In the following, we implement a method for determining
an expression of the form:
\[
\frac{d\rho}{dJ} \frac{\rho_0}{\rho_2} = \langle J \rangle J \nabla
\]

where \(d\rho / dJ\) is the derivative of the population of the state \(\rho_2\) with respect to the transition energy, \(\rho_0 / \rho_2\) is the ratio of the populations of the states \(\rho_0\) and \(\rho_2\), and \(\langle J \rangle J \nabla\) is the average of the transition moment in the population of \(\rho_2\) times the energy of the state \(\rho_2\).
\[
\Delta_{HF} (r) - \Delta_{HF} = \frac{\pi e^2 \gamma_{HF}}{\omega_0^2} \left[ \frac{1}{1 - (\Delta_{HF}/\delta)^2} \right] f(r) \]

(3)

where \(\Delta_{HF}(r)\) is the spatially dependent hyperfine splitting in the presence of the light, and \(\delta \approx (\delta_{r-2} + \delta_{r-3})/2\) measures the laser detuning from the center of the ground state hyperfine splitting. Equation 3 indicates that for \(\delta > \Delta_{HF}\), i.e., for \(\delta_{r-2}\) and \(\delta_{r-3}\) both positive (or both negative), the ground state energy splitting is always reduced by the presence of a light field (see Fig. 1). When the detuning is "between" the hyperfine levels, \(\delta < \Delta_{HF}\), the energy splitting is enlarged.

For a FORT with detuning \(\delta \gg \Delta_{HF}\), Eqs. 2,3 yield:

\[
\Delta_{HF} (r) - \Delta_{HF} \approx \left( \frac{\Delta_{HF}}{\delta} \right) U(r) \]

(4)

where \(U(r)\) is the spatially dependent dipole potential that forms the trap, in frequency units. The fact that the relative ac Stark shift is \(\Delta_{HF}/\delta\) smaller than the dipole potential, is the main motivation for using FORTs for precision spectroscopy. For example in Ref. [5] the relative ac Stark shifts were only \(\Delta_{HF}/\delta \approx 2 \times 10^{-4}\) times the dipole potential. Note that, however small, this relative ac Stark shift is still much larger than the spontaneous photon scattering rate [5].

For a trapped atomic ensemble, \(f(r)\) will cause a shift in the ensemble averaged ground state hyperfine energy splitting, \(\left\langle \Delta_{HF} - \Delta_{HF} \right\rangle\). In addition, the spatial dependence of \(f(r)\) will result in an inhomogeneous broadening, \(\left\langle \Delta_{HF}^2 \right\rangle - \left\langle \Delta_{HF} \right\rangle^2\), which depends also on the atoms' temperature [12]. For example, for a thermal ensemble of atoms with \(\frac{1}{2} k_B T\) kinetic energy in a harmonic trap, we have \(\left\langle \Delta_{HF}^2 \right\rangle - \left\langle \Delta_{HF} \right\rangle^2 = \left( \frac{\Delta_{HF}}{\delta} \right) \left[ U_0 + \frac{3}{7} k_B T \right] \) and

\[
\sqrt{\left\langle \Delta_{HF}^2 \right\rangle - \left\langle \Delta_{HF} \right\rangle^2} = \left( \frac{\Delta_{HF}}{\delta} \right) \sqrt{\frac{2 k_B T}{U_0}}, \quad \text{where } U_0 \text{ is the dipole potential at the trap’s bottom [13].}
\]

In order to cancel these shifts, we introduce an additional laser beam, with intensity \(I'(r)\) and frequency between the resonant frequencies of the two ground state hyperfine levels, say in the middle, i.e. \(-\Delta_{HF}/2\) and \(+\Delta_{HF}/2\) from the lower and higher hyperfine level respectively [14]. For a FORT, the total shift is obtained by adding the shifts from the trap and the compensating beam,

\[
\Delta_{HF} (r) - \Delta_{HF} = \frac{\pi e^2 \gamma_{HF}}{\omega_0^2} \left[ \frac{I(r) - I'(r)}{\delta^2 - (\Delta_{HF}/\delta)^2} \right]
\]

(5)

If the compensating beam is spatially mode-matched with the trap beam, i.e. \(I'(r) = \eta \times I(r)\), then a complete cancellation of the inhomogeneous broadening will occur for

\[
\eta = \left( \frac{\Delta_{HF}}{\delta} \right)^2 \approx \left( \Delta_{HF}/2\delta \right)^2 \]

(6)

Fig. 1 shows the shift of the hyperfine levels (a) and the hyperfine energy difference (b) caused by the trapping beam (dashed line) and compensating beam (dotted line). In the presence of both beams, the levels are shifted by the same amount (full line). As a specific example, with a FORT detuned by 5 nm we have \(\Delta_{HF} = 3.6 \times 10^{-7}\). Hence, with a typical FORT power of 50 mW, the required compensating beam power is 20 mW [15].

Note that the dipole potential created by the compensating beam is \(U'(r) = \pm \frac{1}{2} \Delta_{HF}/\delta U(r)\) for atoms in the upper or lower hyperfine level, respectively, and hence is negligible when compared with the potential of the FORT. Moreover, the photon scattering rate from the compensating beam is \(\gamma_s'\) given by \(h \gamma_s' \approx 2 \gamma U' / \Delta_{HF}\), which can be also written as \(h \gamma_s' \approx \frac{3}{7} U'(r) \approx h \gamma_s\). Hence, the scattering rate from the nearly resonant compensating beam, is of similar magnitude to that of the far-off-resonance trapping beam, \(\gamma_s\).

We implement the proposed scheme with a FORT (a red-detuned gaussian beam [3]), created by focusing a 50 mW laser, detuned 5 nm below resonance, to a waist of \(W_0 = 50 \mu \text{m}\), resulting in a potential depth of \(U_0 \approx 200 E_{\text{rec}}\) (\(E_{\text{rec}}\) is the recoil energy) and oscillation frequencies of \(\nu_r = 2.3 \times 10^3\) Hz and \(\nu_s = 13\) Hz in the radial and axial directions, respectively. An additional laser, with frequency locked close to the middle of the ground state hyperfine splitting, is combined with the FORT laser. To achieve optimal spatial mode-match, both lasers are coupled into a polarization-conserving single-mode optical fiber, and the fiber’s output is passed through a polarizer and focused into the vacuum chamber. Two servo loops are used to control and stabilize the
FIG. 2: Rabi spectrum of the hyperfine splitting of $^{85}\text{Rb}$ with a 3 ms pulse. (a) Spectrum of free falling atoms, $F_0 \approx 3035732430$ Hz is the free-atom line center. (b) Rabi spectrum of trapped atoms, showing a shift in the line center and a broadening. (c) Spectrum of trapped atoms, with an additional compensating beam. The addition of the weak compensating beam, nearly cancels the shift and broadening of the spectrum. Note that since the population of the four $|F = 2, m \neq 0\rangle$ states is included in $N_3/(N_2 + N_3)$, a value of 0.2 represents the maximal possible signal (a $\pi$ pulse) for the $|F = 2, m = 0\rangle \rightarrow |F = 3, m = 0\rangle$ transition.

![Rabi spectrum graphs](image)

The loading procedure is similar to that described in [17]. Briefly, the FORT is loaded by overlapping it with a compressed $^{85}\text{Rb}$ magneto-optical trap (MOT). The MOT beams are shut off after 650 ms of loading, 50 ms of compression, and 5 ms of polarization gradient cooling, leaving $\sim 10^5$ confined atoms with a temperature of $\sim 10\mu K$.

We perform a Rabi spectroscopy measurement on the trapped atoms by driving the ground state $|F = 2, m_F = 0\rangle \rightarrow |F = 3, m_F = 0\rangle$ transition, which is insensitive to magnetic fields, to the first order. A bias magnetic field of $\sim 80\text{mG}$ is applied parallel to the FORT’s polarization axis and to the rf magnetic field direction, in order to Zeeman shift the magnetic sensitive $m_F \neq 0$ levels out of resonance with the rf pulse.

![Compensating beam power graph](image)

FIG. 3: Trapped atoms line center ($F_c$, $\triangle$) and rms width ($\sigma$, $\bullet$) as a function of compensating beam power, for a 3 ms pulse. ($\sigma_{FL} \approx 110$ Hz is the Fourier limited $\sigma$). The spectrum width is minimized to a Fourier limited value, at a compensating beam intensity which corresponds also to a minimal shift from the free-atom line center $F_0$.

A typical sequence is as follows: First, the atoms are prepared in the $|F = 2\rangle$ ground state by turning on the MOT beams, without a repump beam[17], for 1 ms. Then, an rf pulse applied at a variable frequency, using an Anritsu 69317B Signal Generator locked to a high stability oscillator. The intensity and duration of the pulse are adjusted to maximize the $|F = 3\rangle$ population when on-resonance ($\pi$ pulse condition). Following the rf pulse, $N_3$ (the population in the $|F = 3\rangle$ level) is measured by detecting the fluorescence during a short pulse of a laser beam resonant with the cycling transition $[5S_{1/2}, F = 3 \rightarrow 5P_{3/2}, F = 4]$. The population of the $|F = 2\rangle$ level ($N_2$) is then measured by turning on the repumping beam (which is resonant with $[5S_{1/2}, F = 2 \rightarrow 5P_{3/2}, F = 3]$) and applying an additional detection pulse. The normalized signal $N_3/(N_2 + N_3)$ is insensitive to shot-to-shot fluctuations in atom number as well as slow drifting fluctuations of the detection laser frequency and intensity[18].

Figure 2 shows results for the Rabi spectrum with a 3 ms long $\pi$ pulse. A constant background resulting from spontaneous Raman scattering [19] is subtracted. The spectrum of free-falling atoms shows no inhomogenous broadening and a rms width, $\sigma$, which is Fourier limited to 110 Hz. A shift in the peak frequency ($-756\text{Hz}$), and a broadening of the line (to $\sigma = 320\text{Hz}$) are seen in the spectrum of trapped atoms, in fair agreement with the calculated trap depth and atomic temperature. This inhomogeneous broadening is not significantly affected by the duration of the pulse [12]. The addition of the weak compensating beam, nearly cancels the broadening of the spectrum, as well as its shift from the free-atom line center.

Figure 3 shows the measured rms width and shift of the trapped atoms as a function of compensating beam power.
power, again for a 3 ms rf pulse. The spectrum width is minimized to the Fourier broadening limit at a compensating beam power which corresponds also to a minimal shift from the free-atoms line center.

Figure 4 shows the measured rf spectrum for a 25 ms long pulse. A measurement of free atoms with this pulse length is not possible in our setup since the atoms fall due to gravity, and leave the interaction region. A Fourier limited $\sigma = 13$ Hz is measured, representing a 25-fold reduction in the line broadening, as compared to the line broadening of trapped atoms. We performed a similar measurement with a 50 ms pulse, and observed a nearly Fourier limited width (a 25-fold narrowing), at the expense of a much larger spontaneous photon scattering and hence a smaller signal\cite{20}. For even larger measurement times spontaneous photon scattering prevents further narrowing of the line.

We measure the spin relaxation rate\cite{17,19} to be \(3 \times 10^{-3} \text{s}^{-1}\) for atoms trapped in the FORT. The addition of the compensating beam induces an increase of only \(\approx 20\%\).

In summary, we perform an rf spectroscopy measurement of the hyperfine splitting of the ground state of optically trapped atoms. We demonstrate a novel scheme for eliminating the trap-induced inhomogeneous broadening of the transition, by adding a weak “compensating” laser, spatially mode-matched with the trapping laser and with a proper detuning and intensity. Despite being tuned close to resonance, this laser induces a negligible change in the dipole potential, and does not considerably increase the spontaneous scattering rate. With the suppression of inhomogeneous broadening, the atomic coherence time is now limited by the much smaller spontaneous scattering time.

Whereas in a conventional optical trap the ac Stark shift of the line center strongly depends on the temperature of the atoms, which may drift considerably, in the compensated trap the suppression of the line shift is equally effective for all temperatures. Hence, it provides a mean of achieving a higher stability of the line center than that achieved by simply stabilizing the trapping laser detuning and intensity. For relative spectroscopic measurements, such as the proposed measurement of the electron’s permanent electric dipole moment (EDM)\cite{21}, only stability (and not absolute accuracy) of the line center is of importance. For example, for a 10 $\mu$K deep YAG-laser trap, and a compensating beam with a 15 KHz (time averaged) frequency stability, locking the relative intensity between both beams to a $1 \times 10^{-5}$ stability, will result in $1 \times 10^{-14}$ stability of the rf line center.

Finally, a weak compensating beam, spatially mode-matched with the trapping beam and properly tuned near resonance between the upper level of a laser cooling transition and another excited level, can suppress spatially dependent frequency shifts of the cooling transition, and allow simultaneous trapping and cooling with more flexibility than the single frequency method of Ref.\cite{11}.

This work was supported in part by the Israel Science Foundation, the Minerva Foundation, and the United States-Israel Binational Science Foundation. MFA acknowledges help from the Nachemsohn Dansk-Israelisk Studienfond.

\[\text{(4)}\]

<table>
<thead>
<tr>
<th>Reference</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>This is true when the position of each atom is fixed in time, and only an approximation for the more realistic case where the atoms move during the interrogation time.</td>
</tr>
<tr>
<td>14</td>
<td>The detuning of the compensating laser can be chosen in the range $\frac{-2\pi}{T} &lt; \delta &lt; \frac{2\pi}{T}$, yielding a straightforward modification of Eqs. 5,6; however, choosing $\delta = 0$ minimizes spontaneous photon scattering.</td>
</tr>
</tbody>
</table>
[15] A similar calculation can be made which also takes into account also the contribution of the $D_1$ transition, and introduces only a small correction to Eq. 6.

[16] Since typically $\eta \sim 10^{-3}$ in our experiment, the beams are separated by two gratings and two pinholes before their power can be measured independently.


[20] All four $|F = 2, m \neq 0 \rangle$ states are populated and contribute to the spontaneous Raman scattering background, which is hence 5 times larger than that of an ideal 2-level system.