Abstract

We examine the constraints from the recent HEIDELBERG-MOSCOW double beta decay experiment. It leads us to the almost degenerate or inverse hierarchy neutrino mass scenario. In this scenario, we obtain possible upper bounds for the Majorana $CP$ violating phase in the lepton sector by incorporating the data from the neutrino oscillation, the single beta decay experiments, and from the astrophysical observation. We also predict the neutrino mass that may be measurable in the future beta decay experiments. PACS number(s): 14.60.Pq, 13.10.+q, 23.40.-s
The recent neutrino oscillation experiments [1] have shown that neutrinos have masses. On the other hand, the experiments intending to determine directly neutrino mass such as the neutrinoless double beta decay($\beta\beta^{0}\nu$) and the single beta decay experiments are also on going. In a series of papers, we have discussed the $CP$ violation effects in the lepton sector incorporating all these experiments [2] and the other indirect astrophysical observations [3]. Recently, Klapdor-Kleingrothaus et al. [4] argued the evidence for $\beta\beta^{0}\nu$ by analyzing the data of the HEIDELBERG-MOSCOW experiment and reported that

$$0.11 < \langle m_\nu \rangle < 0.56\text{eV (95\% C.L.)}$$

with the best fit value, $\langle m_\nu \rangle = 0.39$ eV.

In this paper we reanalyze our studies in response to this announcement, although some papers [5] have discussed the constraints from this data. We especially consider the $CP$ violation effects in it. Namely, using the data of Eq.(1), we try to derive the constraints on the $CP$ violating phases in the lepton sector by combining the constraints from the neutrino oscillations, the beta decay experiments, and the astrophysical observations. Since the direct test of $CP$ violations from a measurement of such as electric dipole moments of leptons seems to be infeasible at present, the $\beta\beta^{0}\nu$ can be a good channel to detect the $CP$ violation effects, although they are indirectly measurable.

The $\beta\beta^{0}\nu$ experiment gives us the information of the averaged mass $\langle m_\nu \rangle$ for Majorana neutrinos defined by [6]

$$\langle m_\nu \rangle \equiv \left| \sum_{j=1}^{3} U_{e j}^2 m_j \right| = \left| m_1|U_{e 1}|^2 + m_2|U_{e 2}|^2 e^{2i\beta} + m_3|U_{e 3}|^2 e^{2i\rho'} \right|.$$  

(2)

Here $\beta$ and $\rho' \equiv \rho - \phi$ are $CP$ violating phases. The $U_{aj}$ is the Maki-Nakagawa-Sakata (MNS) [7] left-handed lepton mixing matrix that combines the weak eigenstate neutrino ($a = e, \mu$ and $\tau$) with the mass eigenstate neutrino of mass $m_j$ ($j$=1,2 and 3). The $U$ takes the following form in the standard representation [2]:
\[ U = \begin{pmatrix}
    c_1 c_3 & s_1 c_3 e^{i\beta} & s_3 e^{i(\rho - \phi)} \\
    (-s_1 c_2 - c_1 s_2 s_3 e^{i\phi}) e^{-i\beta} & c_1 c_2 - s_1 s_2 s_3 e^{i\phi} & s_2 c_3 e^{i(\rho - \beta)} \\
    (s_1 s_2 - c_1 c_2 s_3 e^{i\phi}) e^{-i\rho} & (-c_1 s_2 - s_1 c_2 s_3 e^{i\phi}) e^{-i(\rho - \beta)} & c_2 c_3
\end{pmatrix}. \quad (3)
\]

Here \( c_j = \cos \theta_j, \ s_j = \sin \theta_j \) (\( \theta_1 = \theta_{12}, \ \theta_2 = \theta_{23}, \ \theta_3 = \theta_{31} \)). Note that three CP violating phases, \( \beta, \ \rho \) and \( \phi \) appear in \( U \) for Majorana particles [8].

The other experimental constraints on neutrino mass and neutrino mixing angles are as follows: The recent beta decay experiments [9] restrict another averaged neutrino mass \( m_{\nu} \) as

\[ m_{\nu} \equiv \sqrt{\sum_{j=1}^{3} |U_{ej}|^2 m_j^2} < 2.2 \text{ eV}. \quad (4)\]

From the solar neutrino oscillation experiment [1], we have

\[ \Delta m_{12}^2 = m_2^2 - m_1^2 = \Delta m_{\text{solar}}^2 = (2 - 20) \times 10^{-5} \text{eV}^2, \quad (5)\]
\[ 0.3 \leq \sin^2 2\theta_{\text{solar}} \leq 0.93 \quad \text{for LMA-MSW}, \quad (6)\]

and

\[ \Delta m_{12}^2 = m_2^2 - m_1^2 = \Delta m_{\text{solar}}^2 = (4 - 9) \times 10^{-6} \text{eV}^2, \quad (7)\]
\[ 8 \times 10^{-4} \leq \sin^2 2\theta_{\text{solar}} \leq 8 \times 10^{-3} \quad \text{for SMA-MSW}, \quad (8)\]

for the large mixing angle (LMA) and small mixing angle (SMA) MSW solutions, respectively. From the atmospheric neutrino oscillation experiment [1], we obtain

\[ \Delta m_{23}^2 = m_3^2 - m_2^2 = \Delta m_{\text{atm}}^2 = \begin{cases} 
    (1 - 7) \times 10^{-3} \text{eV}^2 & \text{for normal hierarchy case} \\
    -(1 - 7) \times 10^{-3} \text{eV}^2 & \text{for inverse hierarchy case}
\end{cases} \quad (9)\]

The astrophysical observation [10] gives

\[ \sum_i m_i < 1.8 \text{ eV} \quad (10)\]

under some reasonable assumptions.
Hereafter we denote the experimental lower and upper bounds in Eq.(1) as $\langle m_\nu \rangle_{\text{min}}$ and $\langle m_\nu \rangle_{\text{max}}$, respectively. ($\langle m_\nu \rangle_{\text{min}} \leq \langle m_\nu \rangle \leq \langle m_\nu \rangle_{\text{max}}$). Let us first show that the data of the HEIDELBERG-MOSCOW ($\beta\beta_{0\nu}$) experiment in Eq.(1) prefers the almost degenerate or inverse hierarchy neutrino mass scenario for the LMA-MSW solution: Irrespectively of the $CP$ violating phases Eq.(2) leads to the inequality that

$$\langle m_\nu \rangle \leq |U_{e1}|^2 + m_1^2|e^{2i\beta}|.$$  \hspace{1cm} (11)

Since we have the constraint $|U_{e3}|^2 < 0.03$ from the oscillation experiments of CHOOZ [11] and SuperKamiokande [1], Eq.(11) becomes

$$\langle m_\nu \rangle < |U_{e1}|^2m_1 + |U_{e2}|^2\sqrt{m_1^2 + \Delta m_{12}^2 + 0.03\sqrt{m_1^2 + \Delta m_{12}^2 + \Delta m_{23}^2}}.$$ \hspace{1cm} (12)

It is apparent from Eqs.(1), (5), (9), and (12) that the normal hierarchy, $m_1 \lesssim m_2 \ll m_3$, is forbidden. We have no way of distinguishing between the almost degenerate and inverse hierarchy neutrino mass scenarios based on Eq.(1) at this stage. Hence we obtain

$$\langle m_\nu \rangle \simeq m|U_{e1}|^2 + |U_{e2}|^2|e^{2i\beta}|,$$ \hspace{1cm} (13)

$$\langle m_\nu \rangle \simeq m,$$ \hspace{1cm} (14)

with $m \equiv m_1 \simeq m_2$. Since $|U_{e3}|^2 < 0.03$, $\sin^2 2\theta_{\text{solar}}$ becomes $4|U_{e2}|^2(1 - |U_{e2}|^2)$ and Eq.(13) is rewritten as

$$\sin^2 \beta = \frac{1}{\sin^2 2\theta_{\text{solar}}} \left( 1 - \frac{\langle m_\nu \rangle^2}{m^2} \right).$$ \hspace{1cm} (15)

For LMA-MSW solution, Eq.(15) gives

$$\frac{1}{(\sin^2 2\theta_{\text{solar}})_{\text{max}}} \left( 1 - \frac{\langle m_\nu \rangle^2}{m^2} \right) \leq \sin^2 \beta \leq \frac{1}{(\sin^2 2\theta_{\text{solar}})_{\text{min}}} \left( 1 - \frac{\langle m_\nu \rangle^2}{m^2} \right).$$ \hspace{1cm} (16)

Here we have denoted the experimental lower and upper limits of Eq.(6) as $(\sin^2 2\theta_{\text{solar}})_{\text{min}}$ and $(\sin^2 2\theta_{\text{solar}})_{\text{max}}$, respectively. The allowed region in the $\sin^2 \beta - m$ plane for the LMA-MSW is shown by the shaded area in Fig.1 with use of the experimental bounds in Eq.(1). Let us superimpose on this allowed region the constraint of the experimental upper bound
$m_{\text{max}}$ for $m$ that is obtained from astrophysical observation and single beta decay. We have $m < 0.6$ eV from Eq.(10) and $m < 2.2$ eV from Eq.(4) using $m_\nu \approx m$. Namely, at present, the following experimental upper bound is obtained:

$$m < 0.6 \text{ eV} \equiv m_{\text{max}}. \quad (17)$$

It turns out from Fig.1 that a meaningful bound on the $CP$ violating phase $\beta$

$$\sin^2 \beta \leq \frac{1}{(\sin^2 2\theta_{\text{solar}})_{\text{min}}} \left( 1 - \frac{\langle m_\nu \rangle_{\text{min}}^2}{m_{\text{max}}^2} \right) \quad (18)$$
is derived for LMA-MSW solution if following condition is satisfied:

$$m_{\text{max}} < \frac{\langle m_\nu \rangle_{\text{min}}}{\sqrt{1 - (\sin^2 2\theta_{\text{solar}})_{\text{min}}}}. \quad (19)$$

Thus the constraint on $\beta$ from the present experiments is rather weak. So next we show in Fig.2 how this constraint is restricted as the future experiments make progress on the precision measurement, that is, as the lower bounds of $\langle m_\nu \rangle$ and $\sin^2 2\theta_{\text{solar}}$ are increased. We also obtain a possible lower bound,

$$\frac{1}{(\sin^2 2\theta_{\text{solar}})_{\text{max}}} \left( 1 - \frac{\langle m_\nu \rangle_{\text{max}}^2}{m_{\text{min}}^2} \right) \leq \sin^2 \beta, \quad (20)$$

if an experimental lower bound $m_{\text{min}}$ for $m$ (i.e. $m_{\text{min}} < m$) is found in the future experiments and the condition $\langle m_\nu \rangle_{\text{max}} < m_{\text{min}}$ is satisfied. The $\beta$ is not restricted in the case of SMA-MSW solution.

Next, following the method used in Ref [2], we discuss the bound on $\sin^2 \beta$ by using numerical analysis. In the following discussions, we assume $m_1 \lesssim m_2 \lesssim m_3$. The results are scarcely changed for the inverse hierarchical case. In order to obtain the constraints among the observable quantities, let us use $m_\nu$, $\Delta m_{12}^2 \equiv m_2^2 - m_1^2$ and $\Delta m_{23}^2 \equiv m_3^2 - m_2^2$ instead of $m_1$, $m_2$ and $m_3$. Namely, Inserting the relations, $m_2 = \sqrt{m_1^2 + \Delta m_{12}^2}$ and $m_3 = \sqrt{m_2^2 + \Delta m_{23}^2} = \sqrt{m_1^2 + \Delta m_{12}^2 + \Delta m_{23}^2}$ into Eq.(4) with the unitarity condition that $|U_{e1}|^2 = 1 - |U_{e2}|^2 - |U_{e3}|^2$, we obtain the following expressions for $m_1$, $m_2$, and $m_3$ [2]:

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\[ m_1 = \sqrt{m_\nu^2 - (|U_{e2}|^2 + |U_{e3}|^2)\Delta m_{12}^2 - |U_{e3}|^2 \Delta m_{23}^2}, \]
\[ m_2 = \sqrt{m_\nu^2 + (1 - |U_{e2}|^2 - |U_{e3}|^2)\Delta m_{12}^2 - |U_{e3}|^2 \Delta m_{23}^2}, \]
\[ m_3 = \sqrt{m_\nu^2 + (1 - |U_{e2}|^2 - |U_{e3}|^2)\Delta m_{12}^2 + (1 - |U_{e3}|^2)\Delta m_{23}^2}. \]

(21)

To show $\beta$ dependence of $m_\nu$, we use the center values; $\langle m_\nu \rangle = 0.39\,\text{eV}$, $|U_{e2}|^2 = 0.29$ (LMA-MSW solution), $|U_{e3}|^2 = 0.03$, $\Delta m_{\text{solar}}^2 = 4.5 \times 10^{-5}\,\text{eV}^2$, and $\Delta m_{\text{atm}}^2 = 3.2 \times 10^{-3}\,\text{eV}^2$ as a typical case. By inserting Eq.(21) into Eq.(2), we get a relation among $m_\nu$, $\beta$, and $\rho'$ as depicted in Fig.3. Also from Eq.(21) and Eq.(10), we obtain the upper bound for $m_\nu$ as

\[ \overline{m}_\nu < 0.6\,\text{eV}. \]

(22)

This has been superimposed on Fig. 3 (b) and (c) giving the upper bound of $\sin^2 \beta$ as

\[ \sin^2 \beta < 0.7. \]

(23)

Of course, this upper bound depends on the input values of $\langle m_\nu \rangle$, $|U_{e2}|^2$, $|U_{e3}|^2$, $\Delta m_{\text{solar}}^2$, and $\Delta m_{\text{atm}}^2$. The Fig.3 also predicts the lower limit of the averaged neutrino mass $\overline{m}_\nu$ as $\overline{m}_\nu > \langle m_\nu \rangle$: Namely, we obtain

\[ \overline{m}_\nu > 0.39\,\text{eV}, \]

(24)

indicating that we have a chance for detecting nonzero $\overline{m}_\nu$ in the future beta decay experiment.

In conclusion, we have obtained the bounds for the Majorana CP violating phases from the recent data of the HEIDELBERG-MOSCOW double beta decay experiment incorporating the data from the neutrino oscillation, the astrophysical observation, and the single beta decay experiments. We have also predicted the lower bound for neutrino mass that may be measurable in the future beta decay experiments.
REFERENCES


See also the recent report: Φ. Elgarφy et al, arXiv:astro-ph/0204152

FIG. 1. The allowed regions in the $\sin^2 \beta - m$ plane for LMA solutions, Eq.(6) with $m \equiv m_1 \simeq m_2$. The dark shaded region is for the center values and the light ones are for the other empirical values.

FIG. 2. The possible upper bounds for $\sin^2 \beta$ shown in the $(\sin^2 2\theta_{\text{solar}})_{\text{min}} - \langle m_\nu \rangle_{\text{min}}$ plane. The dot indicates the present experimental value, implying that it does not restrict $\beta$. The star shows that $\beta$ is constrained as $\sin^2 \beta < 0.7$ if we use the center values.
FIG. 3. The relation among $m_\nu$, $\beta$, and $\rho$ variables for the LMA-MSW solution for the solar neutrino problem. We have fixed the following values: $\langle m_\nu \rangle = 0.39$ eV, $\Delta m_{solar}^2 = 4.5 \times 10^{-5}$ eV$^2$, $|U_{e2}|^2 = 0.29$ (LMA-MSW solution), $\Delta m_{atm}^2 = 3.2 \times 10^{-3}$ eV$^2$, and $|U_{e3}|^2 = 0.03$. The lower (upper) solid lines in (b) and (c) indicates the upper limit of astrophysics Eq.(10) (single beta decay Eq.(4)).