The most conservative explanation of the current cosmological data is that of cold dark matter. That is, the equation of state is: 

\[ p = 0 \]  

and the equation of cosmological evolution is: 

\[ \frac{\dot{a}}{a} = H_0 \]  

where \( H_0 \) is the Hubble constant. This model is consistent with the observation that the universe is flat. 

To explore the possibility of dark energy, which is a repulsive force, we consider a field with the equation of state: 

\[ p = -\rho \]  

where \( \rho \) is the energy density. This model is consistent with the observation that the universe is accelerating. 

The structure of this field can be understood by considering the equations of motion. A rolling Madre de Dios field will have a constant density \( \rho \) and mass \( m \) described by the Lagrangian: 

\[ L = -\frac{1}{2} \dot{\phi} \dot{\phi} - V(\phi) \]  

where \( V(\phi) \) is the potential energy. The field configuration in scalar field models will be described by the potential \( V(\phi) \). 

In this paper, we examine the possibility of dark energy arising from string theory as a rolling Madre de Dios field. The equation of state for dark energy is then: 

\[ p = -\frac{\rho}{3} \]  

which is consistent with the observation that the universe is accelerating. 

Can the clustered dark matter and the smooth dark matter arise from the same scalar field configurations in scalar field models? The clustered dark matter has a different equation of state: 

\[ p = -\frac{\rho}{3} \]  

This model predicts a relation between the cluster mass and the scalar field mass. We explore the possibility of clustered dark matter and smooth dark matter arising from the same scalar field configurations in scalar field models. 

The clustered dark matter and the smooth dark matter are described by the potential \( V(\phi) \). The equations of motion are: 

\[ \ddot{\phi} = -\frac{\partial V}{\partial \phi} - \frac{2}{\rho} \frac{\dot{\rho}}{\dot{\phi}} \]  

The cluster mass is related to the scalar field mass by: 

\[ M_{\text{cluster}} = \int V(\phi) d\phi \]  

The potential for the scalar field is: 

\[ V(\phi) = \frac{1}{2} m^2 \phi^2 + V_0 \]  

The scalar field mass is: 

\[ m = \sqrt{V(\phi)} \]  

The equation of state for dark energy is: 

\[ p = -\frac{\rho}{3} \]  

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\[ M_{\text{cluster}} = \int V(\phi) d\phi \]  

The potential for the scalar field is: 

\[ V(\phi) = \frac{1}{2} m^2 \phi^2 + V_0 \]  

The scalar field mass is: 

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This model is consistent with the observation that the universe is accelerating. 

We explore the possibility of clustered dark matter and smooth dark matter arising from the same scalar field configurations in scalar field models.
To examine this scenario in more detail, we will begin with the action which couples such a scalar field to gravity at low energies:

$$S = \int d^4x \sqrt{-g} \left( \frac{R}{16\pi G} - V(\phi) \sqrt{1 - \partial^\mu \partial^\nu \phi} \right). \tag{1}$$

where $\phi$ is the field and $V(\phi)$ is the potential. Though motivated from string-theoretic considerations, we shall take this action as the starting point and investigate its consequences without worrying about its origin. (In this spirit, we refer to $\phi$ as simply a scalar field, rather than as a tachyonic field). The Einstein equations are

$$R^i_k - \frac{1}{2} \delta^i_k R = 8\pi G T^i_k \tag{2}$$

where the stress tensor for the scalar field can be written in a perfect fluid form

$$T^i_k = (\rho + p) u^i u_k - \rho \delta^i_k \tag{3}$$

with

$$u_k = \frac{\partial_k \phi}{\sqrt{\partial^\mu \partial_\mu \phi}}; \quad u_k u^k = 1$$

$$\rho = \frac{V(\phi)}{\sqrt{1 - \partial^\mu \partial_\mu \phi}}$$

$$p = -V(\phi)\sqrt{1 - \partial^\mu \partial_\mu \phi}.$$ \tag{4}

The remarkable feature of this stress tensor is that it could be considered as the sum of two components (a) and (b) described in the first paragraph. To show this explicitly, we break up the density $\rho$ and the pressure $p$ and write them in a more suggestive form as

$$\rho = \rho_V + \rho_{DM}; \quad p = p_V + p_{DM} \tag{5}$$

where

$$\rho_{DM} = \frac{V(\phi) \partial^\mu \partial_\mu \phi}{\sqrt{1 - \partial^\mu \partial_\mu \phi}}; \quad p_{DM} = 0$$

$$\rho_V = V(\phi)\sqrt{1 - \partial^\mu \partial_\mu \phi}; \quad p_V = -\rho_V \tag{6}$$

This means that the stress tensor can be thought of as made up of two components — one behaving like a pressure-less fluid, while the other having a negative pressure.

If $V(\phi)$ decreases with $\phi$ and has a minimum at $V = 0$ as $\phi \rightarrow \infty$ then it is possible to obtain pressure-less dust solutions by taking the limit $V \rightarrow 0$, $\partial_\mu \partial^\mu \phi \rightarrow 1$ simultaneously and keeping the energy density finite in the $\rho_{DM}$ component. If this happens globally at all scales, then — in this limit — the scalar field will behave as pressure-less dust at all scales. In this limit $\rho_V$ will vanish. Linear perturbation analysis shows [5] that this component will cluster gravitationally somewhat similarly to dust-like particles. In this scenario, the scalar field will merely act as (yet another) candidate for dark matter [5]. (It may be noted that there are still some subtleties related to clustering properties, time-scales etc. which have to be sorted out. But we believe this is indeed possible. For example, some of the problems related to velocities of the condensate particles can be addressed by using solutions which are Lorentz boosted, as explained in [6].)

It is, however, unlikely that such a scenario will be cosmologically acceptable in the absence of another component (b) with negative pressure described in the first paragraph. Unless the clustering property of this scalar field is sufficiently different from that of matter with $p = 0$, we will need to still invoke a separate component to describe cosmological observations. More generally, if one assumes that the field $\phi$ has the same configuration at all length scales, then one would end up getting the same density-pressure relation (equation of state) at all scales. However, in the real universe, we know that the dynamics of structure formation and clustering at galactic scales is dominated by the pressure-less fluid component (dark matter), while at large scales, the dynamics of the expansion of the universe is governed by spatially averaged mean density of a pressure-less component and a smooth component with negative pressure. In order to understand these effects, we need to model the scalar field in such a manner that we get different equations of state at different scales. This is possible if we assume that the field $\phi$ has some sort of stochastic behaviour so that its properties at different scales can be obtained by carrying out an averaging over the corresponding scales.

To tackle this complicated issue, we shall define an average of any quantity $A(\phi(t, \mathbf{x}))$ over a length scale $R$, such that the averaged quantity describes the behaviour of the field at that length scale. (This is a fairly standard practice in the study of structure formation; see, for example, chapter 5 of [7].) The average of $A(\phi)$ over a length scale $R$ is defined by smoothing it with a window function $W_R$. Mathematically, this is expressed as

$$A(R) \equiv \langle A(\phi) \rangle_R = \int \frac{d^3k}{(2\pi)^3} A_k(\phi) W_R(\mathbf{k}),$$

$$A_k(\phi) = \int d^3x A(\phi(\mathbf{x})) e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{7}$$

where $W_R(\mathbf{k}) \propto \exp(-k^2 R^2/2)$ if the window function can be taken to be Gaussian, say. In this case, the behaviour at a scale $R$ will be described by an average potential $\tilde{V}_R(\phi)$ obtained by eliminating $R$ between the average of potential $\tilde{V}(\phi)$ and the average of field $\phi(\mathbf{x})$ when all the average quantities are obtained using the same window function. In such a description, $\phi$ will sample different parts of $V(\phi)$ at different scales and it is possible to have different equations of state at small and large scales.

To see how it works, consider a simple case in which the field configuration evolves as

$$\phi(t, \mathbf{x}) = A(\mathbf{x}) t + \frac{f(\mathbf{x})}{t^3}, \tag{8}$$
which is a simple generalization of the evolution described in some of the previous works (see, e.g. [3], [8], [9], [10]). When averaged over a length scale \( R \) we obtain an effective field

\[
\delta(t, R) = A(R)t + \frac{f(R)}{R^2}
\]

(9)

The dependence of \( A(R) \) and \( f(R) \) on \( R \) will determine the behaviour of the field at different scales. The time dependence of the second term is appropriate if the effective potential at scalar \( R \) behaves as

\[
\overline{V}_R(\delta(t, R)) = V_0 \left( \frac{\phi}{\phi(0, R)} \right)^2
\]

(10)

which was considered earlier in [8], [11]. For a different potential, the time dependence will be different but in general for \( t \gg 1 \), the second term will be small compared to the first. [For example, if the potential has the form \( \overline{V}_R(\phi) \propto \exp(-\delta/\phi) \), the appropriate form of the second term would be \( f(R) \exp(-2t) \).] We shall now show that for a particular choice of \( A(R) \), we shall be able to produce expected behaviour of the equation of state at large as well as galactic scales.

At small scales, evolution could have proceeded to the asymptotic limit so that \( V \to 0 \), \( \frac{\delta}{\phi} \to 0 \) and a dust-like component prevails, which would require \( A(R) \to 1 \). Then we get for the average field

\[
\sqrt{1 - \delta^2} \approx 1 - \frac{\delta^2}{2f(R)} + O \left( \frac{1}{R^2} \right).
\]

(11)

Thus, at these scales, in the limit \( t \to \infty \), we have

\[
\rho_{DM} \approx \frac{V_0 \phi_0^2}{\sqrt{f(R)}} \quad \rho_V \approx 0.
\]

(12)

This means that the dynamics of galactic scales is dominated by the pressure-less component, whose energy density is independent of time [3, 5]. This resembles the non-interacting dark matter, which can cluster and is crucial for structure formation in the universe. The time dependence of the second term in the right hand side of (8) was chosen so as to make the energy density \( \rho_{DM} \) independent of time. In a more general scenario, this energy density will be time dependent and will represent the standard growth of structure in the dust-like component in an expanding universe.

Let us now turn into large scales to study the expansion of the universe. Since the fluctuations are likely to decrease with the averaging scales, \( \phi(R) \) will be a decreasing function of \( R \) and we expect \( A(R) \) to have a value less than unity at large scales. Taking \( \phi(R) = A(R) = \text{constant} \), and \( V = V_0 \phi_0^2 A(R)^{-1} \), one can find consistent set of solutions for an \( \Omega = 1 \) FRW model with a power law expansion \( a(t) \propto t^n \), where (see [8] for a description of this solution):

\[
\phi(t) = \sqrt{\frac{2}{3} n} t + b_0; \quad V(t) = \frac{3n^2}{8 \pi G} \sqrt{1 - \frac{2}{3n} t^2}
\]

(13)

with \( b_0 \) being some constant. Our model reproduces the correct behaviour expected at large scales, provided we identify

\[
A(R) = \sqrt{1 - \frac{2}{3n} R^2} \quad V_0 \phi_0^2 = \frac{n}{4 \pi G} \sqrt{1 - \frac{2}{3n} R^2}
\]

(14)

Thus the average value of \( \phi \) being different at different scales allows the possibility of the same scalar field exhibiting different equations of state at different scales. The rate of expansion of the universe is essentially determined by \( A(R) \) at the larger scales.

Since the same physical entity provides the dark matter at all scales in this scenario, one certainly expects a relation between the energy densities contributed by dark matter (\( \Omega_{DM} \)) and dark energy (\( \Omega_V \)). In our model, the energy densities for the two components are given by

\[
\rho_{DM} \approx \frac{1}{\sqrt{1 - A(R)^2/2}} \quad \rho_V \approx \frac{V_0 \phi_0^2}{A(R)^2/2} = \frac{n^2}{8 \pi G} \left( 1 - \frac{2}{3n} \right) \frac{1}{R^2}
\]

(15)

(It may be necessary to choose the value of \( V_0 \phi_0^2 \) in a particular range to match the values of the energy densities we observe today. This could be considered a fine tuning of the parameters, which we need to resort to at this stage in the absence of a more fundamental understanding of the scalar field. It is no worse or better than the fine tuning which is required in any other model for dark energy.) However, the ratio of the energy densities \( \rho_V / \rho_{DM} \) is independent of time, and is related to the mean value of the scalar field at large scales by

\[
\frac{\rho_V}{\rho_{DM}} = \frac{1}{A(R)^2} - 1.
\]

(16)

In fact, a similar equation holds for the ratio of the two components at all scales. As one proceeds from smaller to larger scales, the dark matter contribution decreases and the dark energy contribution increases.

This result can be converted into a clear prediction for cosmology by expressing the above equation in terms of the rate of expansion \( n \):

\[
n = \frac{2}{3} \left( 1 + \frac{\rho_V}{\rho_{DM}} \right).
\]

(17)

For the values accepted at present \( \rho_V / \rho_{DM} \approx 2 \), we get \( a(t) \propto t^{n \approx 2} \). Such a rate of growth is consistent with supernova observations. (The age of the universe in any accelerating model [with \( \Omega_{st} = 1, a(t) \propto t^n, n > 1 \]) will be \( t_0 \approx n / H_0 \), which is higher than the conventional models with \( t_0 \approx 1 / H_0 \). Any model which agrees with the
SN observations and has entered an accelerating phase in the recent past will have this feature and our model with $n = 2$ is no different.) This relation between (i) the amounts of dark matter and dark energy present in the universe and (ii) the expansion rate is potentially testable by observations. It may be stressed that in our model, the evolution of the single scalar field governs the time dependence of both $\rho_M$ and $\rho_V$. This is equivalent to saying that there is interaction and energy exchange between the two components and the energy is not conserved locally for the dark matter and dark energy components separately (which would imply $\rho_M \propto a^{-3}$ and $\rho_V = \text{constant}$).

Incidentally, it may be possible to put constraints on $n$ from CMB observations as well. The pressure term in the linear perturbation equation in this model has a factor $(1 - \dot{a}^2 k^2)$ where $k$ is the wave number [5]. For the solution (13), this factor is $[1 - (2/3n)k^2]$ and the standard results can be used with a rescaling of $k$. But since the angular scales of features in CMB anisotropy depends on this rescaling, it will lead to an $n$ dependent rescaling of Doppler peaks etc. [12]. Hence, CMB observations can provide another constraint on $n$.

The really serious test of the model will arise from the non-linear small scale dynamics of the clustering and galaxy formation scenarios. This is a hard problem which we have not studied in this paper; instead we have introduced an ansatz for the form of scalar field at different scales by hand. It is necessary to investigate this model further and show that the basic ansatz is correct and the details do not run into any contradiction. While this remains to be done, we consider it very attractive that the single entity can possibly exhibit different equations of state at different scales in the universe. Such a scenario has auras (for example, for CMB observations [12]) which have not been explored in conventional cosmology before.

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[4] This interpretation may be of some historic/pedagogical value. Generalizing the non-relativistic particle Lagrangian $L_{\text{kin}} = (1/2)g^a \phi^a - V(\phi)$ by changing $g(\phi)$ to field $\phi(t, x)$ will lead to standard scalar field theory with a potential $V(\phi)$, while generalizing the relativistic particle Lagrangian leads to the theory we are studying in the paper. Historically, one proceeded from nonrelativistic classical mechanics to relativistic quantum mechanics and attempted to generalize the Schrödinger wave equation to relativistic wave equations. Instead, if one had proceeded from nonrelativistic classical mechanics to relativistic classical mechanics and upgraded the $g$ to a field, one would have naturally led to this Lagrangian. We do not know whether such an attempt was ever made in the early days of quantum field theory. This gives another motivation to study such a scalar field independent of its string-theoretic origin.


[6] A. Sen, hep-th/0204143 [see the discussion after equation (29)].


