mathcal{P}_{\mu\nu}(q)$, which is depicted by the Feynman diagram Fig. 2. With the reduction $\Gamma_\mu = \gamma_\mu$, the operator has the form

$$\mathcal{P}_{\mu\nu}(q) = -i \frac{\alpha}{4\pi^2} \int d^4 k T r \left[ \gamma_\mu G^{(\epsilon)}(k) \gamma_\nu G^{(\epsilon)}(k - q) \right]$$

$$= \left( \lambda_{\mu\nu} - \frac{q_{\parallel\mu} q_{\parallel\nu}}{q_\parallel^2} \right) \mathcal{P}(q_\perp^2, q_\parallel^2) + \ldots,$$

(6)

where dots denote the contribution of the other photon modes. Thus the polarization operator (6) is reduced in fact to the one-loop operator $\lambda_{\mu\nu}$.

The mass operator $M(p_\parallel^2)$ corresponding to the irreducible diagram depicted in Fig. 1, in the gauge $\Gamma_\mu = \gamma_\mu$ and in terms of Eqs. (3) and (4) is reduced to a function $M(p_\parallel^2)$, see Refs. [7, 8]. The following integral equation arises for this function

$$M(p_\parallel^2) = m_0 - i \frac{\alpha}{2\pi^2} \int d^4 k \exp \left( -\frac{k^2}{2eB} \right) \frac{M(k^2)}{(M(k^2))^2 + (k - p)^2 - k_\parallel^2 - \mathcal{P} \left( k_\perp^2, (k - p)^2 \right)}.$$

(7)

Integral in the expression (7) does not contain an ultraviolet divergency, because the integration over the momenta $k_\perp$ transversal to the field direction has the cutoff $k_\perp \leq \frac{1}{eB}$.

On the other hand, the photon polarization operator (6), in general, does contain the ultraviolet divergency. As a result, virtual electrons occupying both the lowest and higher Landau levels contribute to the integral. This fact was not taken into account in all previous publications in the field. However, as will be shown below, it leads to very interesting physical consequences.

The function $\mathcal{P}(q_\perp^2, q_\parallel^2)$ in a strong magnetic field in the one-loop approximation can be extracted, for example,
function $H(z)$ has the form

$$H(z) = \frac{1}{2\sqrt{\pi}} \ln \left( \frac{1 + z}{1 - z} \right).$$

(9)

In the analysis of Eq. (7), we are interested in the region of parameters $q_0^2 < 0, |q_0^2| \gg |M(q_0^2)|^2$. For large negative values of the argument, the function $H(z)$ is simplified, $H(z) \simeq \frac{1}{2\sqrt{\pi}} \ln \left( \frac{1 + z}{1 - z} \right)$. The first term in Eq. (8) acquires in this case the meaning of the photon mass squared, $m^2 = (2\alpha / \pi) eB$, induced by a magnetic field. As for the second term in Eq. (8), containing the contribution from higher Landau levels into the photon polarization operator, its role is in renormalization of the electromagnetic constant $\alpha$ in a magnetic field, $\alpha \rightarrow \alpha_R$. The expression for the renormalized constant $\alpha$ can be obtained from Eq. (2) by the replacement $m_0 \rightarrow M(q_0^2)$.

Turning back to Eq. (7), it should be mentioned that the integral in the right-hand side exactly corresponds to the one-loop field-induced correction to the electron mass, with the replacement of the field-free mass $m_0$ by the soughst mass operator $M(q_0^2)$ in the integrand. In the superstrong field limit, the main contribution into the integral (7) in the form of a big logarithm $\ln(eB/M^2)$ originates from the region $|M(q_0^2)|^2 \ll |k^2_0| \ll m_0^2 \sim \alpha eB, m_0^2 \leq k^2_0 \ll eB$. In view of this, the calculation of the integral in Eq. (7) with logarithmic accuracy gives the following result for the mass operator $M(p_0^2)$ in the Euclidean region $p_0^2 < 0, |p_0^2| \ll m_0^2$:

$$M(p_0^2) \simeq M(0) \left[ 1 + \frac{p_0^2}{4eB} \ln \frac{2\alpha eB}{m_0^2} \right].$$

(10)

The expression (10) shows that the electron physical mass which is defined, strictly speaking, as the solution of the dispersion equation $m = M(-m^2)$, can be taken with a great accuracy in the zero point, $m \simeq M(0)$.

In the leading log approximation, we have obtained the following transcendental equation for the electron physical mass from the integral equation (7):

$$m = m_0 + m_0 \frac{\alpha_R}{2\pi} \left( \ln \frac{\pi}{\alpha_R} - \gamma_E \right) \ln \frac{eB}{m_0^2},$$

(11)

where $\alpha_R$ is taken in the point $q_0^2 \simeq 0$, i.e. obtained from Eq. (2) by the replacement $m_0 \rightarrow M(0) = m$. It is interesting to note that the formula (11) reproduces exactly our result (1), with the substitution of the electron physical mass $m$ instead of the field-free mass $m_0$ under the logarithms. The equation (11) solves the problem of finding the electron physical mass for any large values of the magnetic field. It is free of a singularity, unlike the Eq. (1).

An analysis of Eq. (11) shows that its solution in asymptotically strong fields when $m \gg m_0$, becomes independent on $m_0$ and is reduced in fact to the solution at $m_0 = 0$. This would mean the generation of the dynamical mass of the initially massless electron in a magnetic field. This effect which is also called the dynamical chiral symmetry breaking, was studied in refs. [4, 7, 8, 9, 10], however, the contribution from higher Landau levels into the photon polarization operator was not considered there.

Let us show, that this contribution changes the behaviour of the dynamical mass essentially. Let us extend our analysis to a model with $N$ initially massless fermions ($m_0 = 0$) with equal charges $e$ (in a case of different fermion charges $Q_j e$, the parameter $N$ has the meaning of $\sum_j Q_j^2$). In this case the photon polarization operator (8) is the sum over all fermion loops, i.e. it should be multiplied by $N$. The transcendental equation for the fermion dynamical mass $(m \neq 0)$ can be obtained from Eq. (11) as follows

$$\frac{\alpha_R}{2\pi} \left( \ln \frac{\pi}{N\alpha_R} - \gamma_E \right) \ln \frac{eB}{m^2} = 1,$$

(12)

where

$\alpha_R = \frac{\alpha}{N^3/2\pi} - 1 = \frac{\alpha}{(N\alpha/3\pi) \ln(eB/m^2)}.$

(13)

The expression (12) allows to reproduce the result of Refs. [7, 8], if the actual dependence of the coupling constant $\alpha_R$ on the ratio $eB/m^2$ is formally ignored and $\alpha_R \simeq \alpha$ is taken in it. It is remarkable that the constant $C_1$ obtained there by a numerical calculation as $C_1 \simeq 1.09 \pm 0.06$, appears to be $C_1 = \pi \exp(-\gamma_E) = 1.763877\ldots$.

In Fig. 3 the behaviour of the fermion dynamical mass $m$ divided by $\sqrt{eB}$ is shown versus the number of fermions $N$ and the field-free coupling constant $\alpha$, considered as free parameters of the model.

The dependence is seen to differ essentially from the results of Refs. [7, 8]. Namely, for any value of the coupling constant $\alpha$, such a critical number of fermions $N_{\alpha R}$ exists that for $N < N_{\alpha R}$ two values of the fermion dynamical mass are generated. For $N > N_{\alpha R}$ the equation (12) does not have a solution at all, thus the chiral symmetry is kept unbroken.

The dependence of the critical number $N_{\alpha R}$ on the value of the coupling constant $\alpha$ takes the form

$$N_{\alpha R}(\alpha) = \sqrt{\frac{3\pi}{2\exp(\gamma_E + 1)}} \frac{1}{\sqrt{\alpha}} - \frac{3}{4} + \frac{9}{16} \sqrt{\frac{3\pi}{2\exp(\gamma_E + 1)}} \frac{1}{\sqrt{\alpha} + O(\alpha)}.$$  (14)

It is quite remarkable that the double splitting of the fermion dynamical mass can be rather large. For example, for $\alpha = 0.1$ and $N = 1$ the mass difference is of 15 orders of magnitude. If one considers for the purposes
of illustration the magnetic field value $\sim 10^{33}$ G [16], the two values of the fermion dynamical mass are $\sim 10^3$ GeV and $\sim 10^{-8}$ eV.

We believe that the doublet splitting of the fermion dynamical mass and the conservation of the chiral symmetry at $N > N_c$, as well are new interesting physical phenomena in QED in strong external magnetic field.

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