Electron Neutrino Pair Annihilation: A New Source for Muon and Tau Neutrinos in Supernovae

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ABSTRACT

We show that in a supernova core the annihilation process $\nu_e\bar{\nu}_e \rightarrow \nu_\mu,\tau\bar{\nu}_\mu,\tau$ is always more important than the traditional reaction $e^+e^- \rightarrow \nu_\mu,\tau\bar{\nu}_\mu,\tau$ as a source for muon and tau neutrino pairs. We study the impact of the new process by means of a Monte Carlo transport code with a static stellar background model and by means of a self-consistent hydrodynamical simulation with Boltzmann neutrino transport. Nucleon bremsstrahlung $NN \rightarrow NN\nu_\mu,\tau\bar{\nu}_\mu,\tau$ is also included as another important source term. Taking into account $\nu_e\bar{\nu}_e \rightarrow \nu_\mu,\tau\bar{\nu}_\mu,\tau$ increases the $\nu_\mu$ and $\nu_\tau$ luminosities by as much as 20\% while the spectra remain almost unaffected. In our hydrodynamical simulation the shock was somewhat weakened. Elastic $\nu_\mu,\tau\nu_e$ and $\nu_\mu,\tau\bar{\nu}_e$ scattering is not negligible but less important than $\nu_\mu,\tau e^\pm$ scattering. Its influence on the $\nu_\mu,\tau$ fluxes and spectra is small after all other processes have been included.

Subject headings: neutrinos — supernovae: general

1. INTRODUCTION: neutrinos — supernovae: general

The treatment of $\nu_\mu$ and $\nu_\tau$ transport in numerical supernova (SN) simulations has been somewhat schematic in the past. However, with the advent of numerical Boltzmann solvers for the neutrino transport (Mezzacappa & Bruenn 1993, Mezzacappa & Messer 1999; Yamada, Janka, & Suzuki 1999; Burrows et al. 2000; Rampp & Janka 2002) and their application to the post-bounce phase of stellar core-collapse models (Rampp & Janka 2000; Mezzacappa et al. 2001; Liebendörfer et al. 2001) a new level of accuracy has been achieved. Evidently it is desirable that in consistent state-of-the-art simulations the uncertainties are not dominated by overly crude approximations of the microphysics which governs the neutrino interactions. For example, it has been recognized that nuclear many-body correlations (Burrows & Sawyer 1998; Reddy et al. 1999) or weak-magnetism effects in neutrino-nucleon interactions (Vogel & Beacom 1999; Horowitz & Li 2000; Horowitz 2002) should be included.

Previous simulations used iso-energetic scattering on nucleons $\nu_\mu N \rightarrow N\nu_\mu$ as the main opacity source for $\nu_\mu$ transport, elastic scattering on electrons and positrons $\nu_\mu e^\pm \rightarrow e^\pm\nu_\mu$ as the main energy-exchange reaction, and $e^+e^- \rightarrow \nu_\mu\bar{\nu}_\mu$ as the only source term for $\nu_\mu$ production. (Here and in what follows we use $\nu_\mu$ symbolically for either $\nu_\mu$ or $\nu_\tau$.) However, it is now generally accepted that nucleon bremsstrahlung $NN \rightarrow NN\nu_\mu\bar{\nu}_\mu$ is important or even dominant as a neutrino source reaction (Suzuki 1991,1993; Hannestad & Raffelt 1998; Thompson, Burrows, & Horvath 2000), and that nucleon recoils have a significant impact on the emerging $\nu_\mu$ flux spectrum (Janka et al. 1996; Raffelt 2001; Keil, Raffelt, & Janka 2002).

In this paper we show that in addition $\nu_e\bar{\nu}_e \rightarrow \nu_\mu\bar{\nu}_\mu$ and its inverse reaction should be included because this process is always far more important than $e^+e^- \rightarrow \nu_\mu\bar{\nu}_\mu$ as a neutrino source (Fig. 1). Conversely, $\nu_\mu\nu_e$ and $\nu_\mu\bar{\nu}_e$ scattering turns out to be less important than $\nu_\mu e^\pm$ scattering and thus...
implying that in the traditional process of electron-positron production rates due to $e^+e^-$ and $\nu_e\bar{\nu}_e$ reactions, the new process is included and once it is left out. This allows us to verify that the effects established by the Monte Carlo results are generic and also show up in self-consistent radiation-hydrodynamical models. The use of these two different approaches and independent codes also helps making sure that our results do not depend on details of the technical implementation or the particular numerical resolution.

We begin in Sec. 2 by comparing the $e^+e^-$ and $\nu_e\bar{\nu}_e$ reactions. In Sec. 3 we discuss the results of a Monte Carlo study of neutrino transport while in Sec. 4 we describe the self-consistent hydrodynamical simulations coupled with a Boltzmann transport solver. We summarize our findings in Sec. 5.

2. ELECTRON VS. NEUTRINO PAIR ANNihilation

We begin by comparing the two pair annihilation processes

\[ \nu_e + \bar{\nu}_e \rightarrow \nu_\mu + \bar{\nu}_\mu, \quad (1) \]
\[ e^+ + e^- \rightarrow \nu_\mu + \bar{\nu}_\mu. \quad (2) \]

In the presence of non-vanishing chemical potentials the enhancement is even larger. The pair production rate due to $e^+e^- \rightarrow \nu_\mu\bar{\nu}_\mu$ is shown in Fig. 2 as a function of the degeneracy parameter $\eta_e = \mu_e/T$ for the electrons, and likewise, the rate for $\nu_e\bar{\nu}_e \rightarrow \nu_\mu\bar{\nu}_\mu$ as a function of $\eta_{\nu_e}$. The production rate is a decreasing function of $\eta$. Since in the relevant regions of the SN core $\eta_{\nu_e} < \eta_e$, the traditional process of $e^+e^-$ annihilation is reduced more strongly than the new $\nu_e\bar{\nu}_e$ rate. The latter therefore dominates even more.

In order to estimate the impact of the new source reaction on the neutrino fluxes and spectra we conduct two separate numerical investigations. First, we perform a Monte Carlo transport simulation on a static stellar background model. While this approach has the disadvantage of not follow-

Fig. 1.— Feynman graph for the annihilation processes producing $\nu_\mu\bar{\nu}_\mu$ pairs.

Fig. 2.— Pair production rates by the process $\nu_e\bar{\nu}_e \rightarrow \nu_\mu\bar{\nu}_\mu$ as a function of $\eta_{\nu_e}$ (upper line) and $e^+e^- \rightarrow \nu_\mu\bar{\nu}_\mu$ as a function of $\eta_e$ (lower line). We used $T = 12$ MeV and $\eta_{\nu_e} = 0$. 
In the relativistic limit where the electron mass can be neglected, their squared and spin-summed matrix elements are of the form
\[
\sum_{\text{spins}} |\mathcal{M}|^2 = 8 G_F^2 \left[ (C_V + C_A)^2 u^2 + (C_V - C_A)^2 t^2 \right],
\]
with the Mandelstam variables \( t = -2k_1 \cdot k_3 \) and \( u = -2k_1 \cdot k_4 \). The momenta are assigned to the particles as indicated in Fig. 1. The matrix elements for the two processes differ only in the weak coupling constants \( C_V, A \) shown in Table 1.

The pair production rate is obtained by appropriate phase-space integrations, including particle distributions and blocking factors (Yueh & Buchler 1976; Hannestad & Madsen 1995). We use \( \mu_{\nu_e} = 0 \) even though there could be a small \( \nu_\mu \) chemical potential due to a non-vanishing concentration of muons in the core and due to different transport properties of \( \nu_\mu \) and \( \bar{\nu}_\mu \). Assuming that \( e^\pm, \nu_e, \) and \( \bar{\nu}_e \) are all in LTE, the rates of pair creation vs. degeneracy parameters \( \eta_e \) and \( \eta_{\nu_e} \) are shown in Fig. 2. In the dense regions of a SN core below the neutrino spheres the phase space distribution of electron neutrinos is a Fermi-Dirac function with degeneracy parameter \( \eta_e = \eta_e + \eta_\nu - \eta_n < \eta_n \) so that the new process is always more important than \( e^+e^- \) annihilation.

An interesting difference between the two processes arises in the differential production rates, i.e. the \( \nu_\mu \) and \( \bar{\nu}_\mu \) production rates as functions of neutrino energy \( \epsilon \). As a first case we take \( \eta = 0 \) for both \( e \) and \( \nu_e \) and show the differential production rate \( d^2n/d\epsilon d\Omega \) in Fig. 3. For both processes the differential rate is the same for \( \nu_\mu \) and \( \bar{\nu}_\mu \), i.e. they are produced with the same spectra.

However, in general the “parent particles” will have a significant chemical potential. Taking \( \eta_e = 10 \) we show the differential production rates in Fig. 4. In case of the \( e^+e^- \) process (upper panel) the differential rates are similar for \( \nu_\mu \) and \( \bar{\nu}_\mu \). This is understood by the fact that the values of \( (C_V + C_A)^2 = 0.54^2 \) and \( (C_V - C_A)^2 = 0.46^2 \) are quite similar so that the \( u^2 \) and \( t^2 \) terms in Eq. (3) are almost equally important. Therefore, interchanging \( \nu_\mu \) and \( \bar{\nu}_\mu \), corresponding to an exchange of \( u \) and \( t \), has no big effect.

This is not true for the neutrino reaction, where \( (C_V + C_A)^2 = 1 \) and \( (C_V - C_A)^2 = 0 \) and thus only \( u^2 \) contributes. Replacing \( u \) by \( t \) now changes the kinematics of the process, which in turn modifies the rate if the distribution of \( \nu_e \) differs from that of \( \bar{\nu}_e \), i.e. if \( \eta_{\nu_e} \neq 0 \). The \( \nu_\mu \) and \( \bar{\nu}_\mu \) spectra shown in the bottom panel of Fig. 4 are indeed very different from each other, although the total production rates of \( \nu_\mu \) and \( \bar{\nu}_\mu \) are, of course, equal.

One can easily understand why \( \nu_\mu \) on average have larger energies than \( \bar{\nu}_\mu \). We first look at \( \nu_e\bar{\nu}_e \rightarrow \nu_\mu\bar{\nu}_\mu \) in the center of momentum (CM)

<table>
<thead>
<tr>
<th>Process: ( e^+e^- \leftrightarrow \nu_\mu\bar{\nu}_\mu )</th>
<th>( \nu_e\bar{\nu}<em>e \leftrightarrow \nu</em>\mu\bar{\nu}_\mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_V )</td>
<td>( -\frac{1}{2} + 2 \sin^2 \theta_w + \frac{1}{2} )</td>
</tr>
<tr>
<td>( C_A )</td>
<td>( \frac{1}{2} + \frac{1}{2} )</td>
</tr>
</tbody>
</table>

**Table 1**

**Weak interaction constants.**
The differences of the source spectra, however, do not translate into significant spectral differences of the $\nu_\mu$ and $\bar{\nu}_\mu$ fluxes emitted from the SN core. While pair annihilations and nucleon bremsstrahlung are responsible for producing or absorbing neutrino pairs and thus their equilibration with the stellar medium below the “neutrino-energy sphere,” other processes, notably $\nu_\mu e^\pm$ scattering and nucleon recoils, are more efficient for the exchange of energy between neutrinos and the medium between the equilibration and transport spheres. In our numerical runs we will find in fact that adding the new process to a SN simulation primarily modifies the flux with only minor modifications of the spectrum.

If $\nu_\mu \bar{\nu}_\mu$ is important relative to $e^+e^- \rightarrow \nu_\mu \bar{\nu}_\mu$, one may wonder if processes of the form

$$\nu_\mu + \nu_e \rightarrow \nu_\mu + \nu_e,$$

$$\nu_\mu + \bar{\nu}_e \rightarrow \nu_\mu + \bar{\nu}_e,$$

could be of comparable importance as $\nu_\mu e^\pm$ scattering. Figure 5 shows the rates for $\nu_\mu$ scattering on $\nu_e$ and $\bar{\nu}_e$, and those for scattering on $e^+$ and $e^-$ as functions of $\eta_\nu = \eta_{\nu_e} = 10$, $T = 12$ MeV.

In contrast to the annihilation rates, the scattering rates rise monotonically with $\eta$. Therefore, even though neutrino-neutrino scattering would dominate if all chemical potentials were zero, for realistic situations with $\eta_\nu < \eta$, we expect that scattering on $e^\pm$ has 1–2 times the rate of scattering on $e^\pm$ as a function of $\eta$. The rates are normalized to the scattering rate on $e^\pm$ at $\eta = 0$. We used $T = 12$ MeV and $\eta_\nu = 0$. 

Fig. 5.— Thermally averaged scattering rate for $\nu_\mu$ on $e^\pm$ as a function of $\eta_\nu$ (lower line) and for $\nu_\mu$ on $\nu_e$ and $\bar{\nu}_e$ (upper line) as a function of $\eta_{\nu_e}$. The rates are normalized to the scattering rate on $e^\pm$ at $\eta = 0$. We used $T = 12$ MeV and $\eta_\nu = 0$. 

Fig. 4.— Differential $\nu_\mu$ and $\bar{\nu}_\mu$ production rates $d^2n/dEdt$ for $\eta_\nu = \eta_{\nu_e} = 10$ and $T = 12$ MeV. Upper panel for $e^+e^- \rightarrow \nu_\mu \bar{\nu}_\mu$, lower panel for $\nu_\mu \bar{\nu}_e \rightarrow \nu_\mu \bar{\nu}_\mu$. 

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\( \nu_e \) and \( \bar{\nu}_e \). Therefore, neutrino-neutrino scattering is expected to be a relatively minor correction. In our Monte Carlo studies we will indeed find that this process has only a small effect on the neutrino spectra and fluxes.

3. MONTE CARLO STUDY

To study the impact of the new annihilation process on the \( \nu_\mu \) fluxes and spectra we first use a Monte Carlo method for neutrino transport on a static background stellar model. For this purpose we have adapted the Monte Carlo code of Janka & Hillebrandt (1989a,b) to include additional processes and nucleon recoil. We use this code for an extensive parameter study of \( \nu_\mu \) spectra formation that will be documented elsewhere (Keil et al. 2002).

As a stellar background we employ a model originally provided to us by B. Messer for an earlier study of neutrino spectra formation (Raffelt 2001). Using this model again for our present study facilitates a comparison with this previous work. The model is based on a full-scale Newtonian collapse simulation of the Woosley & Weaver (1995) mass progenitor model labeled s15s7b. We use a snapshot at 324 ms after bounce when the shock wave is at a radius of about 120 km, i.e. the SN core still accretes matter. In Fig. 6 the temperature profile is represented by the steps in terms of \( \langle \epsilon \rangle \equiv 3.15 \times 10^{-2} T \) for each radial zone, i.e. by the average energy of nondegenerate neutrinos \( \langle \eta_{\nu_\mu} = 0 \rangle \) in LTE with the stellar medium.

In Fig. 6 we also show the thermalization depth \( R_{\text{therm}} \) for several processes as a function of neutrino energy \( \epsilon \). The formal definition of \( R_{\text{therm}} \) in terms of an effective mean free path for energy exchange is given by the condition (cf. Shapiro & Teukolsky 1983; Suzuki 1989; Raffelt 2001)

\[
\tau_{\text{eff}}(\epsilon) = \int_{R_{\text{therm}}}^{\infty} dr \frac{1}{\lambda_i} \sum_j \frac{1}{\lambda_j} = \frac{2}{3}
\]

for the effective optical depth \( \tau_{\text{eff}} \) of a particular equilibrating process with mean free path \( \lambda_i(\epsilon) \) among the opacity producing reactions having mean free paths \( \lambda_j(\epsilon) \). The solid line in Fig. 6 is for \( \nu_\mu n \rightarrow p\bar{\nu}_e^- \), i.e. it represents the energy-dependent \( \nu_e \) sphere. The dashed line is the analogous \( \bar{\nu}_e \) sphere due to \( \nu_\mu p \rightarrow n\bar{\nu}_e^+ \). Finally, the dotted line shows where \( \nu_\mu \) or \( \bar{\nu}_\mu \) of given energy last participate in the \( \nu_\mu \bar{\nu}_\mu \rightarrow \nu_e \bar{\nu}_e \) process, assuming that the annihilation partners are distributed according to LTE. Put another way, the dotted line is the energy-dependent freeze-out sphere for our new process. It is always at much smaller radii than the \( \nu_e \) and \( \bar{\nu}_e \) spheres. Therefore, in those regions where \( \nu_\mu \bar{\nu}_\mu \leftrightarrow \nu_e \bar{\nu}_e \) is effective we may assume LTE for \( \nu_e \) and \( \bar{\nu}_e \).

We therefore implemented the new process in our Monte Carlo code by using the same subroutine as for the \( e^+e^- \) pair process, except for inserting the appropriate weak interaction constants \( C_{\nu,A} \) and replacing \( \eta_e \) by \( \eta_{\nu_\mu} \).

We summarize the characteristics of the emerging neutrino flux for our runs in Table 2. As introduced by Janka & Hillebrandt (1989a) we characterize the neutrino spectrum by its first two energy moments \( \langle \epsilon \rangle_{\text{flux}} \) and \( \langle \epsilon^2 \rangle_{\text{flux}} \) and define the “pinching parameter” by following Raffelt (2001) as

\[
\rho_{\text{flux}} = \frac{1}{a} \frac{\langle \epsilon^2 \rangle_{\text{flux}}}{\langle \epsilon \rangle_{\text{flux}}^2},
\]

with the energy moments

\[
\langle \epsilon^n \rangle_{\text{flux}} = \frac{\int_0^\infty d\epsilon \int_{-1}^{+1} d\mu f_\epsilon(\epsilon, \mu) \epsilon^{n+2} \mu}{\int_0^\infty d\epsilon \int_{-1}^{+1} d\mu f_\epsilon(\epsilon, \mu) \epsilon^2 \mu}
\]

5
Table 2
Spectral characteristics of neutrino fluxes from Monte Carlo transport.

<table>
<thead>
<tr>
<th>Energy exchange</th>
<th>$\langle \epsilon \rangle_{\text{flux}}$ [MeV]</th>
<th>$\langle \epsilon^2 \rangle_{\text{flux}}$ [MeV$^2$]</th>
<th>$\rho_{\text{flux}}$ [MeV]</th>
<th>$T$ [MeV]</th>
<th>$\eta$</th>
<th>$L_\nu$ [10$^{51}$ erg s$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_\mu$ transport</td>
<td>original run</td>
<td>17.5</td>
<td>388.</td>
<td>0.97</td>
<td>5.2</td>
<td>1.1</td>
</tr>
<tr>
<td>– s p –</td>
<td>16.6</td>
<td>362.</td>
<td>1.01</td>
<td>5.3</td>
<td>-0.3</td>
<td>15.8</td>
</tr>
<tr>
<td>– s p n</td>
<td>16.9</td>
<td>369.</td>
<td>0.99</td>
<td>5.3</td>
<td>0.4</td>
<td>20.2</td>
</tr>
<tr>
<td>b r s p –</td>
<td>14.2</td>
<td>255.</td>
<td>0.98</td>
<td>4.2</td>
<td>1.1</td>
<td>14.8</td>
</tr>
<tr>
<td>b r s p n</td>
<td>14.4</td>
<td>264.</td>
<td>0.97</td>
<td>4.3</td>
<td>1.2</td>
<td>17.6</td>
</tr>
<tr>
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<td>14.4</td>
<td>263.</td>
<td>0.97</td>
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<td>17.0</td>
</tr>
<tr>
<td>b r s n p n</td>
<td>14.3</td>
<td>260.</td>
<td>0.97</td>
<td>4.3</td>
<td>1.2</td>
<td>17.9</td>
</tr>
<tr>
<td>$\bar{\nu}_\mu$ transport</td>
<td>– s p n</td>
<td>16.9</td>
<td>368.</td>
<td>0.99</td>
<td>5.2</td>
<td>0.6</td>
</tr>
<tr>
<td>b r s p n</td>
<td>14.4</td>
<td>263.</td>
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<td>1.3</td>
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<td>262.</td>
<td>0.98</td>
<td>4.3</td>
<td>1.1</td>
<td>16.8</td>
</tr>
</tbody>
</table>

Note.—For energy exchange, “b” refers to bremsstrahlung, “r” to recoil, “s” to scattering on electrons and positrons, “p” to $e^+e^-$ pair annihilation, “n” to $\nu_e\bar{\nu}_e$ pair annihilation, and “sn” to scattering on both, $e^\pm$ and $\nu_e, \bar{\nu}_e$. 
of the emergent flux spectrum. Here $f_\nu(\epsilon, \mu)$ is the neutrino distribution function in energy-angle space with $\mu$ being the cosine of the angle of neutrino propagation relative to the radial direction. The constant $a$ for a Fermi-Dirac distribution at zero chemical potential is

$$a \equiv \frac{\langle \epsilon^2 \rangle}{\langle \epsilon \rangle^2} = \frac{486000 \zeta_3 \zeta_5}{49 \pi^8} \approx 1.3029.$$  \hspace{1cm} (10)

Then $p = 1$ signifies that the spectrum is thermal up to its second moment, while $p < 1$ signifies a pinched spectrum (high-energy tail suppressed), and $p > 1$ an anti-pinched spectrum (high-energy tail enhanced). For $p < 1.023$ it is common to approximate the spectrum as a nominal Fermi-Dirac distribution characterized by a temperature $T$ and a degeneracy parameter $\eta$ which are chosen such that $\langle \epsilon \rangle_{\text{flux}}$ and $\langle \epsilon^2 \rangle_{\text{flux}}$ are reproduced (Table 2). Finally we show the neutrino luminosity in the last column of Table 2.

We always include elastic $\nu_\mu N$ scattering which provides the dominant opacity contribution. In a given run we additionally include those energy-exchanging processes which are indicated in the first columns of Table 2. We use “b” to indicate bremsstrahlung NN $\rightarrow$ NN$\nu_\mu \bar{\nu}_\mu$, “r” for recoil in $\nu_\mu N \rightarrow N\nu_\mu$, “s” for $\nu_\mu e^\pm$ scattering, “p” for the traditional pair annihilation process $e^+e^- \rightarrow \nu_\mu \bar{\nu}_\mu$, “n” for the new neutrino annihilation process $\nu_\mu \bar{\nu}_e \rightarrow \nu_\mu \bar{\nu}_\mu$, and “sn” for $\nu_\mu e^\pm$ plus $\nu_\mu \nu_\mu$ and $\nu_\mu \bar{\nu}_e$ scattering.

In the first row of Table 2 we show the original flux characteristics of our background model from the calculation of Messer et al. (2002). Running our code with the same neutrino reactions—scattering on $e^\pm$ (s) and $e^+e^-$ pair annihilation (p)—we find the results in the second row. We attribute the small differences to the different numerical approaches. In particular, the Boltzmann solver of Messer et al. (2002) works in practice with a limited number of energy bins. Moreover, there may be differences in the implementation of the microphysical reactions. Finally, in the Monte Carlo code we assume that neutrinos are in thermodynamic equilibrium with the stellar medium at the inner boundary. This choice of boundary condition may have a small effect on our results since the thermalization depth of $e^+e^-$ pair processes is strongly energy dependent and thus low energy $\nu_\mu$ are forced into equilibrium by our boundary condition instead of the pair process. We interpret the first two rows of Table 2 as agreeing satisfactorily well with each other.

Next we add $\nu_e \bar{\nu}_e$ annihilation (n). The spectrum remains almost unchanged, but the luminosity increases by about 30%. Therefore, the new process has a rather significant impact on the predicted $\nu_\mu$ luminosity.

However, other processes are also important which in the past have not been included in numerical simulations. Therefore, we switch off the new process and instead include bremsstrahlung (b) and recoil (r). Compared to the original run we obtain almost the same luminosity, but significantly lowered spectral energies. Now we again include $\nu_e \bar{\nu}_e$ annihilation (n) and find that the spectra remain unaffected, but the luminosity increases by about 20%. Therefore, even with all other energy-exchanging reactions included, the $\nu_e \bar{\nu}_e$ process still has an important effect on the luminosity. We finally switch off the traditional pair process (p), but keep the new one. The spectra remain unaffected, the luminosity slightly drops. It is evident that the $\nu_e \bar{\nu}_e$ process is by far the dominant leptonic source reaction for muon neutrinos. Its importance relative to bremsstrahlung will depend sensitively on the background model (Keil et al. 2002).

As a last step we include the scattering on $\nu_e$ and $\bar{\nu}_e$ in addition to all other processes. The rate for this process is typically half as large as the rate for scattering on $e^\pm$, in agreement with Fig. 5 if we use $\eta_\mu \equiv 3$ and $\eta_{\nu_\mu} \equiv 0.3$. The effect on the $\nu_\mu$, flux and spectrum is minimal and in fact below the numerical resolution of our Monte Carlo runs.

In the second part of Table 2 we finally show several runs for the transport of anti-neutrinos. Recall that $\nu_e \bar{\nu}_e \rightarrow \nu_\mu \bar{\nu}_\mu$ generates different source spectra for $\nu_\mu$ and $\bar{\nu}_\mu$. Of course, the small differences between $\nu_\mu$ and $\bar{\nu}_\mu$ scattering off electrons and positrons are also taken into account, as well as the small differences in $e^+e^-$ pair annihilation. Comparing the $\bar{\nu}_\mu$ runs with those for $\nu_\mu$ we find excellent agreement. Therefore, the detailed spectral distribution of the pair rate is not important, only the total rate of absorption and production of $\nu_\mu \bar{\nu}_\mu$ pairs matters.
4. HYDRODYNAMICAL SIMULATION WITH BOLTZMANN TRANSPORT

The large flux increase found by the Monte Carlo method may not persist in a self-consistent hydrodynamical treatment where the stellar model can adjust in response to the modified transport. For this reason we have performed a Boltzmann transport simulation coupled with a full hydrodynamics code (Rampp & Janka 2002) for the 15 $\odot$ progenitor model s15s7b2 (Woosley, personal communication; Woosley & Weaver 1995).

In order to minimize the required modifications of the code we treat the transport of $\nu_\mu$ and $\bar{\nu}_\mu$ identically. Therefore, we use an average source strength for the $\nu_\mu$ and $\bar{\nu}_\mu$ production from $\nu_e\bar{\nu}_e$ annihilation, ignoring the spectral differences. The Monte Carlo results suggest that this approximation is well justified.

For computing the interaction rates we again assume LTE for $\nu_e$ and $\bar{\nu}_e$ in all regions where the new annihilation process is important. This assumption also justifies introducing an energy source term for the new process directly into the medium energy equation in perfect analogy to the source term of the process $e^+e^- \leftrightarrow \nu\bar{\nu}$. Similarly, the scattering reactions of $\nu_\mu$ and $\bar{\nu}_\mu$ off $\nu_e$ and $\bar{\nu}_e$ can be treated in full correspondence to the scattering off electrons and positrons by a simple change of the weak coupling coefficients. Thus a direct coupling of the $\nu_\mu$ and $\nu_e$ sectors of the neutrino transport code is avoided. For the new process this is achieved only indirectly via the stellar medium as an intermediary.

Our baseline for comparison is a Newtonian simulation which includes the transport of neutrinos of all three flavors. Neutrino-medium interactions for all relevant processes are implemented, notably nucleon bremsstrahlung, as described by Rampp & Janka (2002). Note, however, that nucleon recoils are not taken into account in the results shown for the baseline simulation (thin lines in Figs. 7, 8 and 9). Rather, the charged-current and neutral-current neutrino reactions with nucleons are handled in the “old standard approximation” of infinitely massive nucleons at rest following the treatment by Bruenn (1985) and Mezzacappa & Bruenn (1993).

In a second simulation we included the new leptonic process for $\nu_\mu\bar{\nu}_\mu$ pair production and annihilation. The results for this run are depicted as thick lines in the figures. In Figs. 7 and 8 we show the evolution of the neutrino luminosities and of the root mean squared (rms) energies as functions of time after bounce. The rms energy is defined in the usual way (e.g. Messer et al. 1998) by

$$\langle \epsilon \rangle_{\text{rms}} = \frac{\int_0^\infty d\epsilon \int_{-1}^{+1} d\mu f_\nu(\epsilon, \mu) \epsilon^2}{\int_0^\infty d\epsilon \int_{-1}^{+1} d\mu f_\nu(\epsilon, \mu) \epsilon^3},$$

i.e. using the neutrino energy distribution as a weight function. The results are given for an observer who is comoving with the stellar fluid at a radial location of 500 km. The Doppler-blueshift due to the infall velocity of the matter is rather small at this distance from the neutrino emitting neutron star so that the quantities are close to the observable properties at infinity.

As expected, the $\nu_e$ luminosity is increased by 10–20%, but the spectrum remains essentially unaffected. The enhanced energy loss leads to a somewhat faster proto-neutron star contraction. This causes small changes also in the region where the electron neutrino and antineutrino fluxes are built up and where these neutrinos finally decouple from the stellar background. As a consequence, the mean spectral energies of $\nu_e$ and $\bar{\nu}_e$ are systematically higher by a small amount (Fig. 8). Initially, the $\nu_e$ luminosity is also slightly larger (Fig. 7) but after about 150 ms post bounce the luminosities of $\nu_e$ and $\bar{\nu}_e$ drop below the level of those in the reference simulation. This is caused by the decrease of the radii of the neutrino spheres in response to the accelerated contraction of the proto-neutron star. At 200 ms after bounce the $\nu_{\mu,\tau}$ luminosities of the two models then become nearly equal, probably as a consequence of two competing effects which seem to essentially compensate each other at later times: On the one hand the neutrino emission of the nascent neutron star decays faster with time when the new process is included (as visible from the $\nu_e$ and $\bar{\nu}_e$ luminosities), on the other hand the new process raises the energy loss in $\nu_{\mu,\tau}$ compared to $\nu_e$ and $\bar{\nu}_e$.

The shock positions in both simulations evolve identically until about 100 ms after bounce. Then the shock is somewhat weakened by introducing the new reaction and expands only to a maximum radius of 230 km instead of 250 km (Fig. 9). This can be understood again by the more com-
pact proto-neutron star, which causes higher infall velocities in the region of heating by $\nu_e$ and $\bar{\nu}_e$ absorption behind the shock. This reduces the integral energy deposition by neutrinos as well as the pressure behind the shock front.

Since we completed this study we have implemented the neutrino annihilation process in simulations with an approximate treatment of general relativity (Rampp & Janka 2002). In these new simulations neutrino-nucleon interactions are implemented in terms of dynamical structure functions for correlated nuclear matter and also include weak magnetism corrections (Rampp et al. 2002). Put another way, these simulations include nuclear recoil as well as nuclear correlation effects. The impact of the new process in these more complete simulations is comparable to what we have found here.

5. CONCLUSIONS

We have shown that $\nu_e\bar{\nu}_e$ annihilation is the dominant leptonic source for $\nu_{\mu,\tau},\bar{\nu}_{\mu,\tau}$ pairs in a SN core, far more important than the traditional $e^+e^-$ pair annihilation. Its importance relative to the nucleon bremsstrahlung process, which also has not been included in previous SN simulations, depends on the stellar profile. In our Monte Carlo studies with a background model representing the SN core during the accretion phase, the new process enhanced the $\nu_{\mu}$ luminosity by about 20% in a calculation where nucleon bremsstrahlung and energy exchange by nucleon recoils were taken into account. Without these other new effects, $\nu_e\bar{\nu}_e$ annihilation has an even larger impact.

In self-consistent hydrodynamical simulations with Boltzmann neutrino transport we find that during the first 150 ms the effect is similar to that obtained in the static Monte Carlo simulations and has a noticeable influence on the stellar evolution and structure. Later, the $\nu_{\mu}$ and $\bar{\nu}_{\mu}$ luminosities approach those of the model without the new pro-
cess of $\nu_e\bar{\nu}_e$ annihilation. At that time, however, the $\nu_e$ and $\bar{\nu}_e$ luminosities are smaller. Throughout the simulation the new reaction therefore works in the direction of making the $\nu_e$ and $\bar{\nu}_e$ luminosities more similar to those of $\nu_\mu$ and $\bar{\nu}_\mu$. In the model with the new process the shock is somewhat weakened and reaches only smaller radii.

In all of our Monte Carlo and hydrodynamical runs we confirm the naive expectation that the effect of the new pair production and annihilation process on the neutrino spectra is minimal. The crossed process of $\nu_{\mu, \tau}$ scattering off $\nu_e$ and $\bar{\nu}_e$ also turned out to have a negligible impact on the emitted spectra.

We conclude that state-of-the-art SN simulations should include the $\nu_e\bar{\nu}_e$ annihilation reaction to $\nu_{\mu, \tau}\bar{\nu}_{\mu, \tau}$ pairs. While the effects of this process on the neutrino luminosities and spectra and on the shock propagation are not dramatic, they are nevertheless noticeable and not negligible, even after nucleon bremsstrahlung and nucleon recoils have been included. Implementing the new process is not more difficult and not more CPU-expensive than the traditional $e^+ e^-$ process.

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