Constructing the large mixing angle MNS matrix in see-saw models with right-handed neutrino dominance

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Recent SNO results strongly favour the large mixing angle (LMA) MSW solar solution. We argue that there are only two technically natural low energy neutrino mass matrix structures consistent with the LMA MSW solution, corresponding to either a hierarchy or an inverted hierarchy with pseudo-Dirac neutrinos. We present a model-independent analysis in which we diagonalise each of these two mass matrix structures to leading order in $\theta_{13}$ and extract the neutrino masses, mixing angles and phases. In this analysis we express the MNS matrix to leading order in the small angle $\theta_{13}$ including the neutrino and charged lepton mixing angles and phases, the latter playing a crucial role for allowing the inverted hierarchy case to be consistent with the LMA MSW solution. We then consider the see-saw mechanism with right-handed neutrino dominance and show how the successful neutrino mass matrix structures may be constructed with no tuning and with small radiative corrections, leading to a full, partial or inverted neutrino mass hierarchy. In each case we derive approximate analytic relations between the input see-saw parameters and the resulting neutrino masses, mixing angles and phases, which will provide a useful guide for unified model building. For the hierarchical cases the LMA MSW solution gives a soft lower bound $|U_{e3}| \sim 0.1$, just below the current CHOOZ limit. Both hierarchical and inverted hierarchical cases predict small $\beta\theta_{13}$ with $|m_{ee}| \sim 0.01 \text{ eV}$ within the sensitivity of future proposals such as GENIUS. Successful leptogenesis is possible if the dominant right-handed neutrino is the heaviest one, but the leptogenesis phase is unrelated to the MNS phases.

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Introduction

Recent SNO results on the neutral current (NC) flux Ahmad:2002jz and the day-night effects Ahmad:2002ka, when combined with other solar neutrino data especially that of Super-Kamiokande Fukuda:2001nk strongly favour the large mixing angle (LMA) MSW solar solution MSW with three active light neutrino states, and $\theta_{12} \approx \pi/6$, $\Delta m^2_{21} \approx 5 \times 10^{-5} \text{eV}^2$ Barger:2002iv, Bandyopadhyay:2002xj, Bahcall:2002. The atmospheric neutrino data is consistent with maximal $\nu_\mu - \nu_\tau$ neutrino mixing Fukuda:1998mi with $|\Delta m^2_{32}| \approx 2.5 \times 10^{-3} \text{eV}^2$ and the sign of $\Delta m^2_{32}$ undetermined. The CHOOZ experiment limits $\theta_{13} < 0.2$ over the favoured atmospheric range Apollonio:1999ae. The combined neutrino data is well described by an MNS matrix Maki:1962mu with $\theta_{23} \approx \pi/4$, $\theta_{12} \approx \pi/6$, $\theta_{13} < 0.2$, which we refer to as the LMA MNS matrix. It is clear that neutrino oscillations, which only depend on $\Delta m^2_{ij} \equiv m_i^2 - m_j^2$, gives no information about the absolute value of the neutrino mass eigenvalues $m_i$. Recent results from the 2dfl galaxy redshift survey indicate that $\sum m_i < 1.8 \text{eV}(95\% \text{C.L.})$ Elgaroy:2002bi. Combined with the solar and atmospheric oscillation data this brackets the heaviest neutrino mass to be in the approximate range $0.04-0.6 \text{eV}$. The fact that the mass of the heaviest neutrino is known to within an order of magnitude represents remarkable progress in neutrino physics over recent years.

The basic possible patterns of neutrino mass consistent with this data are: (i) hierarchy (full $m_1 \ll m_2 \ll m_3$, or partial $m_1 < m_2 \ll m_3$), (ii) inverted hierarchy ($m_1 \approx m_2 \gg m_3$, or the pseudo-Dirac form $-m_1 \approx m_2 \gg m_3$), or (iii) degenerate $m_1^2 \approx m_2^2 \approx m_3^2$. Although oscillation data does not distinguish between these possibilities, the theoretical requirement that the neutrino spectrum is generated in a technically natural way, does provide an additional guiding principle. For example it is clear that a degenerate mass scale $m^2 \gg |\Delta m^2_{ij}|$ implies very small fractional neutrino mass splittings. For example, if $m_3 = 0.500 \text{eV}$ then atmospheric oscillations require neutrino masses $m_2 = 0.497 \text{eV}$. The problem is one of tuning, both to set up the small mass splitting at the high energy scale, and to preserve it in the presence of radiative corrections Ellis:1999ny. In general such a neutrino spectrum is not technically natural, since small perturbations in the high energy input parameters will violate the low energy degeneracy. One can envisage a technically natural mechanism which would lead to a degenerate pair of neutrinos with opposite sign masses $m_1 \approx -m_2$, but to achieve the full three-fold degeneracy is much more difficult Barbieri:1999km,Chankowski:2000fp. A similar objection can be raised against the inverted hierarchical spectrum in which the almost degenerate neutrinos have the same sign masses $m_1 \approx m_2$ where for $m_1 =$
0.0500eV we require $m_2 = 0.0497eV$ for the LMA MSW solution. In these kinds of inverted hierarchy models there is no natural mechanism that can set and preserve such mass splittings, however as we shall see, in the inverted hierarchy model with a pseudo-Dirac neutrino pair corresponding to opposite sign masses $m_1 \approx -m_2$, a natural mechanism is possible.

Assuming the LMA MNS matrix plus naturalness implies that there are then just two leading order forms for the light physical effective Majorana neutrino mass matrix $m'_{LL}$ (where LL means that it couples left-handed neutrinos to left-handed neutrinos) corresponding to either a hierarchical spectrum with $m \approx m_3 \gg m_1, m_2$ arising from: equation $m'_{LL} \approx \begin{array}{ccc} a & d & e \\ d & b & f \\ e & f & c \end{array}$

For the hierarchical case the experimental requirement $|\Delta m^2_{32}| \gg |\Delta m^2_{21}|$ implies that $m_3^2 \gg m_2^2$ which again looks technically unnatural since we would expect two roughly equal eigenvalues once the lower block is diagonalised. For the LMA MSW solution we only require a mild hierarchy, $|m_2|/|m_3| \sim 0.1$, and this will require accidental cancellations of order 10% to take place in the diagonalisation of the lower block of the neutrino mass matrix. Although the degree of tuning necessary to achieve the hierarchy is not large, it does imply that the radiative corrections of the matrix elements will be competitive with the amount by which they need to be tuned, and hence that radiative corrections will be very important in determining the low energy spectrum. In the case of see-saw models seesaw the radiative corrections may be sufficient to destroy (or create) the cancellations necessary to achieve the desired hierarchy Babu:1993qv, Ellis:1999nk. In all cases radiative corrections will be important and the neutrino masses and mixings calculated in a given high energy unified theory will not be simply related to the low energy ones.

The above situation could be improved if it were possible to make the neutrino mass hierarchy completely natural. If the hierarchy $m_3 \gg m_2$ could emerge without any tuning at all, not even at the level of 10% accidental cancellations, then the low energy spectrum would faithfully preserve the nature of the spectrum calculated at high energy, without being severely affected by radiative corrections. If this could be achieved then the low energy measurements would provide a direct window into the nature of the high energy theory. Therefore it is interesting to ask whether it is possible for the neutrino mass hierarchy to arise in a completely natural way? Indeed this is possible if the see-saw mechanism seesaw is supplemented by a mechanism known as single right-handed neutrino dominance King:1998jw, King:1999cm, King:2000mb. According to single right-handed neutrino dominance one of the right-handed neutrinos makes the dominant contribution to the lower block of $m'_{LL}$ causing its determinant to approximately vanish, and thereby leading to $|m_2| \ll |m_3|$ without relying on accidental cancellations which are subject to important radiative corrections. Single right-handed neutrino dominance does not mean that there is only a single right-handed neutrino, only that one of the right-handed neutrinos is making the dominant contribution. If the dominant right-handed neutrino is denoted $\nu R_3$ with a heavy Majorana mass $Y$ and Dirac couplings $d, e, f$ to the weak eigenstate neutrinos $\nu e, \nu \mu, \nu \tau$ given by $\nu R_3(d\nu_e + e\nu_\mu + f\nu_\tau)$ then according to the see-saw mechanism this will result in a light physical neutrino $\nu_3 \approx d\nu_e + e\nu_\mu + f\nu_\tau$ of mass $m_3 \approx (d^2 + e^2 + f^2)/Y$. King:1998jw together with two light orthogonal combinations of neutrinos which would be massless in the limit that there is only a single right-handed neutrino. The requirements of a maximal atmospheric angle $\theta_{23} \approx \pi/4$ and a small CHOOZ angle $\theta_{13} \ll 1$ imply the relation $d \ll e \approx f$ King:1998jw.

In order to account for the solar data we must consider the effect of the sub-leading right-handed neutrinos. These perturb the spectrum leading to a small second neutrino mass $m_2^2 < m_3^2$. The strength of the hierarchy is controlled by the relative importance of the sub-leading right-handed neutrinos, rather than relying on accidental cancellations. If the sub-leading contribution is dominated by a single sub-leading right-handed neutrino $\nu R_2$ with mass $X$ and couplings $\nu R_2(a\nu_e + b\nu_\mu + c\nu_\tau)$, then this leads to a second neutrino mass of order $m_2 \sim (1/2)(b - c)^2/X$ which only depends on the subleading parameters because we must have $m_3 = 0$ in the limit of there being only a single right-handed neutrino King:2000mb. The solar angle was given by $\tan \theta_{12} \sim 2a/(b - c)$ King:2000mb. Note that the solar angle is completely determined by the sub-leading couplings, due to a natural cancellation of the leading contributions King:2000mb. The lightest neutrino mass $m_1$ is generated by the sub-sub-leading couplings due to the right-handed neutrino $\nu R_1$ with mass $X'$ leading to a full neutrino mass hierarchy $m_1 < m_2 < m_3$ King:2000mb. We shall refer to this as sequential sub-dominance.

The sub-leading contribution may alternatively result from two equally contributing sub-dominant right-handed neutrinos in which case only a partial neutrino mass hierarchy results $m_1 < \cdots m_2 < m_3$ and the results above will be different King:2000mb. While the full hierarchy results from an approximately diagonal right-
handed neutrino mass matrix, a partial hierarchy may result from three possible textures for the right-handed neutrino mass matrix namely diagonal, democratic or off-diagonal where the nomenclature refers to the upper block King:1999cm: equation $M^{\text{diag}}_{RR} = (arrayccX')^0$

The neutrino mass matrix in Eq.I corresponding to an inverted hierarchy with opposite sign masses $m_1 \approx -m_2$ and $-m_1 \approx m_2 \gg m_3$, can be reproduced by three right-handed neutrinos with the texture in Eq.off-diag King:2001ce. However an important difference is that now it is the off-diagonal pair of right-handed neutrinos with pseudo-Dirac mass $X$ that dominates the neutrino mass matrix leading to the pseudo-Dirac structure of neutrino masses in Eq.I, with the right-handed neutrino of mass $Y$ now giving the sub-dominant contributions King:2001ce. We shall refer to this as off-diagonal right-handed neutrino dominance. As in the hierarchical cases based on single right-handed neutrino dominance, the resulting inverted hierarchical spectrum does not rely on any accidental cancellations and is technically natural. The radiative corrections arising to the inverted neutrino mass spectrum arising from off-diagonal right-handed neutrino dominance have been studied and shown to be only a few per cent King:2001ce.

In this paper we perform a two-part analysis:

(i) We consider neutrino mass matrices with the leading order structures in Eqs.H,I, and diagonalise each of them to leading order in $\theta_{13}$ to extract the neutrino masses, mixing angles and phases. The MNS matrix is then constructed to leading order in the small angle $\theta_{13}$ including the neutrino and charged lepton mixing angles and phases, the latter playing a crucial rôle for allowing the inverted hierarchy solution to be consistent with the LMA MSW solution.

(ii) We then go on to show how the neutrino mass matrix structures in Eqs.H,I may be constructed naturally from the see-saw mechanism with right-handed neutrino dominance, with no tuning and with small radiative corrections, leading to a full, partial or inverted neutrino mass hierarchy. In each case we derive approximate analytic relations between the input see-saw mass matrices and the resulting neutrino masses, mixing angles and phases. The goal of this analysis is to provide a useful and reliable guide for constructing the LMA MNS matrix.

The analysis builds on that in King:2000nb and King:2001ce, by including the effects of phases and the charged lepton mixing angles, which we did not previously consider. It is sufficient in a top-down approach to work to order $\theta_{13}$, since although the radiative corrections are only a few per cent (due to right-handed neutrino dominance), this is sufficient to wash out the order $\theta_{13}^2$ corrections. By contrast in a bottom-up approach the order $\theta_{13}^2$ corrections may be considered Lavignac:2002gf. However the effect of charged lepton mixing angles was not considered in Lavignac:2002gf and this can significantly affect the conclusions based on naturalness arguments. The leading order results which we present here give the simple relations between the see-saw parameters necessary in order to obtain the phenomenologically successful LMA MNS matrix. Ultimately the masses and mixing angles in unified models must be calculated numerically, including the radiative corrections. The purpose of the analytic results we present here is to provide insight into the construction of unified models which must then be studied numerically.

The paper is organised as follows. In section 2 we construct the MNS matrix starting from general complex neutrino and charged lepton mass matrices. The results in section 2 assume the leading order neutrino mass matrices in Eqs.H,I but are otherwise completely model-independent. In section 2.1 we discuss the equivalence of different parametrisations of the MNS matrix. In section 2.2 we discuss the charged lepton contributions to the MNS matrix, where the natural expectation is that the charged lepton mixing angles are all small, and give an expansion of the MNS matrix to leading order in the small angles. In section 2.3 we give analytic results for the diagonalisation of the neutrino mass matrix whose leading order form is in Eq.H, working to leading order in the small angle $\theta_{13}$, but allowing large mixing angles $\theta_{23}, \theta_{12}$. The general method for diagonalising hierarchical complex mass matrices is described in Appendix hierarchical.

In section 2.4 we present analogous results for the case of inverted hierarchical mass matrices of the leading order form in Eq.I. In section 3 we construct the successful neutrino mass matrix structures in a natural way using the see-saw mechanism with right-handed neutrino dominance. In section 3.1 we derive results for single right-handed neutrino dominance with sequential sub-dominance, corresponding to a full neutrino mass hierarchy. In section 3.2 we derive results for single right-handed neutrino dominance with off-diagonal sub-dominance, corresponding to a partial neutrino mass hierarchy. In section 3.3 we derive results for off-diagonal right-handed neutrino dominance, corresponding to an inverted neutrino mass hierarchy, which we...
show is consistent with the LMA MSW solution once the effect of charged lepton mixing angles is taken into account. For each type of right-handed neutrino dominance we derive useful analytic expressions for neutrino masses and mixing angles in terms of the see-saw mass matrices, and show that for the hierarchical cases the LMA MSW solution gives a soft lower bound $|U_{e3}| > 0.1$, just below the curent CHOOZ limit. In section 4 we give two physical application of the results. In section 4.1 we consider $\beta\beta_{0\nu}$, and show that both hierchical and inverted hierarchical cases predict small $\beta\beta_{0\nu}$ with $|m_{ee}| \sim 0.01$ eV within the sensitivity of future proposals such as GENIUS. In section 4.2 we discuss leptogenesis, and show that successful leptogenesis is possible if the dominant right-handed neutrino is the heaviest one, but the leptogenesis phase is unrelated to the MNS phases. Section 5 concludes the paper.

Constructing the MNS matrix

The charged lepton masses and the neutrino masses are given by the eigenvalues of the complex charged lepton mass matrix $m_{LR}^E$ and the complex symmetric neutrino Majorana matrix $m_{LL}^\nu$, obtained by diagonalising these mass matrices, equation $V^E_L m_{LR}^E V^{E,R \dagger} = (\begin{array}{ccc} c_1 & c_2 & c_3 \\ s_1 & s_1 & 0 \\ 0 & 0 & 0 \end{array})$

Equivalence of different parametrisations

A $3 \times 3$ unitary matrix may be parametrised by 3 angles and 6 phases. We shall find it convenient to parametrise a unitary matrix $V^\dagger$ by It is convenient to define the parametrisation of $V^\dagger$ rather than $V$ because the MNS matrix involves $V^{\nu L \dagger}$ and the neutrino mixing angles will play a central rôle. equation $V^\dagger = P_2 R_{23} R_{13} P_1 R_{12} P_3 V^1$