We present some piloting calculations of the masses of the doubly heavy baryons in the framework of the simple approximation within the nonperturbative string approach. The simple analytical results for dynamical masses of heavy and light quarks and eigenvalues of the effective QCD Hamiltonian are presented.

The purpose of this talk is to present the results of the calculation of the masses and wave functions of the heavy baryons in a simple approximation within the nonperturbative QCD (see and references therein). The starting point of the approach is the Feynman-Schwinger representation for the three quark Green function in QCD in which the role of the time parameter along the trajectory of each quark is played by the Fock-Schwinger proper time. The proper and real times for each quark related via a new quantity that eventually plays the role of the dynamical quark mass. The final result is the derivation of the Effective Hamiltonian, see Eq. (1) below. In contrast to the standard approach of the constituent quark model the dynamical mass is not a free parameter but it is expressed in terms of the current mass defined at the appropriate scale of from the condition of the minimum of the baryon mass as function of : \[ \frac{\partial \langle M_B \rangle}{\partial m_i} = 0. \] Technically, this has been done using the einbein (auxiliary fields) approach, which is proven to be rather accurate in various calculations for relativistic systems.

This method was already applied to study baryon Regge trajectories and very recently for computation of magnetic moments of light baryons. The essential point of this talk is that it is very reasonable that the same method should also hold for hadrons containing heavy quarks. As in we take as the universal parameter the QCD string tension fixed in experiment by the meson and baryon Regge slopes. We also include the perturbative Coulomb interaction with the frozen coupling .

Consider the ground state baryons without radial and orbital excitations in which case tensor and spin-orbit forces do not contribute perturbatively.
survives in the perturbative approximation. The EH has the following form

\[ H = \sum_{i=1}^{3} \left( \frac{m_i^{(0)}^2}{2m_i} + \frac{m_i}{2} \right) + H_0 + V, \]  

where \( H_0 \) is the non-relativistic kinetic energy operator, \( m_i^{(0)} \) are the current quark masses and \( m_i \) are the dynamical quark masses to be found from the minimum condition, and \( V \) is the sum of the perturbative one gluon exchange potential \( V_c \) and the string potential \( V_{\text{string}} \). The string potential has been calculated in \(^3\) as the static energy of the three heavy quarks:

\[ V_{\text{string}}(r_1, r_2, r_3) = \sigma R_{\text{min}}, \]

where \( R_{\text{min}} \) is the sum of the three distances \( |r_i| \) from the string junction point, which for simplicity is chosen as coinciding with the center-of-mass coordinate.

We use the hyper radial approximation (HRA) in the hyper-spherical formalism approach. In the HRA the three quark wave function depends only on the hyper-radius \( R^2 = \rho^2 + \lambda^2 \), where \( \rho \) and \( \lambda \) are the three-body Jacobi variables: \( \rho_{ij} = \sqrt{\mu_{ij}} (r_i - r_j) \), \( \lambda_{ij} = \sqrt{\mu_{ij,k}} \left( \frac{m_i r_i + m_j r_j}{m_i + m_j} - r_k \right) \), where \( \mu_{ij} = \frac{m_i m_j}{m_i + m_j} \), \( \mu_{ij,k} = \frac{(m_i + m_j) m_k}{m_i + m_j + m_k} \), and \( \mu \) is an arbitrary parameter with the dimension of mass which drops off in the final expressions. Introducing the reduced function \( \chi(R) = R^{5/2} \psi(R) \) and averaging \( V = V_c + V_{\text{string}} \) over the six-dimensional sphere one obtains the Schrödinger equation

\[ \frac{d^2 \chi(R)}{dR^2} + 2\mu \left[ E_n + \frac{a}{R} - bR - \frac{15}{8\mu R^2} \right] \chi(R) = 0, \]  

where \( a = \frac{2\alpha_s}{3} \cdot \frac{16}{3\pi} \sum_{i<j} \sqrt{\mu_{ij}} \), \( b = \sigma \cdot \frac{32}{15\pi} \sum_{i<j} \sqrt{\mu(m_i + m_j)} \).

We use the same parameters as in Ref. \(^5\): \( \sigma = 0.17 \text{ GeV}, \alpha_s = 0.4, m_q^{(0)} = 0.009 \text{ GeV}, m_s^{(0)} = 0.17 \text{ GeV}, m_c^{(0)} = 1.4 \text{ GeV}, \) and \( m_{\text{b}}^{(0)} = 4.8 \text{ GeV} \). We solve Eq. (2) by the variational method introducing a simple variational Ansatz \( \chi(R) \sim R^{5/2} e^{-\mu p R^2} \), where \( p \) is the variational parameter. Then the three-quark Hamiltonian admits explicit solutions for the energy and the ground state eigenfunction: \( E \approx \min_p E(p) \), where

\[ E(p) = \langle \chi|H|\chi \rangle = 3p^2 - \left( a\sqrt{\mu} \right) \cdot \frac{3}{4} \sqrt{ \frac{\mu}{2} } \cdot p + \left( b\sqrt{\mu} \right) \cdot \frac{15 \sqrt{\pi}}{16} \cdot p^{-1}. \]
Table 1. Comparison of results of analytical and numerical variational calculations for $\Lambda_b$ and $\Lambda_c$ baryons (all quantities are in units of GeV)

<table>
<thead>
<tr>
<th>Baryon</th>
<th>$\Lambda_b$ Numerical calculation</th>
<th>$\Lambda_b$ Analytical calculation</th>
<th>$\Lambda_c$ Numerical calculation</th>
<th>$\Lambda_c$ Analytical calculation</th>
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<tbody>
<tr>
<td>$E_0$</td>
<td>1.06</td>
<td>1.08</td>
<td>1.18</td>
<td>1.16</td>
</tr>
<tr>
<td>$m_q$</td>
<td>0.56</td>
<td>0.56</td>
<td>0.52</td>
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</tr>
<tr>
<td>$m_Q$</td>
<td>4.84</td>
<td>4.82</td>
<td>1.50</td>
<td>1.47</td>
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The dynamical masses $m_i$ and the ground state eigenvalues $E_0$ are given for various baryons in Table 1 of Ref. 1. The dynamical values of light quark mass $m_q \sim \sqrt{\sigma} \sim 450 - 500$ MeV ($q = u, d, s$) qualitatively agree with the results of Ref. 5 obtained from the analysis of the heavy–light ground state mesons. For the heavy quarks ($Q = c$ and $b$) the variation in the values of their dynamical masses $m_Q$ is marginal. This is illustrated by the simple analytical results for Qud baryons. These results were obtained from the approximate solution of equation $\frac{dE}{dp}_{|p=p_0} = 0$ in the form of expansion in the small parameters $\xi = \sqrt{\sigma}/m_Q^{(0)}$ and $\alpha_s$. Omitting the intermediate steps one has

$$E_0 = 3\sqrt{\sigma} \left( \frac{6}{\pi} \right)^{1/4} \left( 1 + A \cdot \xi - \frac{5}{3} B \cdot \alpha_s + \ldots \right)$$

$$m_q = \sqrt{\sigma} \left( \frac{6}{\pi} \right)^{1/4} \left( 1 - A \cdot \xi + B \cdot \alpha_s + \ldots \right),$$

$$m_Q = m_Q^{(0)} \left( 1 + 2A \cdot \xi^2 + \ldots \right)$$

where for our variational Ansätze $A = \frac{\sqrt{2} - 1}{2} \left( \frac{6}{\pi} \right)^{1/4}$, $B = \frac{4 \sqrt{2}}{18} \sqrt{\frac{6}{\pi}}$. Accuracy of this approximation is illustrated in Table 1.

To calculate hadron masses we, as in Ref. 3, first renormalize the string potential: $V_{\text{string}} \rightarrow V_{\text{string}} + \sum_i C_i$, where the constants $C_i$ take into account the residual self-energy (RSE) of quarks. In what follows we adjust the RSE constants $C_i$ to reproduce the center-of-gravity for baryons with a given flavor. As a result we obtain $C_q = 0.34$, $C_s = 0.19$, $C_c \sim C_b \sim 0$.

We keep these parameters fixed to calculate the masses given in Table 2, namely the spin–averaged masses (computed without the spin–spin term) of the lowest double heavy baryons. In this Table we also compare our pre-
Table 2. Masses of baryons containing two heavy quarks

<table>
<thead>
<tr>
<th>State</th>
<th>present work</th>
<th>Ref. 6</th>
<th>Ref. 7</th>
<th>Ref. 8</th>
<th>Ref. 9</th>
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<tr>
<td>Ξ{qcc}</td>
<td>3.69</td>
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<td>Ω{scv}</td>
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<tr>
<td>Ω{sbb}</td>
<td>10.34</td>
<td>10.30</td>
<td>10.32</td>
<td>10.37</td>
<td>10.19</td>
</tr>
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</table>

dictions with the results obtained using the additive non–relativistic quark model with the power-law potential \(^6\), relativistic quasipotential quark model \(^7\), the Feynman-Hellmann theorem \(^8\) and with the predictions obtained in the approximation of double heavy diquark \(^9\).

In conclusion, we have employed the general formalism for the baryons, which is based on nonperturbative QCD and where the only inputs are \(\sigma\), \(\alpha_s\) and two additive constants, \(C_q\) and \(C_s\), the residual self–energies of the light quarks. Using this formalism we have also performed the calculations of the spin–averaged masses of baryons with two heavy quarks. One can see from Table 2 that our predictions are especially close to those obtained in Ref. \(^6\) using a variant of the power–law potential adjusted to fit ground state baryons.

Acknowledgements

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References