Partial conservation of axial current and axial exchange currents in the nucleon

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Abstract

We discuss the axial form factors of the nucleon within the context of the nonrelativistic chiral quark model. Partial conservation of axial current (PCAC) imposed at the quark operator level enforces an axial coupling for the constituent quarks which is smaller than unity. This leads to an axial coupling constant of the nucleon $g_A$ in good agreement with experiment. PCAC also requires the inclusion of axial exchange currents. Their effects on the axial form factors are analyzed. We find only small exchange current contributions to $g_A$, which is dominated by the one-body axial current. On the other hand, axial exchange currents give sizeable contributions to the axial radius of the nucleon $r_A^2$, and to the non-pole part of the induced pseudoscalar form factor $g_P$. 

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I. INTRODUCTION

Several attempts have been made to reconcile the nonrelativistic quark model (NRQM) prediction [1] for the nucleon axial vector coupling constant

$$g_A(0) = \frac{5}{3} g_{Aq}(0)$$  \hspace{1cm} (1)

with the experimental value \( (g_A(0)/g_V(0))_{(exp)} = 1.2670(35) \) [2], where \( g_V(0) = 1 \) according to the conserved vector current hypothesis. This 25% deviation between theory and experiment contrasts sharply with the successful NRQM prediction of nucleon magnetic moments \( \mu_p/\mu_n = -\frac{3}{2} \) \( (exp.-1.46) \). However, many quark model calculations tacitly assume that the axial coupling of the constituent quark, \( g_{Aq}(0) = 1 \). In 1990, Weinberg [4] has offered an explanation for why \( g_{Aq}(0) = 1 \).

In 1979 Glashow [5] suggested that a deformation of the valence quark distribution in the nucleon could reduce the NRQM result of Eq.(1) while leaving the successful NRQM prediction for the nucleon magnetic moments intact [6]:

$$g_A(0) = \frac{5}{3} \left( 1 - \frac{6}{5} P_D \right),$$ \hspace{1cm} (2)

where \( P_D = 0.21 \) is the admixture probability of \( D \)-waves in the nucleon. This is a rather large \( D \)-state probability compared to the \( P_D = 0.0016 \) calculated from gluon exchange induced tensor forces between quarks [7].

A different solution is provided by relativistic bag model calculations [8] in which

$$g_A(0) = \frac{5}{3} \left( 1 - \frac{4}{3} \int_0^\infty dr f^2(r) \right),$$ \hspace{1cm} (3)

The lower component \( f(r) \) of the Dirac spinor is responsible for a 30% reduction of the NRQM result. Furthermore, these bag-model calculations show that the axial vector coupling constant is completely determined by the quark core. Pion cloud effects turned out to be zero or negligibly small.
Similarly, in the NRQM, the relativistic corrections to the usual one-body axial current considerably reduce $g_A(0)$ [9]. However, the corresponding relativistic correction terms in the one-body electromagnetic current spoil the good agreement for baryon magnetic moments, as we have pointed out in Ref. [10]. In the case of the magnetic moments, the success of the NRQM is closely related to the cancellation of the various electromagnetic two-body currents required by the continuity equation

$$ q \cdot J(q) - [H, J^0(q)] = 0 $$

for the electromagnetic current $(J^0, J)$. Recently, it has been shown that the axial two-body current contributions to $g_A$ cancel each other [11], similar to the cancellation of the electromagnetic exchange current contributions in the case of the magnetic moments. As a result of this almost complete cancellation, the NRQM prediction of Eq.(1) is not modified when axial exchange currents are included.

Before one can draw any conclusion concerning the failure of the NRQM to accurately predict $g_A(0)$ one should investigate the implications of the PCAC constraint for the axial operators at the quark level. PCAC can be formulated as [12]

$$ q \cdot A(q) - [H, A^0(q)] = -i \sqrt{2} f_\pi M^\pi(q), $$

where $H$ is the full Hamiltonian of the three-quark system, including the center of mass motion, $q$ is the three-momentum transfer, $A^\mu = (A^0, A)$ stands for the axial current operator, $f_\pi = 92.4$ MeV is the pion decay constant, and $M^\pi$ is the pion absorption/emission operator. Eq.(5) is the weak axial current analogue of Eq.(4) for the electromagnetic current. In contrast to the electromagnetic current $J$, which is exactly conserved, the axial current $A$ is only partially conserved. The PCAC condition states that the four-divergence of the axial nucleon current is not zero but proportional to the pion absorption/emission amplitude. It also implies that the two-body potentials in $H$ be accompanied by two-body contributions to the axial current and absorption operators if PCAC is to hold. Thus, two-body axial exchange currents consistent
with the nonrelativistic quark model Hamiltonian have to be constructed, and their effect on the axial form factors of the nucleon has to be investigated.

The paper is organized as follows. In sect. 2 we introduce the notation for the various axial form factors appearing in the general expression for the axial current. Sect. 3, briefly reviews the chiral quark potential model. The axial quark current operators are derived from Feynman diagrams and their consistency with PCAC is investigated in sect. 4. Numerical results for the axial form factors are presented and discussed in sect. 5. Finally, a short summary of the results achieved is presented.

II. AXIAL FORM FACTORS OF THE NUCLEON

As in the case of the electromagnetic current one can write down the allowed Lorentz structures for an axial current operator. The most general form for the nucleon axial current can be written as [13]

\[ A^\mu_a = \bar{u}'(p') \left( g_A(q^2) \gamma^\mu \gamma_5 + 2 \frac{M_N}{m_\pi^2} g_P(q^2) q^\mu \gamma_5 + g_T(q^2) P^\mu \gamma_5 \right) \frac{\tau^a}{2} u(p), \tag{6} \]

where \( u(p) \) and \( u'(p') \) are the Dirac spinors of the nucleon in the initial and final state with three-momenta \( p \) and \( p' \), and \( \tau^a \) is the nucleon isospin operator. The masses of the nucleon and pion are denoted by \( M_N \) and \( m_\pi \). Here, the \( q \) is the four-momentum transfer and \( P \) the total four momentum defined as

\[ q = p' - p, \quad P = p' + p. \tag{7} \]

The three form factors in Eq.(6) \( g_A(q^2), g_P(q^2), \) and \( g_T(q^2) \) are scalar functions of the four-momentum transfer. They are called the axial form factor \( g_A \), the induced pseudoscalar form factor \( g_P \), and the tensor form factor \( g_T \). All three form factors are real because of the time-reversal (\( T \)) invariance of the weak interaction.

We work in the Breit frame where a clear separation of form factors is achieved. The
nonrelativistic reduction of Eq.(6), including the normalization factors of the Dirac spinors, lead, in the Breit frame, to the following axial operators at the baryon level

\[ A^0 = \frac{\sigma \cdot q}{2M_N} \left( -P^0 g_T(q^2) \right) \frac{\tau^+}{2}, \]

\[ A = \sigma \left[ g_A(q^2) \left( 1 - \frac{q^2}{24M_N^2} \frac{g_P(q^2)}{3m_\pi^2} \right) - \left( \frac{q^2}{3m_\pi^2} g_P(q^2) \right) \right] \frac{\tau^+}{2}, \]

\[ + [\sigma^{[1]} \otimes q^{[2]}] \left[ \sqrt{\frac{5}{3}} \left( \frac{1}{8M_N^2} g_A(q^2) + \frac{1}{m_\pi^2} g_P(q^2) \right) \frac{\tau^+}{2} \right], \]  

where \( \sigma \) and \( \tau \) are nucleon spin and isospin operators. \( \tau^+ \) is the usual ladder operator defined as \( \tau^+ = \tau^x + i\tau^y \) appropriate for the \( n \rightarrow p \) transition.

Note that \( g_T(q^2) \) is zero if the strong interactions are invariant under \( G \)-parity transformations and for transitions within the same isospin multiplet, as is the case here. This would imply that the time component of the axial current is also zero in the Breit frame. The experimental evidence for \( g_T \) is very scarce. We quote here \( g_T(0) = (-2.94 \pm 5.88) \times 10^{-4} \text{ MeV}^{-1} \) taken from Ref. [14], which is compatible with zero.

The induced pseudoscalar form factor of the nucleon consists of two parts [15]

\[ g_P(q^2) = f_\pi g_{\pi NN}(q^2) \frac{m_\pi^2}{M_N} m_\pi^2 - q^2 + g_P^{\text{non-pole}}(q^2) \]  

The first term is the so-called pion-pole contribution. For \( q^2 = 0 \) the non-pole part is given by

\[ g_P^{\text{non-pole}}(0) \simeq -\frac{1}{6} g_A m_\pi^2 r_A^2 \]  

where \( r_A \) is the axial radius of the nucleon. This result is sometimes referred to as Adler-Dothan-Wolfenstein (ADW) correction [16].

Evaluating the time component of the axial current at the quark level, we obtain \( g_T(q^2) \), while the spatial part of the axial operators will give us both \( g_P^{\text{non-pole}}(q^2) \) and \( g_A(q^2) \). In order to obtain \( g_P^{\text{pion-pole}}(q^2) \) in the quark model, one would have to couple the weak gauge boson first to the pion and then the pion to the constituent quarks in the nucleon.
III. THE CHIRAL QUARK MODEL

In this section we briefly discuss the Hamiltonian of the chiral quark model (χQM). The χQM model was devised to effectively describe the low-energy properties of QCD. Theoretical reasons for such an approach are given in Ref. [17]. Here, the chiral symmetry of QCD is introduced via a linear σ model with pseudoscalar (π) and scalar (σ) degrees of freedom. The spontaneous breakdown of chiral symmetry at the 1 GeV scale leads to a quasi-particle picture of the constituent quark [18], which is an extended object with finite hadronic and e.m. size, and with a mass of about 1/3 the mass of the nucleon. At the same time the axial coupling of constituent quarks $g_{Aq}$ gets renormalized in the transition from QCD to the effective theory and deviates from its QCD value $g_{Aq} = 1$ as we will argue below.

The Hamiltonian for the internal motion of three quarks with identical mass is given by:

$$H_{int} = \sum_j \left( m_q + \frac{p_j^2}{2m_q} \right) - \frac{P^2}{6m_q} + \sum_{j<k} \left( (V^{conf})_{j,k} + (V^g)_{j,k} + (V^\pi)_{j,k} + (V^\sigma)_{j,k} \right),$$

(11)

where $m_q$ is the constituent quark mass for which we take the value $m_q = 313$ MeV. The momentum operator of the j-th quark is denoted by $p_j$, and $P$ is the center of mass momentum of the three quark system. The kinetic energy associated with the center of mass motion is subtracted from the total Hamiltonian. Apart from the confinement potential ($V^{conf}$), the Hamiltonian includes two-body interactions from one-gluon ($V^g$), one-pion ($V^\pi$), and one-sigma ($V^\sigma$) exchange. These are obtained from the Feynman diagrams in Fig. 1. For the π and σ meson exchange potentials we introduce a short distance regulator by means of the static vertex form factor

$$F(k^2) = \left( \frac{\Lambda^2}{\Lambda^2 + k^2} \right)^{1/2}.$$  

(12)

Here, $k$ is the three-momentum of the exchanged meson and $\Lambda$ is the cut-off parameter. In coordinate space this leads to a very simple form for the potential, where a second Yukawa term with a fictitious meson mass $\Lambda$ appears.
The \( \chi \)QM model also contains the one-gluon exchange potential introduced by de Rújula et al. [19], and originally used to explain certain regularities in the spectrum of excited baryon states [20,21]. The expressions used in the following are:

\[
(V_\pi)_{j,k} = \frac{g_{\pi q}^2}{4\pi} \frac{\Lambda_\pi^2}{\Lambda_\pi^2 - m_\pi^2} \frac{\tau_j \cdot \tau_k}{4m_q^2} \sigma_j \cdot \nabla_q \sigma_k \cdot \nabla_q \left( \frac{e^{-m_\pi r}}{r} - \frac{e^{-\Lambda_\pi r}}{r} \right) \tag{13}
\]

\[
(V_\sigma)_{j,k} = -\frac{g_{\sigma q}^2}{4\pi} \frac{\Lambda_\sigma^2}{\Lambda_\sigma^2 - m_\sigma^2} \left( \frac{e^{-m_\sigma r}}{r} - \frac{e^{-\Lambda_\sigma r}}{r} \right) \tag{14}
\]

\[
(V_g)_{j,k} = \frac{\alpha_s}{4} \frac{\lambda_j \cdot \lambda_k}{m_q^2} \left\{ \frac{1}{r} - \frac{\pi}{m_q^2} \left( 1 + \frac{2}{3} \sigma_j \cdot \sigma_k \right) \delta(r) - \frac{1}{4m_q^2} \left( 3 \sigma_j \cdot \hat{r} \sigma_k \cdot \hat{r} - \sigma_j \cdot \sigma_k \right) \frac{1}{r^3} \right\} \tag{15}
\]

where \( r_j, \sigma_j, \tau_j, \) and \( \lambda_j^c \) are the position, spin, isospin, and color operators of the \( j \)-th quark. The relative coordinate of the two interacting quarks is given by \( r = r_j - r_k \) with modulus \( r = |r| \) and unit vector \( \hat{r} = r/r \).

In the gluon exchange potential \( V^g \), \( \alpha_s \) is the effective quark-gluon coupling constant, which we consider as a free parameter. Following Isgur and Karl [20] spin-orbit terms in \( V^g \) are neglected. In the meson exchange interactions, the quark-meson couplings \( (g_{\pi q}, g_{\sigma q}) \) and the cut-off parameters \( (\Lambda_\pi, \Lambda_\sigma) \) are related via the chiral symmetry constraints [22]

\[
g_{\sigma q} = g_{\pi q}, \quad \Lambda_\sigma = \Lambda_\pi. \tag{16}
\]

For the pion \((m_\pi)\) and sigma \((m_\sigma)\) masses, the bosonization technique applied to the chirally symmetric Nambu-Jona-Lasinio (NJL) Lagrangian [18] gives

\[
m_\sigma^2 = 4m_q^2 + m_\pi^2. \tag{17}
\]

We use \( m_\pi = 139 \text{ MeV} \) from which \( m_\sigma = 641 \text{ MeV} \) results. The pion-quark coupling is determined by the experimental \( \pi N \) coupling strength \( f_{\pi N}^2 / 4\pi = 0.0749 \) via [23]:

\[
g_{\pi q}^2 = \left( \frac{3}{5} \right)^2 \frac{f_{\pi N}^2}{4\pi} \left( \frac{2m_q}{m_\pi} \right)^2. \tag{18}
\]

and a cut-off value \( \Lambda = 4.2 \text{ fm}^{-1} \) is obtained [23], by fitting the size of the \( q\bar{q} \) component of the pion, assumed to be 0.4 fm [24].
The constituent quarks are confined by a long-range, spin-independent, scalar two-body potential. For convenience a harmonic oscillator (h.o.) potential is often used

\[
(V_{\text{conf}})_{j,k} = -a \lambda^c_j \cdot \lambda^c_k r^2. \tag{19}
\]

However, from lattice calculations we know that a linear confinement, which at larger distances is screened by quark-antiquark pair creation is more realistic. The effect of these color screening potentials on the baryon spectrum has been investigated by Zhang et al. [25]. Here we consider both the standard h.o. potential and a color screening potential of the form

\[
(V_{\text{conf}})_{j,k} = -a \lambda^c_j \cdot \lambda^c_k (1 - e^{-\mu r}) + C. \tag{20}
\]

We use the h.o. confinement potential when working with unmixed (UM) wave functions. On the other hand, a pure h.o. potential without any anharmonicity cannot reproduce the baryon mass spectrum [20,21]. Therefore, we employ a color screened confinement potential for mixed (CM) wave functions. For a discussion of the wave functions and the model parameters we refer the reader to Ref. [26].

IV. THE AXIAL CURRENT OPERATORS

A. Impulse approximation

Traditionally, in nonrelativistic quark models, the study of electromagnetic and weak properties of hadrons is done in the so-called impulse approximation, in which only one-body operators are considered. In this approximation the axial charge \( (A^0) \) and axial current \( (A) \) operators corresponding to Fig. 2(a) are:

\[
A^0_{\text{imp}} = -\frac{1}{\sqrt{2}} g_{Aq} \sum_{j=1}^{3} \tau^1_j e^{iq \cdot r_j} \frac{1}{2m_q} \sigma_j \cdot (q + 2p_j),
\]

\[
A_{\text{imp}} = -\frac{1}{\sqrt{2}} g_{Aq} \sum_{j=1}^{3} \tau^1_j e^{iq \cdot r_j} \sigma_j. \tag{21}
\]
where $\mathbf{q}$ is the three-momentum transfer imparted by the $W$ boson. In the spherical basis used here, the isospin operator of the $j$-th quark is given by

$$
\tau_j^{\pm 1} = \mp \frac{1}{\sqrt{2}} (\tau_j^x \pm \tau_j^y) = \mp \frac{1}{\sqrt{2}} \tau_j^z,
\tau_j^0 = \tau_j^z,
$$

(22)

where $\tau^\pm$ are usual isospin raising and lowering operators, and $\tau_j^\lambda$ with $\lambda = \pm 1, 0$ are the spherical components of the Pauli isospin matrix. We take the $+1$ component appropriate for the $n \to p$ transition.

As the axial current is not exactly conserved there is nothing to prevent the constituents quarks from having an effective axial charge $g_{Aq}$ different from current quarks. In his second paper on $g_{Aq}$ Weinberg has proven that while constituent quarks have no anomalous magnetic moments, their axial coupling may be considerably renormalized by the strong interactions [27]. In fact, explicit calculation shows that

$$
g_{Aq}^2 = 1 - \frac{m_q^2}{8\pi^2 f^2_\pi},
$$

(23)

which leads to a 10% reduction of $g_{Aq}$. Weinberg’s arguments have recently been re-investigated. Using Witten’s large $N_C$ counting rules, it has been shown that, in contrast to the magnetic moment of the quarks, corrections to $g_{Aq}$ appear already at order $N_C^0$ [28]. Further investigation in the constituent quark structure in the NJL model shows that $g_{Aq} \approx 0.78$ [29]. In a different approach, Peris [30] shows that $g_{Aq}$ is renormalized by pion loops and obtains

$$
g_{Aq} = 1 - \frac{m_q}{4\pi f_\pi} \ln\left(\frac{m^2_\pi}{m^2_q}\right).
$$

(24)

Thus, by now different models of constituent quark structure agree concerning the value $g_{Aq} \approx 3/4$.

As explained in the next section, a value of $g_{Aq} \approx 3/4$ is also obtained after imposing the constraints of PCAC ( Eq.(5) ) on the axial current and pion absorption operators.
In order to check PCAC for the one-body axial operators we need to know the impulse contribution to the pion absorption operator. This can be derived from the Feynman diagram in Fig. 2(b) and is given by

$$M_{\text{imp}}^\pi = -i \sum_j \tau_j \frac{g_{\pi q}}{2m_q} e^{i\mathbf{q} \cdot \mathbf{r}_j} \mathbf{\sigma}_j \cdot \mathbf{q}. \quad (25)$$

It is now straightforward to prove that the relation

$$\mathbf{q} \cdot \mathbf{A}_{\text{imp}} - [T, A^0_{\text{imp}}] = -i\sqrt{2} f_\pi M_{\text{imp}}^\pi, \quad (26)$$

is satisfied*, up to $O(g_{Aq}/m_q^2)$ provided

$$g_{Aq} = f_\pi \frac{g_{\pi q}}{m_q}. \quad (27)$$

Thus, one obtains a Goldberger-Treiman relation at the quark level as a consequence of imposing PCAC on the axial current and pion absorption operators. Using the phenomenological pion-quark coupling from Eq.(18) $g_{\pi q} = 2.62$, the empirical pion decay constant $f_\pi = 92.4$ MeV, and the constituent quark mass $m_q = 313$ MeV one gets $g_{Aq} = 0.774$ in accord with the result [29] quoted above. If we use this value of $g_{Aq}$ in the axial one-body current operators of Eq.(21) we obtain for unmixed wave functions

$$g_A(0) = g_{Aq} \frac{5}{3} = 1.29 \quad (28)$$

in much better agreement with the experimental number. Obviously, the often quoted “failure of the NRQM” in reproducing $(g_A)_{exp} = 1.267$ is related to the incorrect assumption that the axial coupling constant of the constituent quarks is the same as for current quarks, namely $g_{Aq} = 1$.

*Here, $T$ is the total kinetic energy operator given by the first three terms in Eq.(11).
The result of Eq. (28) seems to leave little room for configuration mixing effects and for exchange current contributions to $g_A(0)$. This notwithstanding, consistency requires that both effects be included in the present theory. From our analysis of axial exchange currents we will again see that the internal consistency of the constituent quark model requires $g_A \approx 3/4$.

C. Axial exchange current operators

The PCAC condition not only requires a value of $g_A q$ different from unity, but also, as indicated in the introduction and the preceding section, the inclusion of two-body axial current and absorption operators consistent with the two-body potentials in the Hamiltonian.

We obtain the two-body axial current and absorption operators from a nonrelativistic reduction of the Feynman diagrams of Fig. 3 and Fig. 4 respectively. We have gluon, pion and scalars ($\sigma$ and confinement) exchange currents, plus the pion-sigma axial exchange current. As in the quark-quark potentials and the electromagnetic currents, we keep only the local terms in the operators. We hope that the nonlocal terms are to some extent included in the effective parameters of the model.

The axial gluon exchange current and absorption operators are obtained as

$$A^0_g = g_A q \sum_{j<k} \frac{\alpha_s}{8m_q^2} \lambda^c_j \cdot \lambda^c_k \left\{ \frac{\tau^1_j}{\sqrt{2}} e^{iqr_j} (\sigma_j \times \sigma_k) \cdot r + (j \leftrightarrow k) \right\} \frac{1}{r^3}$$

$$A_g = g_A q \sum_{j<k} \frac{\alpha_s}{16m_q^3} \lambda^c_j \cdot \lambda^c_k \left\{ -\frac{\tau^1_j}{\sqrt{2}} e^{iqr_j} \left[ -i(\sigma_j \cdot r) q + \left( 3(\sigma_j + \sigma_k) \cdot \hat{r} - (\sigma_j + \sigma_k) \right) \right] \frac{1}{r^3} + \frac{8\pi}{3} (\sigma_j + \sigma_k) \delta(r) \right\} + (j \leftrightarrow k) \right\}$$

$$M^\pi_g = -g_A q \sum_{j<k} \frac{\alpha_s}{8m_q^2} \lambda^c_j \cdot \lambda^c_k \left\{ (-\tau^1_j) e^{iqr_j} \frac{1}{r^3} \sigma_j \cdot r + (j \leftrightarrow k) \right\}$$

The axial pion-pair exchange current and absorption operators resulting from pseudoscalar pion-quark coupling are given next

$$A^0_\pi = g_A q \frac{g_{\pi q}^2}{4\pi} \frac{1}{2m^2_\pi} \frac{\Lambda^2}{\Lambda^2 - m^2_\pi} \sum_{j<k} \left\{ \frac{(\sigma_j \times \sigma_k)}{\sqrt{2}} e^{iqr_j} \frac{1}{r} \gamma^1_j (r) \sigma_k \cdot r + (j \leftrightarrow k) \right\}.$$
\[ A_\pi = g_\pi q \frac{g_\pi^2}{4 \pi} \frac{1}{8 m^3_q} \Lambda^2 - m^2_\pi \frac{i \sqrt{2}}{2} \sum_{j<k} \left\{ e^{i \mathbf{q} \cdot \mathbf{r}_j} \left[ \frac{1}{r} \mathbf{r}_j \cdot \mathbf{r} \mathbf{q} - i (\mathbf{\tau}_j \times \mathbf{\sigma}_k) \frac{1}{r} \mathbf{Y}_1(r) \mathbf{\sigma}_k \cdot \mathbf{r} \right. \mathbf{q} \\
\left. + i (\mathbf{\tau}_j \times \mathbf{\sigma}_k) \frac{1}{r} \mathbf{Y}_1(r) \mathbf{\sigma}_k \cdot \mathbf{r} \right] + (j \leftrightarrow k) \right\}. \]

In Eq.(30) we have used the following abbreviations

\[ \mathbf{Y}_1(r) = m^2_\pi Y_1(m_\pi r) - \Lambda^2 Y_1(\Lambda r), \]
\[ \mathbf{Y}_2(r) = m^3_\pi Y_2(m_\pi r) - \Lambda^3 Y_2(\Lambda r), \]

and

\[ Y_1(x) = \frac{e^{-x}}{x} \left( 1 + \frac{1}{x} \right), \]
\[ Y_2(x) = \frac{e^{-x}}{x} \left( 1 + \frac{3}{x} + \frac{3}{x^2} \right). \]

As in the one-pion exchange potential of Eq.(13), we have introduced the short distance regulator via the static vertex form factor in Eq.(12).

The axial current and absorption operators connected with scalar exchange reads

\[ A^0_S = 0, \]
\[ A_S = -\frac{1}{\sqrt{2}} \frac{ig_\pi q}{4 m^3_q} \sum_{j<k} \left\{ \mathbf{r}_j e^{i \mathbf{q} \cdot \mathbf{r}_j} \left[ -i (\mathbf{\sigma}_j \cdot \mathbf{q}) + (\mathbf{q} \cdot \nabla_r) \mathbf{\sigma}_j - (\mathbf{\sigma}_j \cdot \nabla_r) \mathbf{q} \right] + (j \leftrightarrow k) \right\} V_S(r) \]
\[ M^S_S = ig_\pi q \frac{1}{2 m^2_q} \sum_{j<k} \left\{ \mathbf{r}_j e^{i \mathbf{q} \cdot \mathbf{r}_j} (\mathbf{\sigma}_j \cdot \mathbf{q}) + (j \leftrightarrow k) \right\} V_S(r). \]

Here, \( V_S(r) \) stands for either the one-sigma exchange potential or the confinement potential introduced in the previous section. \( A^0_S = 0 \) because it is purely nonlocal in lowest order.

Finally, for the pion-sigma contribution we get the following axial charge, current and pion absorption operators
\[ A_{\pi-\sigma}^0 = 0, \quad (34) \]

\[ A_{\pi-\sigma} = \sqrt{2} \frac{g_{\pi g}^2 g_{Aq}}{4\pi 2m_q} \sum_{j<k} \left\{ \tau_k^1 \sigma_k \cdot \nabla_k \left( \int_{-1/2}^{1/2} dv \ e^{i\mathbf{q} \cdot (\mathbf{R} - \mathbf{r} v)} \left( \mathbf{r} + i\mathbf{q} v \frac{1}{L_v} \frac{e^{-L_v r}}{r} \right) + (j \leftrightarrow k) \right) \right\}, \quad (35) \]

\[ M_{\pi-\sigma}^\pi = -\frac{g_{\pi q}^3}{4\pi} \frac{m_\pi^2 - m_q^2}{4m_q^2} \sum_{j<k} \left\{ \tau_k^1 \sigma_k \cdot \nabla_k \left( \int_{-1/2}^{1/2} dv \ e^{i\mathbf{q} \cdot (\mathbf{R} - \mathbf{r} v)} \frac{e^{-L_v r}}{L_v} \right) + (j \leftrightarrow k) \right\}, \]

where

\[ \mathbf{R} = \frac{\mathbf{r}_j + \mathbf{r}_k}{2}, \quad L_v^2 = m_\sigma^2 \left( v + \frac{1}{2} \right) + m_\pi^2 \left( \frac{1}{2} - v \right) + q^2 \left( \frac{1}{4} - v^2 \right). \quad (36) \]

**D. PCAC and exchange currents**

The axial exchange currents and pion absorption operators have been listed in the preceding section but their consistency with PCAC remains to be considered. Here, we check to what extent the axial current and pion absorption operators derived from the Feynman diagrams in Figs.2-4 satisfy the PCAC condition of Eq.(5).

We start with the one-gluon exchange contributions. Considering \( g_{Aq} \) as a \( \mathcal{O}(1/m_q) \) quantity, as suggested by Eq.(27), one immediately gets

\[ [T, A_{g}^0] = \mathcal{O}(1/m_q^4), \]

\[ \mathbf{q} \cdot \mathbf{A}_g = \mathcal{O}(1/m_q^4), \quad (37) \]

while

\[ M_g^\pi = \mathcal{O}(1/m_q^2). \]

Ignoring the \( \mathcal{O}(1/m_q^4) \) contributions in Eq.(37), PCAC requires that to order \( \mathcal{O}(1/m_q^2) \)

\[ \sqrt{2} \ i f_\pi \ M_g^\pi = \left[ \sum_{j<k} (V_h)_{jk} , A_{imp}^0 \right] \]

is satisfied. The commutator is given as
\[
\left[ \sum_{j<k} (V_g)_{jk} , A_{imp}^0 \right] = -i \sum_{j<k} g_{Aq} \frac{\alpha_s}{4m_q} \lambda_j^c \cdot \lambda_k^c \left\{ \frac{-\tau_j^1}{\sqrt{2}} e^{i q \cdot r_j} \frac{1}{r^3} \sigma_j \cdot r + (j \leftrightarrow k) \right\}, \]
\]

where terms of higher order than \( O(1/m_q^2) \) are neglected. Comparing Eq. (39) with the pion absorption operator in Eq. (29) we see that Eq. (38) is satisfied in this order if \( g_{Aq} = f_\pi \frac{g_\pi q}{m_q} \) holds.

Thus, the PCAC constraint for the axial gluon exchange current leads again to the Goldberger-Treiman relation as in Eq. (27). Note that the commutator in Eq. (38), and those appearing in the following, generate higher order three-body operators. These are not taken into account here.

We turn now to the pion and sigma exchange contributions. A direct calculation shows that

\[
[T, A_\pi^0] = O(1/m_q^4),
\]
\[
q \cdot A_\pi = O(1/m_q^4),
\]
\[
M_\pi^2 = O(1/m_q^2),
\]
\[
(40)
\]

while

\[
\left[ \sum_{j<k} (V_\pi)_{jk} , A^0 \right] = O(1/m_q^4).
\]
\[
(41)
\]

This clearly shows that pion exchange contributions alone are not sufficient to satisfy the PCAC condition. Neither the commutator nor the spatial divergence generates a \( O(1/m_q^2) \) term that would correspond to the pion absorption operator in Eq. (30).

Similarly, one finds for the one-sigma exchange contributions (we replace \( S \rightarrow \sigma \) in Eq. (33))

\[
[T, A_\sigma^0] = O(1/m_q^4),
\]
\[
q \cdot A_\sigma = O(1/m_q^4).
\]
\[
(42)
\]

In addition, to order \( O(1/m_q^2) \)

\[
\left[ \sum_{j<k} (V_\sigma)_{jk} , A^0 \right] = -i \frac{1}{\sqrt{2}} \frac{g_Aq}{m_q} \sum_{j<k} \left\{ \frac{\tau_k^1}{r^3} e^{i q \cdot r_k} \sigma_k \cdot \nabla_k + (j \leftrightarrow k) \right\} V_\sigma(r),
\]
\[
(43)
\]

whereas
\[
\sqrt{2} f_\pi M_\sigma^\pi = - \frac{1}{\sqrt{2}} g_{Aq} f_\pi \frac{g_{\pi \sigma}}{m_q} \sum_{j<k} \left\{ \boldsymbol{\tau}_k e^{i \mathbf{q} \mathbf{r}_k} \mathbf{\sigma}_k \cdot \mathbf{q} + (j \leftrightarrow k) \right\} V_\sigma(r).
\] (44)

Thus, as in the pion exchange case, the sigma exchange contributions alone are not sufficient to recover PCAC.

Without regularization terms in the \( \pi \) and \( \sigma \) exchange pieces everything would fall into place once the pion-sigma exchange contribution in Eq.(35) (including the factor \( g_{Aq} \)) is considered. In that case

\[
\mathbf{q} \cdot \mathbf{A}_{\pi-\sigma} - \left[ \sum_{j<k} (V_\sigma)_{jk} A^0 \right] = -i \sqrt{2} f_\pi \left\{ M_{\pi-\sigma}^\pi + M_{\pi}^\pi + M_{\sigma}^\pi \right\}
\] (45)

is satisfied to order \( \mathcal{O}(1/m_q^2) \) whenever the quark level Goldberger-Treiman relation Eq.(27) holds. However, PCAC is broken by the regularization terms proportional to \( \Lambda \) in the pion and sigma exchange contributions.

There is also a problem with the axial confinement current for which one has

\[
\left[ \sum_{j<k} (V_{\text{conf}})_{jk} A^0 \right] - \sqrt{2} i f_\pi M_{\text{conf}}^\pi = \mathcal{O}(1/m_q^2).
\] (46)

There is no microscopic axial current at this order, whose divergence would cancel this contribution. Therefore, PCAC is also broken at order \( \mathcal{O}(1/m_q^2) \) due to confinement.

V. RESULTS AND DISCUSSIONS

In Table I we give the results for \( g_A(0), g_{P^{\text{non-pole}}}^{\text{pole}}(0) \) and \( g_T(q^2) \). In the configuration mixing case, we have not considered the small D- and P-wave state contributions. Starting with \( g_T(q^2) \) we obtain exactly 0 for all value of \( q^2 \). This holds both with unmixed and mixed wave functions, in impulse approximation or for the total axial current including two-body exchange currents. This is a welcome result and a reflection of the \( SU(2) \) isospin symmetry underlying our model.
A. The axial form factor \( g_A(q^2) \)

Turning now to \( g_A(0) \), we find that its value is dominated by the one-body axial current. This is true both for mixed and unmixed wave functions. Different exchange currents contributions cancel to a large extent giving rise to a small 3 – 7% increase in the total value. Also the effect of the wave function is very small with variations of the order of 2% (see table I). These results are in good agreement with experiment.

The axial radius is discussed next. It is defined as the slope of the axial form factor

\[
  r_A^2 = -\frac{6}{g_A(0)} \frac{dg_A(q^2)}{dq^2} \bigg|_{q^2=0}, \tag{47}
\]

and has been measured in (quasi)elastic scattering of (anti)neutrinos on nucleons and from charged pion-electroproduction on protons. A one-parameter dipole form is used for \( g_A(q^2) \) [31]

\[
  g_A(q^2) = g_A(0) \frac{1}{(1 - q^2/M_A^2)^2}. \tag{48}
\]

\( M_A \) is the so called axial mass which is fitted to experiment. From Eq.(48) one obtains using the definition in Eq.(47)

\[
  r_A^2 = \frac{12}{M_A^2}. \tag{49}
\]

The world averages for \( r_A^2 \) extracted from Ref. [32] are: \( r_A^2 = (0.444 \pm 0.015) \) fm\(^2\) from neutrino reactions and \( r_A^2 = (0.449 \pm 0.031) \) fm\(^2\) from pion-production reactions. The latter number contains the chiral correction evaluated in [33]. Our results are compiled in Table II under \((r_A^2)_0\). In contrast to \( g_A(0) \), we find that exchange currents give a sizeable contribution to the axial radius which amounts to 25% of the total in the unmixed case and 44% in the configuration mixing case.

The reader may object that we have not considered the possible \( q^2 \) dependence associated with \( g_{Aq} \) itself. In our previous work [10,26], we assumed a vector meson dominance form factor for the photon-quark coupling. Here, it would seem just as appropriate to use for \( g_{Aq} \) a form
factor as given by axial-vector meson dominance [34]. This has already been done with good results at the nucleon level [35]. One then has

\[ g_{Aq}(q^2) = \frac{g_{Aq}}{1 - q^2/m_{a1}^2} \]  \hspace{1cm} (50)

with \( m_{a1} = 1260 \) MeV. This leads to an axial radius of a constituent quark

\[ r_{Aq}^2 = -\frac{6}{g_{Aq}(0)} \frac{dg_{Aq}(q^2)}{dq^2} \bigg|_{q^2=0} = \frac{6}{m_{a1}^2} \]  \hspace{1cm} (51)

and a numerical value \( r_{Aq}^2 = 0.147 \) fm\(^2\).

With the axial form factor of the constituent quark included we get larger values for the axial radius \( r_A^2 \) as shown in table II. Obviously, the impulse approximation agrees better with the data. The total result, including exchange currents, gives an axial nucleon radius close to the electromagnetic radius of the proton, i.e., a value that is too large compared with present experimental data. The reason for this deviation between theory and experiment is mainly due to the uncertain confinement contribution to the axial current.

In Figs. 5 and 6 we show our results for \( g_A(q^2)/g_A(0) \), with and without configuration mixing respectively. We have included the \( q^2 \) dependence of \( g_{Aq} \) as given by axial-vector meson dominance. We compare our impulse approximation (dotted line) and our total results (solid line) with experimental data and with the dipole fit (dashed line), using for the latter an axial mass given by \( M_A = 1.025 \) GeV. We see that in the very low \( q^2 \) region the data are best described when exchange current contributions are included, although the absence of data points below 0.08 GeV\(^2\) and the big error bars do not allow to be very conclusive. At higher momentum transfers, exchange currents give sizeable contributions that worsen the agreement with the data. We also show another line (dashed-dotted) that corresponds to our total results but with no \( q^2 \) dependence in \( g_{Aq} \). A better agreement with data is achieved in this latter case.
B. The induced pseudoscalar form factor $g_P(q^2)$

The non-pole contribution $g_P^{\text{non-pole}}(0)$ to the pseudoscalar coupling constant is mainly given by exchange current, and its value is dominated by the axial confinement current. This form factor is sensitive to the wave function, being some 25% larger in absolute value with configuration mixing. Our results in Table I agree in sign and magnitude with the ADW result in Eq.(10). There is a proposal to measure $g_P$ at PSI with a 2% accuracy [37]. This would allow to separate the non-pole from the dominant pion-pole contribution to $g_P$ and to test our quark model prediction for this quantity.

VI. CONCLUSIONS

The PCAC relation requires an effective axial coupling of the constituent quarks $g_{Aq} \approx 0.75$. As a result, the nucleon axial coupling, $g_A$, evaluated in the quark model, is in good agreement with experiment without invoking $D$-states or lower components in Dirac spinors.

The PCAC relation also requires that axial exchange currents consistent with the interquark potentials be included in the theory. We have investigated the influence of exchange currents on the axial form factors of the nucleon. We find that exchange currents do not contribute significantly to $g_A(0)$. However, they give sizeable contributions to the axial radius. When combined with axial-vector meson dominance we get axial radii that are too large compared to the experimental data. The lack of exact PCAC in the model could be a source of disagreement with data.

We have seen that exchange currents also give the main contribution to the non-pole part of the induced pseudoscalar coupling constant $g_P^{\text{non-pole}}(0)$. Our prediction agrees in sign and magnitude with the ADW result.

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There is an analogy to this suggestion in nuclear physics. In impulse approximation, the axial coupling of triton is given in terms of the free neutron axial coupling constant as follows [12]:

\[ G_A(0) = -g_A(0)(P_S - 1/3P_{S'} + 1/3P_D), \]

where \( P_S, P_{S'}, \) and \( P_D \) are the symmetric \( S \)-wave and the mixed symmetric \( S \)- and \( D \)-wave probabilities, respectively, which satisfy \( P_S + P_{S'} + P_D = 1 \).

The deviation of the triton wave function from a pure \( S \)-state leads to some reduction of the axial coupling of \( ^3H \) with respect to \( g_A(0) \) of the free neutron.
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[34] The authors of Ref. [29] find an axial quark radius of 0.036 fm$^2$, and that the ratio of the electromagnetic and axial quark radius is approximately given by $m_{a_1}/m_{\rho}$, where $m_{a_1}$ and $m_{\rho}$ are the masses of the $a_1$ axial-vector and $\rho$ vector meson respectively. This ratio is also obtained from the vector meson dominance model.


[37] D. V. Balin et al., PSI proposal R-97-05.
TABLE I. Axial couplings $g_A(0)$, $g_{P}^{\text{non-pole}}(0)$ and $g_T(q^2)$ calculated in the Breit frame. Results evaluated with unmixed (configuration mixed) wave functions are denoted by UM (CM). The individual axial current contributions are labelled as follows: Impulse (Imp); gluon exchange (g); pion exchange ($\pi$); sigma exchange ($\sigma$); pion-sigma exchange ($\pi - \sigma$); confinement (Conf); total result (Total).

<table>
<thead>
<tr>
<th></th>
<th>Imp.</th>
<th>g</th>
<th>$\pi$</th>
<th>$\sigma$</th>
<th>$\pi - \sigma$</th>
<th>Conf.</th>
<th>Total</th>
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<tr>
<td>$g_A(0)$</td>
<td>1.290</td>
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<td>0.154</td>
<td>0</td>
<td>0.105</td>
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<td>$g_{P}^{\text{non-pole}}(0)$</td>
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<td>-0.0045</td>
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<td>0.0011</td>
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<tr>
<td>$g_T(q^2)$ [MeV$^{-1}$]</td>
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<td>0</td>
<td>0</td>
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<tr>
<td>CM</td>
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<tr>
<td>$g_A(0)$</td>
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<td>$g_{P}^{\text{non-pole}}(0)$</td>
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<td>$g_T(q^2)$ [MeV$^{-1}$]</td>
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<td>0</td>
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TABLE II. Axial radius of the nucleon obtained in the Breit frame. Notation as in Table I. Results using a constant $g_{Aq}$ are denoted by $(r_A^2)_0$. Axial radii calculated with a $q^2$ dependence for $g_{Aq}$ as given by axial-vector meson dominance (see Eq.(50)) are denoted by $r_A^2$. The experimental results for the axial mass $M_A$ [32] give according to Eq.(49) $r_A^2 = (0.444 \pm 0.015)$ fm$^2$ (from neutrino scattering) and $r_A^2 = (0.449 \pm 0.031)$ fm$^2$ (from electro-pion production).

<table>
<thead>
<tr>
<th></th>
<th>Imp.</th>
<th>g</th>
<th>π</th>
<th>σ</th>
<th>$\pi - \sigma$</th>
<th>Conf.</th>
<th>Total</th>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(r_A^2)_0$ [fm$^2$]</td>
<td>0.364</td>
<td>-0.006</td>
<td>0.008</td>
<td>0.021</td>
<td>0.029</td>
<td>0.075</td>
<td>0.491</td>
</tr>
<tr>
<td>$r_A^2$ [fm$^2$]</td>
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<td>-0.031</td>
<td>0.025</td>
<td>0.021</td>
<td>0.041</td>
<td>0.075</td>
<td>0.640</td>
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<tr>
<td>CM</td>
<td></td>
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<tr>
<td>$(r_A^2)_0$ [fm$^2$]</td>
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<td>0.023</td>
<td>0.040</td>
<td>0.204</td>
<td>0.744</td>
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FIG. 1. Feynman diagrams for the two-body potentials: (a) one-gluon exchange potential, (b) one-pion exchange potential, (c) one sigma exchange potential.
FIG. 2. Feynman diagrams for one-body operators: (a) axial current operator, (b) pion absorption operator.
FIG. 3. Feynman diagrams for the axial exchange current operators: (a) one-gluon exchange, (b) one-pion exchange, (c) scalar exchange (sigma plus confinement), (d) pion-sigma exchange. The wavy line represents the weak gauge boson W.
FIG. 4. Feynman diagrams of the pion absorption exchange operators: (a) one-gluon exchange, (b) one-pion exchange, (c) scalar exchange (sigma plus confinement), (d) pion-sigma exchange.
FIG. 5. $g_A(q^2)/g_A(0)$ evaluated with configuration mixing. With the exception of the dashed-dotted line a $q^2$ dependence for $g_{Aq}$ as given by axial-vector meson dominance in Eq.(50) is included. The dotted line is obtained in impulse approximation. The solid line is our total result including the contribution of axial exchange currents. The long-dashed line is the dipole fit with $M_A = 1.025$ GeV. Experimental points are adapted from Ref. [36]. The dashed-dotted line is our total result calculated with a $q^2$-independent axial quark coupling constant $g_{Aq} = 0.774$. 
FIG. 6. $g_A(q^2)/g_A(0)$ evaluated without configuration mixing. Notation as in Fig. 5.