Magnetically warped discs in close binaries

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ABSTRACT

We demonstrate that measurable vertical structure can be excited in the accretion disc of a close binary system by a dipolar magnetic field centred on the secondary star. We present the first high resolution hydrodynamic simulations to show the initial development of a uniform warp in a tidally truncated accretion disc. The warp precesses retrogradely with respect to the inertial frame. The amplitude depends on the phase of the warp with respect to the binary frame. A warped disc is the best available explanation for negative superhumps.

Key words:
accretion, accretion discs — instabilities — hydrodynamics — methods: numerical — binaries: close — novae, cataclysmic variables.

1 INTRODUCTION

Much of the variability of cataclysmic variables can be traced to instabilities in the accretion disc. Thermal-viscous instability leads to dwarf nova outbursts, and tidal instability leads to superhumps and superoutbursts. More recently, people have been giving thought to warp-type instabilities. Pringle (1996) showed that radiation from the central accreting object will exert an asymmetric force on either surface of the disc that can be sufficient to warp it. However, relatively extreme conditions are required to generate a warp in this way (Ogilvie & Dubus 2001). In this paper we show numerically that stellar magnetic fields can warp the disc of a cataclysmic variable. In particular we shall focus on the effects of the secondary (donor) star’s magnetic field.

The light-curves of many cataclysmic variables exhibit periodicities close to but distinct from the orbital period. These signals are categorised as positive superhumps if they are longer than orbital and negative superhumps if they are less than orbital. Both are thought to emanate from the disc with the well established explanation for the former being that they are the visible consequences of an eccentric disc precessing with respect to the binary’s tidal field (see O’Donoghue 2000 for a review). Negative superhumps have not been studied in nearly as much detail. The observations of negative superhumps are summarised in Patterson (1999) in which 11 systems are listed. With periods typically a few per cent less than orbital, a key feature of negative superhumps is that they appear to be independent of positive superhumps. They have been observed both in the absence of and in conjunction with their better studied brethren.

Patterson et al. (1993) suggested that negative superhumps occur when the accretion disc is tilted to the binary plane. Just as the line of nodes of the Moon’s orbit about the Earth regresses, so too the intersection of the disc and binary planes precesses retrogradely. The effect is present in the purely dynamical problem and is analogous to the eccentric disc case. A test particle in circular orbit around a single star has three natural frequencies, that of its azimuthal motion (ω), and those of any radial or vertical motion. Around a single star these frequencies are identical and any perturbation to a circular orbit causes the particle to orbit in an ellipse. For a particle in orbit around a star in a binary we find that the tidal field breaks the symmetry of the three periods. The radial frequency becomes slightly lower than (ω) and the vertical frequency slightly higher. As a result, the axis of any perturbation to the orbit will precess. Analytical calculations (e.g. Papaloizou & Terquem 1995) and numerical simulations (e.g. Larwood et al. 1996; Wood et al. 2000) showed that an initially rigidly tilted disc in a binary system will precess retrogradely, approximately as a rigid body, as a result of the gravitational torque acting on the disc.

The major stumbling block for the warped disc hypothesis has been the lack of a viable mechanism for generating it. Three possibilities can be explored. A warp might have developed as a result of an impulsive torque or out of the plane initial condition, and then precessed freely (see e.g. Larwood et al. 1996). A second possibility is that the warp developed from a co-planar disc via an instability. Lubow (1992) and Pringle (1996) discussed dynamical and radiative disc instabilities respectively. The third possibility, the one explored in this paper, is that the warp is excited by continual forcing out of the plane.

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Both impulsively and resonantly generated warps would be forced back into the plane as the disc evolved viscously. Murray & Armitage (1998) presented a series of calculations suggesting that the dynamical and radiative instabilities were too weak to overcome viscosity. Whilst continual forcing could overcome the problem of viscous flattening, the long term disc response will not be free precession but be with the same period as the forcing.

The inspiration for this paper was the idea espoused by Terquem and Papaloizou (2000), that a warp might be induced in a circumstellar disc by the stellar magnetic field. Should the axis of the magnetic field be misaligned with the binary rotation axis, then the magnetic field will exert a torque in the plane of the accretion disc. Terquem and Papaloizou were interested in T Tauri stars, however the principle could equally be applied to a cataclysmic variable with a moderately magnetised accreting white dwarf. Lai (1999) described in detail the mechanisms by which the stellar magnetic field would couple with the disc. Again however his attention was directed more at X-ray accreting pulsars and not at cataclysmic binaries.

We carried out an initial series of smoothed particle hydrodynamics simulations and did indeed find that vertical structure was induced in the disc. However this structure co-rotated with the accreting star and could not possibly generate the periods characteristic of negative superhumps. We will describe these simulations in a subsequent paper (Wynn, Murray & Chakrabarty in preparation). Pringle (private communication) suggested we allow the donor rather than the accretor to be magnetised. Whereas the primary star in a cataclysmic variable typically rotates several times faster than the binary, the secondary is tidally locked and hence co-rotates with the binary. This paper describes a series of simulations with which we sought to determine whether the resultant accretion disc structure was approximately stationary in the binary or in the inertial frame.

Several authors have already performed detailed analysis of the suitability of the smoothed particle hydrodynamics (SPH) technique for modelling three dimensional structure in accretion discs. Murray (1997) and Yukawa, Boffin & Matsuda (1997) demonstrated that tidally excited spiral structure is resolved in well constructed two dimensional and three dimensional SPH simulations respectively. Larwood et al. (1996) modelled the precession of tilted discs in close binaries. They followed the long term evolution of discs tilted at 45 and 90 degrees to the binary plane using 17500 particle simulations. Nelson & Papaloizou (1999) studied the propagation of bending waves in accretion discs. They specifically tested SPH simulations against the linear theory of warped discs and found good agreement. Calculation resolution ranged from 20000 to 102000 particles. They later (Nelson & Papaloizou 2000) studied the warp produced by the Bardeen-Petterson effect using simulations ranging up to 200000 particles in size. Wood, Montgomery & Simpson (2000) used simulations with 25000 particle resolution to follow the precession of a pre-tilted disc in a close binary.

The calculations described later in this paper involve over 500000 particles. At this resolution we are not limited to resolving the gross features of the warp itself. We can also determine the the vertical structure caused by the tidal forces of the secondary, and then follow the interaction between the two.

2 MAGNETISED SECONDARIES

The secondary stars in cataclysmic variables have rotational periods which are tidally locked to the binary’s orbital period, and are typically of the order of a few hours. The connection between rapid stellar rotation rates and chromospheric, and by implication magnetic, activity has long been established (eg. Kraft, 1967; Skumanich, 1972). This connection is often invoked as an argument in favour of the magnetic nature of the rapidly rotating secondary stars in CVs. Direct observational evidence for magnetic activity remains elusive however. Some of the strongest evidence comes from observations of star spots (and their close association with magnetic activity) in the CV ST Leonis Minoris (Howell et al., 2000) and the pre-CV V471 Tau (Applegate & Patterson, 1987) as well as in a number of other types of close binary systems. The evidence for solar-type activity cycles in the secondary stars of CVs is discussed by Bianchini (1990). Any such cycle would imply the presence of significant magnetic fields. The existence of these magnetic fields has strong theoretical support. Angular momentum loss by magnetic braking of the secondary star is the generally accepted mechanism driving the orbital evolution of CVs above the orbital period gap (Verbunt & Zwan, 1981; Mestel & Spruit, 1987). Moreover, theories which attempt to explain the synchronism between the binary orbital period and the rotational periods of magnetic white dwarfs in CVs require secondary magnetic moments of the order $\mu_2 \sim 10^{33}$ - $10^{34}$ G cm$^3$ (eg. King, Whitehurst & Frank, 1990).

For the purposes of the simulations presented in this paper we assume that the secondary stars in CVs have a significant dipolar magnetic field, and that the magnetic moment is similar to that quoted above. This implies that the (unperturbed) field strengths close to the edge of the accretion disc are of order $\sim 10^2$ G. As the gas in the accretion disc orbits the white dwarf in the binary it will twist the field of the secondary star and induce currents within the disc. The interaction of these currents and the imposed magnetic field gives rise to a variety of forces. King (1993) developed a drag prescription to parametrise the effects of these magnetic forces on accreting gas in the AM Herculis sub-class of cataclysmic variables. Pearson, Wynn and King (1997) later made use of this prescription to study the effects of the azimuthal force arising from the interaction of radial surface currents and the vertical component of the magnetic field on the structure of a two dimensional disc in the orbital plane of the binary. This study showed that disc-field interaction caused the disc to lose angular momentum (to the secondary star via the field lines) causing the disc radius to decrease, and enhancing the mass accretion rate. Moreover, resonance phenomena similar to those in the SU UMa systems caused the disc to become eccentric. In this paper we extend the analysis of the effect of the magnetic field on the accretion disc to three dimensions.

Lai (1999) made a detailed analytic study of the torques to which a disc is subjected to when an external stellar field is imposed upon it. In the general case he found that a warping torque resulted from the interaction of the az-
magnetic stresses over the disc. In order to examine the response of a fluid disc to an applied warping torque we implemented a simplified version of the magnetic interaction in which the gas within the disc is subject to an acceleration of the general form

\[ a_{\text{mag}} \simeq -k[v - v_f]_\perp \]  

where \( v \) and \( v_f \) are the velocities of the material and field lines respectively, and the suffix \( \perp \) refers to the velocity components perpendicular to the field lines. The parameter \( k \sim 1/t_{\text{mag}} \) reflects the details of the plasma-magnetic field interaction, and for the purposes of this paper we adopt the simple scaling \( k \propto B^2(r, \theta, \phi) \) and assume that the magnetic field has a purely dipolar form. We can estimate the magnitude of \( k \) in the vicinity of the accretion disc in a cataclysmic variable with a magnetic secondary star from the comparison of (1) and the calculations of Lai (1999). Equating the force per unit area in each case we have

\[ \Sigma k[v - v_f]_\perp \sim \frac{\mu_2^2}{2\pi r_2^2} \]  

where \( r_2 \) is the distance from the secondary star and \( \Sigma \) is the surface density of the accretion disc. Typically \( r_2 \sim 10^{10} \) cm, \( \mu_2 \sim 10^{34} \) G cm \(^3\), \( \Sigma \sim 100 \) g cm \(^{-2}\) and \( |v - v_f|_\perp \sim 100 \) km s \(^{-1}\) in a CV giving \( k(10^{10} \text{ cm}) \sim 10^{-2} \) s \(^{-1}\). This simplistic form of the magnetic force preserves all of the components required to induce a warp within the disc and has a magnitude of the order of that expected of magnetic stresses induced on the accretion disc by a magnetic secondary in a CV.

3 \hspace{1em} \text{CODE ASPECTS}

We use a smoothed particle hydrodynamics code that has been described extensively elsewhere (Truss et al. 2000, Murray 1996). The code uses operator splitting to separate the calculation of the gravitational and magnetic forces from that of the gas-dynamical processes. The code has been parallelised using OpenMP to run on shared memory parallel supercomputers. The viscosity is slightly modified from Murray (1996). We are free to choose the length scale in the viscosity term so we set it to equal the disc scale height, and so are able to reproduce a Shakura-Sunyaev \( \alpha \) viscosity. The viscosity term also produces a bulk viscosity of similar magnitude in regions where the velocity field is significantly divergent. A variable smoothing length \( h \) is used. To ensure the disc is everywhere resolved \( h \) must be less than the scale height whilst maintaining an adequate number of neighbours.

We use an isothermal equation of state. In other words we assume any viscously dissipated energy is instantly radiated away.

The magnetic interaction is implemented by including the acceleration given by equation 2 along with the gravitational terms in the operator split equations of motion.

As in previous papers we have normalised units so that the binary separation \( d \), the total system mass \( M \), and the binary angular velocity \( \Omega_b \) are all equal to one.
Figure 3. Disc silhouette at times t=5,6,7,8,9 and 10 (from top left to bottom right). Azimuth is measured as in figure 2.
Figure 4. Vertical structure of the disc at times t=5, 6, 7, 8, 9 and 10 (from top left to bottom right). Blue hues indicate structure below the disc plane, green shows regions that are symmetric about $z = 0$ whilst yellow and red show regions that are progressively more lifted above the disc plane. The panels in this figure correspond to those in figure 3.
Table 1. Simulation parameters. The magnetic dipole on the secondary is oriented so as to be $\phi_d$ to the vertical and $\theta_d$ prograde of the binary axis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass ratio</td>
<td>$q$</td>
<td>0.4</td>
</tr>
<tr>
<td>sound speed</td>
<td>$c$</td>
<td>0.05 $d\Omega_b$</td>
</tr>
<tr>
<td>viscosity parameter</td>
<td>$\alpha$</td>
<td>1.0</td>
</tr>
<tr>
<td>magnetic parameter</td>
<td>$k$</td>
<td>$10^{-5}$ ($10^{-2}$ s$^{-1}$)</td>
</tr>
<tr>
<td>dipole declination</td>
<td>$\phi_d$</td>
<td>20 deg</td>
</tr>
<tr>
<td>dipole azimuth</td>
<td>$\theta_d$</td>
<td>15 deg</td>
</tr>
</tbody>
</table>

4 CALCULATIONS

The system and code parameters for the calculations are listed in Table 1. We constructed a three dimensional initial state for the calculation by running a two dimensional non-magnetic simulation to equilibrium. The particles were then replicated in $z$ to produce a three dimensional disc which was then followed for several orbits to allow vertical oscillations to die out. The resulting disc comprised 536033 particles. It is shown in plan view, shaded according to surface density, in figure 1. Tidally inspired spiral shocks are clearly visible. We see no evidence that spiral shocks are suppressed or any weaker in three dimensions than in two dimensional calculations.

We also show the silhouette of the initial disc (figure 2) as would be seen by an observer on the disc plane. This figure was obtained by plotting all the particles on the $\theta - z$ plane. The two spiral arms produced 50 per cent increases in the disc height which should be observable in X-ray and EUV observations of eclipsing systems (see e.g. Billington et al., 1996).

As we shall see below, the starting disc was not in equilibrium with the mass addition stream from the secondary star. However the time-scale for mass evolution is much longer than that for magnetically induced evolution and this did not affect our results.

We completed two simulations; a reference calculation with no magnetic field, and a second in which the magnetic field was added instantaneously. The dipole orientation

$$\vec{\mu} = \sin(\phi_d)(\cos(\theta_0 + \Omega_b t) \hat{i} + \sin(\theta_0 + \Omega_b t) \hat{j}) + \cos(\phi_d) \hat{k}$$

(3)

where the angular speed of the binary $\Omega_b = 1$. $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the cartesian unit vectors. In our coordinates the disc lies in the $x$-$y$ plane with the angular momentum vector of the binary in the positive $z$ direction.

Mass transfer from the secondary was incorporated into the simulations by adding single particles from $L_1$ at intervals $\Delta t = 0.01 P_{orb}$. As in previous simulations (e.g. Murray, 1996), the initial velocity of injected particles was taken from Lubow & Shu (1977). The disc was not in equilibrium with the mass transfer stream which was only incorporated into the simulation as a useful tracer of the secondary magnetic field. Gas flow on the secondary star was not resolved in the calculations.

The calculations described here (and their antecedents) took the order of one week of wall time on 8 processors of an SGI Origin 2000 or 3000.

5 RESULTS

In this section we show that the drag on an accretion disc, exerted by a tilted magnetic dipole anchored to the secondary star, results in a warp. Figure 3 is a sequence of six disc silhouettes, each generated in the same manner as figure 2. The snapshots show the disc at equally spaced times from $t = 5 - 10$ (in our units $P_{orb} = 2\pi$). Whilst the disc is approximately symmetric about its midplane in the first three panels, by the fourth ($t = 8$) it is warped.

We have plotted six plan views of the disc vertical struc-
ture in figure 4, corresponding to the snapshots in figure 3. Each image is constructed by interpolating particle $z$ (averaged over all particles in a region) to a grid. Blue regions of the disc are predominantly below the $z = 0$ plane. Green regions are approximately symmetric about the midplane, and yellow-red regions are more extended above $z = 0$. The magnetic torque deflects the stream of gas from the secondary above the plane (redwards). Contours have also been drawn to aid the eye. Figures 3 and 4 therefore show the disc’s initial response to an instantaneously applied out of plane force.

Less than one binary period after the magnetic force was applied, the disc warp was a coherent global structure (panel 1). However one sixth of a binary period later (panel 2) the warp was no longer apparent. It reappeared again, much stronger, in panels three and four. In each image the warp is a disc-wide structure, with the spiral density waves in the initial disc having been rapidly diminished by the warp.

If one follows, for example, the yellow-red zone from panel to panel one sees that the warp has precessed retrogradely in the binary frame. The disc precession with respect to the inertial frame occurred on a much longer time scale so quantitative analysis of the calculations was required.

We Fourier transformed the disc’s vertical structure in both azimuth and time, as in Murray & Armitage (1998).

The sine transform of the vertical displacement,

$$S_{\sin}(t) = \frac{1}{\pi N} \int_{t}^{t+2\pi} \left( \sum_{p=1}^{N} z_p(t') \sin(\theta_p) \right) dt',$$  

(4)

where $\theta_p$ is the angular position of particle $p$ (measured in the inertial frame) and $N$ is the total number of particles. The cosine transform is defined similarly and the mode strength is set equal to the root mean square of the two. The mode strength has units of length, and so a large response would be of the order of the disc height $(0.01 \, d)$. As a check, we returned to the initial disc (as shown in figure 1) and applied a uniform tilt of five degrees to it. We then evolved the tilted disc for one orbital period, and applied equation 4. We found $S^z \approx 0.02$ which agreed with the average vertical displacement of the particles.

Figure 5 overlays the warp mode strength for the two calculations. For convenience we have normalised both curves, by a factor $10^3$. No warp developed in the reference calculation, and the corresponding curve (dashed line) is noisy but with typical magnitude of only $10^{-4}$ or a factor two hundred smaller than for the five degree warp.

When the magnetic field was switched on in the second calculation, the disc immediately responded (solid line). Instead of increasing to a steady value however, the warp mode strength oscillated with a period $P_w = 0.987 \pm 0.003 \, P_{orb}$ as the warp slowly precessed retrogradely in the inertial frame, and the structure of the warp changed as it moved relative to the binary. High time resolution animations of the disc motion were also made that confirmed this to be the precession period. Typical amplitudes of $S^z$ for the magnetic calculation were of the order of $10^{-3}$, about a factor ten or twenty less than for the tilted disc. These amplitudes then agree with the vertical structure shown in figure 9.

There was a secular decline in warp amplitude. Now in these simulations we had $\alpha = 1$ and disc scale height $H \simeq 0.04r$. One can estimate the time scale for viscous forces to flatten a warp $t_l \simeq \alpha(r/H)^2 \Omega^{-1}$. At the disc outer edge $t_l \simeq 300$, not entirely inconsistent with the decay time scale apparent in figure 5, particularly when one considers the disc shrinkage over the course of the magnetic calculation. Reducing the effective $\alpha$ of the disc would result in more rapid damping of the warp.

The applied magnetic torque swiftly removed angular momentum from the outer disc, causing it to shrink. Figure 6 shows the evolution of the average disc radius ($R_d$), which we define to be the radius of a circle with the same area as the
disc. The figure clearly shows that the disc shrinks after the application of the magnetic field. This was predicted by the earlier treatment of Pearson, Wynn and King (1997) and is a consequence of the relation \( t_{\text{visc}}(R_d) \sim t_{\text{mag}}(R_d) \). Figure 7 shows the mass declined more rapidly for the torqued disc than for the reference calculation. Energy dissipation (which can be equated to bolometric luminosity for the disc) due to viscosity in the two discs is plotted in figure 8. The sharp rise in dissipation correlates with an accelerated rate of accretion onto the white dwarf. It is no surprise therefore that the warp strength declined in figure 5.

Figures 9 and 10 show the changes in silhouette and plan view respectively over the time period \( t = 22 - 27 \). The retrograde precession of the warp can clearly be seen in figure 10. The variations in warp amplitude are also apparent. The warp almost disappeared when it was 180° out of phase with the magnetic torque (in between panels 3 and 4). Even then however the disc was not flat, but the vertical structure that was present at that time was symmetric about the mid-plane (figure 9 panels 3 and 4). Half a binary period later (between panels 6 and 1 as the structure changed cyclically) the warp was reinforced by the magnetic torque and the disc was most distorted. In panel 1 for example the scale height varied by as much as 30% around the circumference of the disc. Such variations are sufficiently large to be observable in nearly edge on systems. Again we refer to the Billington et al. (1996) analysis of ultraviolet superdips in OY Car.

A simple toy model, constructed by the anonymous referee, is very helpful in understanding the results shown in figures 4, 5 and 10. Consider a single annulus of the disc, and neglect pressure and viscosity. If the inclinations are small, then the tilt of the annulus can be described by a variable \( W(t) = l_x + il_y \) (where \( x \) and \( y \) are spatial coordinates measured in the binary frame). \( W(t) \) obeys a linear equation of the form

\[
\frac{dW}{dt} = i\omega_p W - \gamma (W - W_B),
\]

where \( \omega_p < 0 \) is the tidal precession rate in the binary frame, and \( \gamma \) represents the effect of the magnetic drag. In the absence of precession, the drag forces would bring the disc tilt \( W \) to coincide with \( W_B \) (determined by the orientation of the secondary magnetic field) on a timescale \( 1/\gamma \). Starting with zero initial tilt, we have

\[
W(t) = W_B \frac{\gamma}{\gamma - i\omega_p} [1 - \exp(i(\omega_p - \gamma)t)].
\]

The oscillations in tilt amplitude in figure 5 are neatly explained as \( |W| \) is oscillatory with frequency \( \omega_p \). Equation 6 also shows that the warp’s precession is a transient response to the initial kick of the magnetic field. The steady state disc response will be a disc warp that is stationary in the binary frame.

Negative superhumps have not been subject to the same degree of scrutiny as their positive counterparts. We are confident that positive superhumps arise in a precessing disc (see most recently Rolfe, Haswell & Patterson, 2001). Observations of negative superhumps are limited to photometry, with no information regarding either temperature or location of the emitting region. Patterson et al. (2001) describes a persistent negative superhump observed in the nova-like variable V751 Cygni, and states that “... many nova-like variables in their bright state and with \( P_{\text{orb}} = 3 - 4 \) hr show negative superhumps”. They do not show the close association with superoutbursts that normal superhumps do, and they do not appear in very low mass ratio systems (i.e. those below the period gap).

Positive superhumps are caused by a tidal resonance which can only act upon an accretion disc if the ratio of secondary to primary stellar masses \( q \approx 1/3 \) (Lubow 1991; Murray, Warner & Wickramasinghe 2000). Lubow (1992) showed that a second resonance that acts to tilt the disc, almost exactly overlays the eccentric resonance. The tilt resonance is however much weaker, and simulations described in Murray & Armitage (1998) failed to produce a disc warp, but with 30,000 particles in the calculations any detection of a warp would have been marginal.

It is worth revisiting the problem, making use of the much higher resolution now available to us (10⁶ particles) and also considering a higher mass ratio. The strength of the tilt resonance goes as \( q^2 \) and mass ratios of observed negative superhumpers are closer to 1/3 rather than the 3/17 used in Murray & Armitage. The calculations presented in this paper give us confidence that smoothed particle hydrodynamics simulations are an appropriate tool for studying the evolution of warp modes in discs, however the warps are generated.

6 CONCLUSIONS

We have followed the evolution of a warp in a close binary accretion disc. An initially planar disc was subjected to a force that modelled the effects of an inclined magnetic dipole centred on the mass donor star. The magnetic field was assumed to be fixed in the binary frame.

Other authors have modelled the evolution of discs with pre-existing warps, or of discs that were misaligned with the binary plane. However these are the first hydrodynamic simulations to follow the initial development of a warp, and the first to show how the warp structure varies as it precesses.

The disc’s initial response to the inclined field was complicated by spiral shocks excited by the secondary’s tidal field. However the spiral shocks are strongly damped by the developing warp. In turn the warp structure rapidly simplified and steady, uniform retrograde precession (in the inertial frame) occurred.

The amplitude and structure of the warp depended upon its phase with respect to the perturbing force (and hence of the binary). The warp was maximum when it was reinforced by the magnetic force. However 180 deg later, when the warp and the force were anti-phased, the disc was almost flat. We measured the period of the motion of the warp with respect to the binary to be \( P_w = 0.987 \pm 0.003 \) hr.

We suggest therefore that negative superhumps could be explained not simply via the retrograde precession of a warped disc, but in terms of changes in the discs vertical structure that occur on the same period, \( P_w \). Because of the very limited observations of negative superhumps we can but hypothesise as to the precise location of the luminosity variations. One possibility is the changing disc aspect presented to the accretion stream as the warp precesses. Alternatively, negative superhumps may be the result of radiation from
Figure 9. Disc silhouette at times $t=22, 23, 24, 25, 26$ and $27$. Panels 4 and 5 show an almost flat disc and correspond to the minimum in warp strength (see figure 5).
Figure 10. Vertical structure of the disc at times t=22,23,24,25,26 and 27 (from top left to bottom right). This figure corresponds to figure 9.
the central source being reprocessed over a changing disc surface.
In subsequent papers already in preparation we shall discuss both the evolution to steady state of discs in magnetised systems, and the influence of magnetised accretors on disc structure.

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