Does the Rapid Appearance of Life on Earth Suggest that Life is Common in the Universe?

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ABSTRACT

It is sometimes assumed that the rapidity of biogenesis on Earth suggests that life is common in the Universe. Here we critically examine the assumptions inherent in this if-life-evolved-rapidly-life-must-be-common argument. We use the observational constraints on the rapidity of biogenesis on Earth to infer the probability of biogenesis on terrestrial planets with the same unknown probability of biogenesis as the Earth. We find that on such planets, older than \( \sim 1 \text{ Gyr} \), the probability of biogenesis is > 13% at the 95% confidence level. This quantifies an important term in the Drake Equation but does not necessarily mean that life is common in the Universe.

1. The Biogenesis Lottery

Much of current astrobiological research is focused on learning more about the early evolution of the Earth and about the origin of life. We may be able to extrapolate and generalize our knowledge of how life formed here to how it might have formed elsewhere. Indirect evidence suggesting that life may be common in the Universe includes:

- Sun-like stars are common.
- Sources of free energy such as starlight and reduction-oxidation pairs are common (Nealson and Conrad 1999).

It is difficult to translate this circumstantial evidence into an estimate of how common life is in the Universe. Without definitive detections of extraterrestrial life we can say very little about how common it is or even whether it exists. Our existence on Earth can tell us little about
how common life is in the Universe or about the probability of biogenesis on a terrestrial planet because, even if this probability were infinitesimally small and there were only one life-harboring planet in the Universe we would, of necessity, find ourselves on that planet. However, the rapidity with which life appeared on Earth gives us more information. If life were rare it would be unlikely that biogenesis would occur as rapidly as it seems to have occurred on Earth.

Although we do not understand the details of how life originated, we have some useful observational constraints on how long it took. Carbon isotopic evidence suggests that life existed on Earth more than 3.85 billion years ago (Mojzsis et al. 1996). High temperatures and large frequent sterilizing impacts may have frustrated an earlier appearance of life (Maher and Stevenson 1988, Sleep et al. 1989). If life originated on Earth, then increasingly tight observational constraints indicate that biogenesis was rapid (Oberbeck and Fogleman 1989, Sleep et al. 2001). The extraterrestrial implications of rapid biogenesis on Earth and the extent to which this rapidity suggests that life is common in the Universe have not been looked at carefully and are the focus of this paper.

The basic concept is simple; over a given time period, more probable events happen more often (and thus more rapidly) than less probable events. Thus, the probability of winning a lottery can be inferred from how quickly a lottery winner has won. For example, suppose we have no idea about the probability $q$, of winning a daily lottery ($0 \leq q \leq 1$). Suppose a gambler buys a lottery ticket every day for three days, losing on the first two days and winning on the third. We can use this information to infer something about $q$. Specifically, in this case, we can say that $q$ is more likely to be about one third than one hundredth, and is unlikely to be close to 1 (see $\mathcal{L}(n = 3; q)$ in Fig. 1). If the gambler can only tell us that he won at least once within 3 days then we can no longer exclude high values of $q$ with such confidence, but the likelihood of $q$ can still tell us that $q > 0.16$ at the 95% confidence level (see $\mathcal{L}(\leq 3; q)$ in Fig. 1 and Eq. 9 of Appendix).

Suppose there is a group of gamblers, all of whom have won at least once within $N$ (e.g. 12) days. A gambler is chosen from this group at random and after carefully examining his tickets, he tells us that he won at least once within the first 3 days – relatively early in the 12 days that he had to have won by, to be in the group. This is analogous to our situation on Earth. We find ourselves in the group of planets on which biogenesis has necessarily occurred – we have of necessity won the biogenetic lottery some time in the past. And we also find that biogenesis has occurred rapidly. We won soon after life became possible on Earth. Given the above information about the gambler, the likelihood of $q$ is plotted in Fig. 1 as $\mathcal{L}(n \leq 3, N \leq 12; q)$ and allows us to conclude that $q > 0.12$ at the 95% confidence level. This statistical result applies to a group of gamblers or to a group of terrestrial planets on which the probability $q$ of biogenesis is unknown, as long as $q$ is approximately the same for each planet in the group and approximately the same as it was on Earth. In the next section we review the observational constraints on when and how quickly life appeared on Earth. In Section 3 we use these constraints to identify and critically examine selection effects that complicate this result. In Section 4 we discuss the relationship between our result, the Drake Equation, and the larger question: ‘How common is life in the Universe?’ Mathematical details are relegated to the appendix.
Fig. 1.— Let the unknown probability of winning a daily lottery be $q$. Suppose a gambler buys a ticket each day and wins on the third day. From this information we can calculate the likelihood of $q$ (see $\mathcal{L}(n = 3; q)$ above). The most likely value is, as expected, 1/3, but we can also conclude that $0.06 < q < 0.80$ at the 95% confidence level (hatched area). Different scenario: suppose that after three days the gambler tells us that he won at least once. The likelihood of $q$ then becomes $\mathcal{L}(n \leq 3; q)$ plotted above. High values of $q$ can no longer be excluded and we can say only that $0.16 < q$ at the 95% confidence level. Different scenario (and one more analogous to our situation on Earth): we have a group of gamblers, all of whom have won at least once, on or before the $N$th (e.g. 12th) day. One of them, chosen at random (analogous to the Earth), won at least once, on or before the 3rd day. This is quite early since it could have happened any time during the 12 days. Given this information, the likelihood of $q$ becomes the thick curve labelled $\mathcal{L}(n \leq 3, N \leq 12; q)$. In this case, we can say that $q > 0.12$ at the 95% confidence level (grey area). See Appendix for computational details.
2. Observational Constraints on the Timing of Terrestrial Biogenesis

If life originated on Earth, then during and immediately following the Earth’s formation there was a period without life ($\Delta t_{\text{frustrated}}$), followed by a period during which life evolved ($\Delta t_{\text{biogenesis}}$), followed by a period during which life has been present ($\Delta t_{\text{life}}$). The sum of these intervals adds up to the age of the Earth (see Fig. 2):

$$\Delta t_{\text{frustrated}} + \Delta t_{\text{biogenesis}} + \Delta t_{\text{life}} = \Delta t_{\text{Earth}}$$

where $\Delta t_{\text{Earth}} = 4.566 \pm 0.002$ Gyr (Allègre et al. 1995). As older fossils and biosignatures have been found, $\Delta t_{\text{life}}$ has gotten longer. The significance of large impacts in frustrating or sterilizing proto-life has only recently been appreciated and assessed ($\Delta t_{\text{frustrated}}$). Combined, these observations indicate that biogenesis was rapid since $\Delta t_{\text{biogenesis}}$ is caught in the middle—the longer $\Delta t_{\text{frustrated}}$ and $\Delta t_{\text{life}}$ get, the shorter $\Delta t_{\text{biogenesis}}$ must get. The distinction between how rapid biogenesis was and when it was, is important because our result, the inferred probability of biogenesis, depends on how rapid it was, while only a marginal selection effect depends on when it was (Section 3).

The majority of the Earth’s mass accreted from planetesimals within the first 100 million years of the Earth’s formation (Halliday 2000). With an initially molten surface, life could not have appeared. The transition from accretion to heavy bombardment included the formation of the Moon by the collision with a Mars-sized object $\sim$ 4.5 Gyr ago (Hartmann and Davis 1975, Halliday 2001, Canup and Asphaug 2001). We can infer from the dates and sizes of lunar impact craters, whose record goes back to when the Moon formed a solid crust ($\sim$ 4.44 Gyr ago, Sleep et al. 1989) that the surface of the Earth was periodically vaporized. Since the mass of the Earth is 80 times the mass of the Moon, impacts on the Earth were more numerous, more energetic and periodically produced 2,000 K rock vapor atmospheres which lasted for several thousand years (Hartmann et al. 2000, Sleep et al. 2001). These conditions were probably an effective and recurring autoclave for sterilizing the earliest life forms or more generally frustrating the evolution of life. A steadily decreasing heavy bombardment continued until $\sim$ 3.8 Gyr ago.

Estimates of the time of the most recent sterilizing impact range between 4.44 and 3.7 Gyr ago (Maher and Stevenson 1988, Sleep et al. 1989, Oberbeck and Fogleman 1989, Halliday 2001). These estimates span the time from the solidification of the Moon’s crust to the end of the late heavy bombardment. Thus, life was frustrated for at least the first 0.1 Gyr and possibly as long as the first 0.9 Gyr of the Earth’s existence. We take our preferred value as the middle of this range: $\Delta t_{\text{frustrated}} \approx 0.5 \pm 0.4$ Gyr. The range of these estimates reflects the large uncertainties due to small number statistics for the largest impactors and the uncertainty of the energy required to sterilize the Earth completely. We do not know where biogenesis happened or the extent to which it was protected from the effects of impacts. Tidal pools have little protection, hydrothermal vents have some protection, while autotrophic thermophiles in sub-surface rock under several kilometers of crust were probably in effective bomb shelters.
The roots of the universal tree of life point to a thermophilic origin (or at least a thermophilic common ancestor) for all life on earth (Pace 1991, Stetter 1996). This suggests a hot biogenesis in hydrothermal vents or possibly sub-surface rock and/or selection for thermophilia by periodic temperature pulses from large impacts. If we knew that life evolved on the surface of the Earth and was therefore more susceptible to impact sterilizations, life would have been frustrated longer and our preferred value would be more precise: \( \Delta t_{\text{frustrated}} \approx 0.7 \pm 0.2 \) Gyr.

If we accept the carbon isotopic evidence for life more than 3.85 billion years ago (Mojzsis et al. 1996) then life has been on Earth at least that long, i.e., \( \Delta t_{\text{life}} \) is at least 3.85 Gyr. In addition, because of the Earth's tectonic history, this time is also the earliest time we could reasonably hope to find biological evidence from rocks on Earth – even if life existed earlier. With this selection effect in mind (which we know exists at some level), our preferred value for the time life has existed on Earth is: \( \Delta t_{\text{life}} \approx 4.0^{+0.4}_{-0.2} \) Gyr.

It is possible that biogenesis occurred several times on Earth. For example, during the period \( \Delta t_{\text{frustrated}} \), life could have evolved and been sterilized multiple times. We do not know if this happened. Similar potential sterilizations and biogeneses could have occurred during the period \( \Delta t_{\text{life}} \) but we do not know. For the purposes of this analysis we can ignore this complication. We are only interested in the shortest period within which the observations can constrain biogenesis to have occurred. Thus, \( \Delta t_{\text{biogenesis}} \) is our best observational constraint on any epoch of biogenesis and this happens to be on the period between the most recent sterilizing impact that is older than the oldest evidence we have for life on Earth.

Since our preferred values yield \( \Delta t_{\text{frustrated}} + \Delta t_{\text{life}} \approx \Delta t_{\text{Earth}} \), there is little time left for biogenesis to have occurred. This is the basis for the statement that biogenesis occurred rapidly. Substituting our three preferred values, \( \Delta t_{\text{Earth}} \), \( \Delta t_{\text{frustrated}} \) and \( \Delta t_{\text{life}} \) into Eq. 1 and solving for \( \Delta t_{\text{biogenesis}} \) yields,

\[
\Delta t_{\text{biogenesis}} = 0.1^{+0.5}_{-0.1} \text{ Gyr.}
\]

Thus we take 600 Myr as a crude estimate of the upper limit for the time it took life to appear on Earth. Assuming biogenesis took place on the surface of the Earth, Oberbeck and Fogleman (1989) found this maximum time to be \(~ 25\) Myr. Maher and Stevenson (1988) assumed that biogenesis took somewhere between 0.1 Myr and 10 Myr depending on environment, while Sleep et al. (2001) find a similar range for evolutionarily significant periods of clement surface conditions. Independently, biologists specializing in the chemistry of the origin of life have estimated that the time required for biogenesis is potentially quite short (Miller 1982) and could be less than 8 Myr (Lazcano and Miller 1994). Thus, several lines of evidence indicate that biogenesis was geologically rapid. If biogenesis occurred in the fissures around a hydrothermal vent or in subsurface rocks, it was well-protected and we can only constrain biogenesis to have taken less than \(~ 600\) Myr. If biogenesis occurred on the surface, it probably took less than \(~ 25\) Myr. In this analysis we consider 600 and 25 Myr to represent the high and low ends of a plausible range for \( \Delta t_{\text{biogenesis}} \) (see Figs. 2 and 3 respectively).

These observational constraints on \( \Delta t_{\text{frustrated}} \), \( \Delta t_{\text{biogenesis}} \) and \( \Delta t_{\text{life}} \) are important because
they quantify how rapidly biogenesis occurred on Earth and enable us to put limits on the probability that biogenesis occurred on other planets. For example, consider a group of terrestrial planets with approximately the same probability of biogenesis 'q' as Earth. Suppose \( q = 0.30 \). At their formation, none of these planets had life. As time passed, life arose on more and more of them. The thick line in Fig. 2 shows the increasing percentage of these planets with life as time passes. After \( \Delta t_{\text{biogenesis}} \), 30% will have life (that is how \( q = 0.30 \) is defined). After 4.566 Gyr, 93% will have life (7% still will not). Of that 93%, 32% will, like the Earth, have had biogenesis within \( \Delta t_{\text{biogenesis}} \). The histogram on the far right of Fig. 2 represents these numbers. Suppose \( q \) is only 0.03. Then only 20% will have life after 4.566 Gyr and only 14% of those will have had biogenesis, like the Earth, within \( \Delta t_{\text{biogenesis}} \). Assuming Earth is a random member of the planets with life, the single observation that biogenesis occurred within \( \Delta t_{\text{biogenesis}} \) on Earth indicates that larger values of \( q \) are more likely than small values. This is the basic idea behind our analysis. It is illustrated in Figs. 1, 4 and 5 which also show quantitative constraints on \( q \) under the various assumptions discussed in Section 3.

The histograms in Figs. 2 and 3 also show that if \( q \) is large, the fraction of gamblers or terrestrial planets who have won after a certain time is large. Suppose the gambler did not know how many gamblers had won by the 12th day, but only that he is a random member of the group that had won. From the fact that he won quickly, he can infer that \( q \) is large. This tells the gambler that after a few days a large fraction of lottery ticket buyers are in the winners group. He is not alone. For the biogenesis lottery, finding large \( q \) means that after a few times \( \Delta t_{\text{biogenesis}} \), a large fraction of the terrestrial planets with probability of biogenesis similar to the Earth, have (or have had) life.
Fig. 2.— We divide the history of the Earth into three epochs: Impact frustration, Biogenesis and Life. The black curves show the percentages of terrestrial planets with life as a function of time assuming two different probabilities of biogenesis ($q = 0.30, 0.03$), within $\Delta t_{\text{biogenesis}} = 600$ Myr. The percentages in the histograms on the right (14% and 32%) are obtained from comparing the subset of planets that have formed life within $\Delta t_{\text{biogenesis}}$ (middle grey) with the total number of planets that have life (or have had biogenesis) within 4.566 Gyr of their formation (cross-hatched). If $q$ is high (0.30, thick line) a large fraction (32%) of the planets which have evolved life within 4.566 Gyr of formation, have life that evolved rapidly – within $\Delta t_{\text{biogenesis}}$ – on their planets. If $q$ is low (0.03, thin line) then a smaller fraction (14%) will have life that evolved rapidly. These different percentages illustrate the principle that a single observation of rapid terrestrial biogenesis is more likely to be the result of high $q$. This allows us to compute the relative likelihood of $q$ and to constrain $q$ (see Fig. 4).
Fig. 3.— Zoomed in version of the previous figure with two differences: 1) The window for biogenesis shown here is at the short end of the range permitted by observations: $\Delta t_{\text{biogenesis}} = 25$ Myr. 2) In the previous figure, by normalizing the histograms to 4.566 Gyr, we have assumed that Earth is a random member of a group of planets at least 4.566 Gyr old upon which biogenesis could have happened anytime up to 4.566 after formation, including one million years ago. This ignores the non-observability-of-recent-biogenesis selection effect (Section 3.2). In this figure, to minimize the influence of this selection effect, we only allow biogenesis to occur anytime earlier than 3.77 Gyr ago, i.e., within ‘$\Delta t(N = 4)$’. The influence of this normalization time $\Delta t(N)$ is illustrated in Fig. 4 and discussed in Section 3.2.
3. Selection Effects

3.1. Daily Lottery → Biogenesis Lottery

In Section 1 we drew parallels between a daily lottery and a biogenesis lottery. Explicitly, these parallels are:

- The first day of the lottery corresponds to the end of $\Delta t_{\text{frustrated}}$, the time when conditions become clement enough for biogenesis.

- All the gamblers had the same (but unknown) chance of winning the lottery each time they bought a ticket ($q$ is the probability of winning per day). This corresponds to a group of terrestrial planets with approximately the same (but unknown) chance of biogenesis as the Earth ($q$ is the probability of biogenesis within a period of time called here $\Delta t_{\text{biogenesis}}$).

- We selected a gambler at random from those who had won on or before the $N$th day. Thus, we conditioned on winning before a certain time. This corresponds to assuming that the Earth is a random member of the group of terrestrial planets that has had biogenesis on or before the end of $\Delta t(N) = N \times \Delta t_{\text{biogenesis}}$. Conditioning on biogenesis before this time corresponds to correcting for the selection effect that biogenesis had to have occurred for us to be here.

- A gambler chosen at random, from the group that has won within $N$ days, found that he had won on the first day ($n = 1$). This corresponds to finding that biogenesis has occurred rapidly on Earth, that is, within $\Delta t_{\text{biogenesis}}$.

- If we set $N = 1$, see Fig. 4, $L(1, 1; q)$, we are conditioning on rapid biogenesis. We are considering a group, all of whose members have had rapid biogenesis. In this case, a random member having rapid biogenesis can tell us nothing about the probability $q$. To infer something about $q$ we must have $N > 1$.

3.2. Non-observability of recent biogenesis

If our conclusions from the daily lottery are to apply to biogenesis on terrestrial planets we need to correct for the fact that the evolution of an observer takes some time. How long it takes observing equipment, or complex life or multicellular eukaryotes to evolve, is difficult to say. On Earth it took $\sim 4$ Gyr. A limited pace of evolution has prevented us from looking back at our own history and seeing that biogenesis happened last year or even more recently than $\sim 2$ Gyr ago (we consider a plausible range for the requisite time elapsed since biogenesis to be between 2 and 10 billion years, or $\Delta t_{\text{evolve}} = 4^{+6}_{-2}$ Gyr).

This selection effect for non-recent biogenesis is selecting for biogenesis to happen a few billion years before the present regardless of whether it happened rapidly. It is not a selection effect for
rapid biogenesis since the longer it took us to evolve to a point when we could measure the age of
the Earth, the older the Earth became. Similarly, if biogenesis took 1 Gyr longer than it actually
did, we would currently find the age of the Earth to be 5.566 Gyr (= 4.566 + 1) old; ‘Why is the
Earth 4.566 billion years old?’, ‘Because it took that long to find it out.’ The generalization of
this plausible assumption to the ensemble of terrestrial planets is necessary if the likelihoods and
constraints in Figs. 1, 4 and 5 are to be applicable to the group of terrestrial planets.

3.3. Potential Problems

Any effect that makes rapid biogenesis a prerequisite for life would undermine our inferences
for $q$. For example, although it is usually assumed that the heavy bombardment inhibited
biogenesis, energetic impacts may have set up large chemical and thermal disequilibria that play
some crucial role in biogenesis. We know so little about the details of the chemical evolution that
led to life that heat pulses and rapid cooling after large impacts may be part of the preconditions
for biogenesis. If true, the timescale of biogenesis would be linked to the timescale of the
exponential decay of bombardment and biogenesis would (if it occurred at all) be necessarily
rapid; most extant life in the Universe would have rapid biogenesis and little could be inferred
about the absolute value of $q$ from our sample of one.

In a panspermia scenario, the rapid appearance of life on Earth is explained not by rapid
terrestrial biogenesis as assumed here, but by the ubiquity of the ‘seeds of life’ (e.g. Hoyle and
Wickramasinghe 1999). An analysis in the context of a panspermia scenario would be subject to
the same observational constraints as terrestrial biogenesis and would therefore lead to the same
inferred probability for the appearance of life on other terrestrial planets.

Another potential problem: suppose the autocatalytic chemical cycles leading to life are
exponentially sensitive to some still unknown peculiarity of the initial conditions on Earth. In
this case, to have the same $q$ as the Earth, our group of terrestrial planets may have to be almost
indistinguishable rare clones of Earth. That is, conditioning on planets identical to Earth, (‘same
$q$’) would be conditioning on rapid biogenesis ($N = 1$) and would prevent us from inferring much
about $q$ from the observations of rapid biogenesis on Earth (see $\mathcal{L}(1, 1; q)$ in Fig. 4).

This is an example of the more general issue of the status of the rapidity of biogenesis on
Earth. Did it have to be rapid? If we assume it could have been otherwise then we can infer
something about $q$. If it had to be that way, we can not. The middle ground might be the most
plausible option: biogenesis did not necessarily have to happen as rapidly as it did, but (to be
consistent with our existence) it may have had to happen within one or two billion years of
the Earth’s formation. If this is true, we need to look carefully at the influence of varying the
somewhat arbitrary and counterfactual duration ($\Delta t(N) = N \times \Delta t_{\text{biogenesis}}$), that biogenesis could
have taken on Earth. What values of $\Delta t(N)$ are plausible? and how do they effect the results?
This is done in Figure 4 which quantifies the degree of variability one can assume for the duration
of biogenesis and still have interesting constraints on $q$. The lower $N$ is, the less variability is
assumed. For example, if \( N = 2 \), see \( \mathcal{L}(1,2;q) \), we are assuming that biogenesis could only have taken as long as \( 2 \times \Delta t_{\text{biogenesis}} \) – enough variability to be able to say something about \( q \) but small enough to maintain the non-observability of recent biogenesis.

Figure 5 allows us to generalize our inferences about \( q \) (the probability of biogenesis within \( \Delta t_{\text{biogenesis}} \)) to inferences about \( q_N \) (the probability of biogenesis within an arbitrary time, \( \Delta t(N) \)). Specifically, it shows that we only need to be able to assume that biogenesis could have been twice as long as \( \Delta t_{\text{biogenesis}} \) to have interesting constraints: \( q_2 > 0.13 \) at the 95% confidence level (see \( \mathcal{L}(1,2;q_N) \)). This is the result reported in the abstract.

4. How Common is Life?

4.1. Relation of our analysis to the Drake Equation

The Drake Equation was devised to address the question of ‘How many communicative civilizations are in our Galaxy?’ (e.g. Sagan 1973). It has been criticized as ‘a way of compressing a large amount of ignorance into a small space’ (Oliver, cited in Dicke 1998). Despite its short-comings, it continues to focus the efforts of the SETI community. An important goal of the SETI community is to turn its subjective probabilities into mathematical probabilities. We have done that here for one of the most important terms.

We are interested in a simpler question: ‘How common is life in the Universe?’ Our question is simpler because life is more generic than intelligent or technological life. We modify the Drake Equation to address our question and introduce a parameter \( F \) which is a measure of how common life is in the universe. \( F \) is the fraction of stars in our galaxy today, orbited by planets that have had independent biogenesis,

\[
F = \frac{N_I}{N_*} = f_p f_e f_l
\]

where,

- \( N_I \): number of stars in our Galaxy orbited by planets that have had independent biogenesis.
- \( N_* \): number of stars in our Galaxy.
- \( f_p \): fraction of stars in our Galaxy with planetary systems.
- \( f_e \): fraction of these planetary systems that have a terrestrial planet suitable for life in the same way as the Earth, that is, they have approximately the same probability \( q \) as the Earth.
- \( f_l \): fraction of these suitable planets on which biogenesis has occurred.

Many recent observations of the frequency and age dependence of circumstellar disks around young stars in star-forming regions support the widely accepted idea that planet formation is a common by-product of star formation and that the fraction of stars with planetary systems is close to unity, \( f_p \approx 1 \) (e.g. Habing et al. 1999, McCaughrean et al. 2000, Meyer and Beckwith 2000). Equation 3 then becomes,

\[
F \approx f_e f_l
\]
If $F > 10^{-2}$ then more than 1% of all stars has (or has had) life, and we conclude that life is ‘common’ in the Universe. If $F < 10^{-11}$, we may be the only life in the Galaxy and life is ‘rare’.

There has been little agreement on the value of $f_l$. Michael Hart (1996) writes “The value of $f_l$ is extremely speculative,” but argues based on the concatenation of low probabilities that it must be extremely small and thus, life elsewhere is improbable. However Shostak (1998) assumes quite the opposite: “On the basis of the rapidity with which biology blossomed on Earth, we can optimistically speculate that this fraction ($f_l$) is also one (100%)”. Our analysis is a close statistical look at this optimistic speculation.

The relation between the fraction of suitable planets on which biogenesis has occurred (‘$f_l$’ in the Drake Equation) and the $q$ analyzed here is,

$$f_l(q, N) = 1 - (1 - q)^N$$  \hspace{1cm} (5)

$$f_l(q, \Delta t(N)) = 1 - (1 - q)^{\Delta t(N) / \Delta t_{biogenesis}}$$  \hspace{1cm} (6)

$$f_l(q, t) = 1 - (1 - q)^{(t - \Delta t_{frustrated}) / \Delta t_{biogenesis}}$$  \hspace{1cm} (7)

where Eq. 5 refers to the daily lottery (see Appendix, Eq. 12), Eq. 6 is the translation to the biogenesis lottery and we obtain Eq. 7 by using $t = \Delta t_{frustrated} + \Delta t(N)$. Equation 7 is plotted in Figs. 2 and 3. Thus, in Eq. 7 we have expressed the ‘$f_l$’ term of the Drake Equation as a function of time, of the observables $\Delta t_{frustrated}$ and $\Delta t_{biogenesis}$, and of the probability $q$, for which we have derived the relative likelihood. In addition, since $f_l(q, N) = q_N$ (see Eq. 12), Fig. 5 shows the relative likelihood of $f_l$ and establishes quantitative but model-dependent (specifically $N > 1$ dependent) constraints on $f_l$. For example, for terrestrial planets older than $\sim 1$ Gyr ($\sim 2 \times \Delta t_{biogenesis}$), $f_l > 13\%$ at the 95% confidence level.

However, to go from $f_l$ to an answer to the question ‘How common is life?’, i.e., to go from $f_l$ to $F$ in Eq. 4, we need to know $f_e$: the fraction of these planetary systems with a terrestrial planet suitable for life in the same way as the Earth. Our understanding of planet formation is consistent with the idea that ‘terrestrial’ planet formation is a common feature of star formation (Wetherill 1996, Lissauer and Lin 2000). The mass histogram of detected extrasolar planets peaks at low masses: $dN/dM \propto M^{-1}$ and is also consistent with this idea (Lineweaver and Grether 2002, Zucker and Mazeh 2001, Tabachnik and Tremaine 2002).

Our analysis assumed the existence of a group of terrestrial planets with approximately the same, but unknown ($q \in [0, 1]$) probability of biogenesis. It is reasonable to postulate the existence of such a $q$-group since, although we do not know the details of the chemical evolution that led to life, we have some ideas about the factors involved: energy flux, temperatures, the presence of water, planet orbit, residence time in the continuously habitable zone, mass of the planet, atmospheric composition, bombardment rate at the end of planetary accretion and its dependence on the masses of the large planets in the planetary system, metallicity of the prestellar molecular cloud, crust composition, vulcanism, basic chemistry, hydration, pH, presence of particular clay minerals, amino acids and other molecular building blocks for life (Lahav 1999). Since all or many
of these physical variables determine the probability of biogenesis, and since the Earth does not seem to be special with respect to any of them (i.e., the Earth probably does not occupy a thinly populated region of this multi-dimensional parameter space), the assumption that the probability of biogenesis on these planets would be approximately the same as on Earth is plausible. This is equivalent to assuming that $q$ is a “slowly” varying function of environment. If true, $f_e$ and $F$ are not vanishingly small. This can be contrasted with the “exponentially sensitive” case discussed in Section 3.3.

4.2. Discussion

Carter (1983) has pointed out that the timescale for the evolution of intelligence on the Earth ($\sim 5$ Gyr) is comparable to the main sequence lifetime of the Sun ($\sim 10$ Gyr). Under the assumption that these two timescales are independent, he argues that this would be unlikely to be observed unless the average timescale for the evolution of intelligence on a terrestrial planet is much longer than the main sequence lifetime of the host star (see Livio (1999) for an objection to the idea that these two timescales are independent). Carter’s argument is strengthened by recent models of the terrestrial biosphere that indicate that the gradual increase of solar luminosity will make Earth uninhabitable in a billion years or so – several billion years before the Sun leaves the main sequence (Rampino and Caldiera 1994, Caldeira and Kasting 1992).

Our analysis is similar in style to Carter’s, however we are concerned with the appearance of the earliest life forms, not the appearance of intelligent life. Subject to the caveats raised in Section 3.3 and by Livio (1999), the implications of our analysis and Carter’s are consistent and complementary: the appearance of life on terrestrial planets may be common but the appearance of intelligent life may be rare.

4.3. Summary

• Our existence on Earth does not mean that the probability of biogenesis on a terrestrial planet, $q$, is large, because if $q$ were infinitesimally small and there were only one life-harboring planet in the Universe we would, of necessity, find ourselves on that planet. However, such a scenario would imply either that Earth has a unique chemistry or that terrestrial biogenesis has taken a long time to occur. Neither is supported by the evidence we have. Since little can be said about the probability $q$, of terrestrial biogenesis from our existence, we assume maximum ignorance: $0 \leq q \leq 1$. We then use the observation of rapid terrestrial biogenesis to constrain $q$ (Fig. 4).

• We convert the constraints on $q$ into constraints on the $f_l$ term of the Drake Equation (the fraction of suitable planets which have life). For terrestrial planets older than $\sim 1$ Gyr we find that $f_l$ is most probably close to unity and $> 13\%$ at the 95% confidence level.
• If terrestrial planets are common and they have approximately the same probability of biogenesis as the Earth, our inference of high $q$ (or high $q_N$) indicates that a substantial fraction of terrestrial planets have life and thus life is common in the Universe.

However, there are assumptions and selection effects that complicate this result:

• Although we correct the analysis for the fact that biogenesis is a prerequisite for our existence, our result depends on the plausible assumption that rapid biogenesis is not such a prerequisite.

• Although we have evidence that the fraction of planets which are ‘terrestrial’ in a broad astronomical sense (rocky planet in the continuously habitable zone) is large, this may be different from the fraction of planets which are ‘terrestrial’ in a more detailed chemical sense. Although we can make reasonable estimates of what the crusts and atmospheres are made of, without detailed knowledge of the steps of chemical evolution, we cannot be sure that astronomically terrestrial planets have the same $q$ as Earth. That is, the fraction of planets belonging to the Earth’s $q$-group is uncertain. Thus, although we have been able to quantify the $f_l$ term of the Drake Equation using rapid biogenesis, our knowledge of the $f_e$ term is still only qualitative and inhibits our ability to draw stronger conclusions about how common life is in the Universe.

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Fig. 4.— The effect on the likelihood of varying the somewhat arbitrary number of days that the gamblers have to have won by to be in the group. In contrast to Fig. 1, all the likelihoods of \( q \) here are based on the information that a gambler (chosen at random from the group whose members have won within \( N \) days) has won on the first day. These likelihoods are plotted and labeled \( \mathcal{L}(1,N;q) \) with \( N \in \{1,2,4,\infty\} \). In the biogenesis lottery (just as in the daily lottery) \( N \) defines the group and is a measure of the duration biogenesis could have taken \( (\Delta t(N) = N \times \Delta t_{\text{biogenesis}}) \). For example, if \( N = 1 \) we can not say anything about \( q \) since the likelihood \( \mathcal{L}(1,1;q) \) is flat (we have conditioned on ‘rapid’ biogenesis). While if \( N = \infty \) we can put the strongest constraint on \( q \). If \( n = 1 \) (when it could have been much larger, \( 1 \leq n \leq N \)), we have more information about \( q \) and the likelihood for large \( q \) is higher (see Appendix, Eq. 10). The ratio of the probabilities in the histograms in Fig. 3 corresponds to the ratio of the \( \mathcal{L}(1,4;q) \) likelihoods here: \[
\frac{39\%}{26\%} = \frac{\mathcal{L}(1,4;q=0.30)}{\mathcal{L}(1,4;q=0.03)} = \frac{0.49}{0.30} = 1.63.
\] That is, with \( n = 1 \) and \( N = 4 \), values of \( q \approx 0.30 \) are 50% more likely to be the case than \( q \approx 0.03 \) (in Fig. 3, \( \Delta t(N) = 4 \times 25 \text{ Myr} \)). Notice that for \( 2 \leq N \leq \infty \), the 95% lower limit for \( q \) varies only between 0.07 and 0.23.
Fig. 5.— Likelihood of $q_N$. In Figs. 1 and 4 the likelihood of $q$ is shown, where $q$ is the probability of winning on any one day. Here we show likelihoods of $q_N$, the unknown probability of winning on or before the $N$th day. This figure shows the effect of varying $N$. The information used to compute these likelihoods is that a gambler chosen at random from the group whose members have won within $N$ days has won on the first day (see Appendix, Eqs. 13, 14, 15). Translated this becomes; a planet chosen at random, from the group of planets that has had biogenesis within $\Delta t(N) = N \times \Delta t_{biogenesis}$, has had biogenesis within $\Delta t_{biogenesis}$. As in the previous figure, if $N = 1$ we can say nothing meaningful about $q_1 (= q)$. However, even if $N = 2$, we can make a stronger statement about $q_2$ than we could about $q$: $q_2 > 0.13$ at the 95% confidence level. The 95% lower limit on $q_N$ increases dramatically as $N$ increases, constraining $q_N$ to be close to 1. The ability to extract a useful constraint even if $N$ is low reduces the influence of the selection effects discussed in Section 3.
5. Appendix: Likelihood Computations

Let the unknown but constant probability of winning a daily lottery be \( q \). Given the information that a gambler who buys one ticket each day for \( n \) days, lost on the first \( n - 1 \) days, and won on the \( n \)th day, we can compute the likelihood function for \( q \) (probability of the data, given the model \( q \)):

\[
L(n; q) = (1 - q)^{n-1} q. \tag{8}
\]

This is equal to the fraction of all gamblers who first experienced \((n - 1)\) losses and then won. Given only the information that the gambler won at least once on or before the \( n \)th day, the likelihood function for \( q \) is:

\[
L(\leq n; q) = 1 - (1 - q)^n. \tag{9}
\]

This is equal to the fraction of all gamblers who have won at least once on or before the \( n \)th day. Given the information that a group of gamblers have won at least once on or before the \( N \)th day, and that a gambler chosen at random from this group won at least once on or before the \( n \)th day (\( n \leq N \)), the likelihood of \( q \) is:

\[
L(\leq n, \leq N; q) = \frac{L(\leq n; q)}{L(\leq N, q)} = \frac{1 - (1 - q)^n}{1 - (1 - q)^N} \tag{10}
\]

Out of all the gamblers who have won on or before the \( N \)th day, this is the fraction who have won on or before the \( n \)th day. Notice that as \( N \to n \) the likelihood of low \( q \) increases and that if \( N = n \) the likelihood is the same for all \( q \) (see Fig. 4). As \( N \to \infty \) we have \( L(\leq n, \leq N; q) \to L(\leq n; q) \) which yields the tightest constraints on \( q \). A normalized likelihood \( \mathcal{L} \) (or probability density) is defined such that \( \int_0^1 \mathcal{L}(q) dq = 1 \). Thus the renormalization conversion is,

\[
\mathcal{L}(x) = \frac{L(x)}{\int_0^1 L(x) dx}. \tag{11}
\]

The normalized likelihoods for Eqs. 8, 9 and 10 are plotted in Fig. 1 for the cases \( n = 3 \) and \( N = 12 \). The 95% confidence levels cited are Bayesian credible intervals based on a uniform prior for \( q \).

Although \( q \) is the probability of winning the lottery in one day, we would like to generalize and ask what is the probability \( q_N \) of winning the lottery within \( N \) days. In the analogous biogenesis lottery, \( q \) is the probability of biogenesis on a terrestrial planet with the same unknown probability of biogenesis as the Earth. The time window for biogenesis constrained by observations on Earth, \( \Delta t_{\text{biogenesis}} \), corresponds to one day for the gambler. However we would like to compute the likelihood of biogenesis after an arbitrary period of time \( \Delta t(N) = N \times \Delta t_{\text{biogenesis}} \). Let \( q_N \) be the probability of winning on or before the \( N \)th day:

\[
q_N = 1 - (1 - q)^N = L(\leq N, q), \tag{12}
\]

(see Eqs. 9 and 10). We would like to know the likelihood of \( q_N \) rather than limit ourselves to the likelihood of \( q \) (= \( q_1 \)). Suppose the information is the same that was available to compute
$L(\leq n, \leq N; q)$. That information is: a randomly chosen gambler from the group of gamblers who have won after $N$ days, won on or before the $n$th day. We want $L(\leq n, \leq N; q_N)$. The relationship between the likelihood of $q_N$ and the likelihood of $q$ is

$$L(\leq n, \leq N; q_N) dq_N = L(\leq n, \leq N; q) dq$$

(13)

which, with $dq_N/dq = N(1 - q)^{N-1}$ (from Eq. 12) becomes

$$L(\leq n, \leq N; q_N) = \frac{L(\leq n, \leq N; q)}{dq_N/dq} = \frac{L(\leq n, \leq N; q)}{N(1 - q)^{N-1}},$$

(14)

(15)

which is plotted in Fig. 5 and has, as expected, relatively larger likelihoods for larger values of $q_N$. 