Non–Gaussian Signatures in the Cosmic Background Radiation
from Warm Inflation

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Abstract

We calculate the bispectrum of the gravitational field fluctuations generated
during warm inflation, where dissipation of the vacuum potential during in-
flation is the mechanism for structure formation. The bispectrum is non–zero
because of the self–interaction of the scalar field. We compare the predictions
with those of standard, or ‘supercooled’, inflationary models, and consider
the detectability of these levels of non–Gaussianity in the bispectrum of the
cosmic microwave background. We find that the levels of non–Gaussianity
for warm and supercooled inflation are comparable, and over–ridden by the
contribution to the bispectrum due to other physical effects. We also con-

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clude that the resulting bispectrum values will be undetectable in the cosmic microwave background for both the MAP and Planck Surveyor satellites.

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I. INTRODUCTION

The most commonly adopted model of the early Universe pictures the large scale structure present in the Universe today to be seeded by small scale fluctuations in the matter distribution of the primordial universe during an inflationary phase. In order to realize an inflationary regime, the generic dynamical model is based on a single scalar field, often termed the inflaton. The homogeneous, zero-mode component of this field is pictured to roll on a ultraflat potential, thereby sustaining a large potential energy and negligible kinetic energy, which are the necessary conditions for realizing inflation. These models assume there is no radiation production during the inflation period, with any radiation present prior to inflation rapidly diluting away. We refer to this picture as supercooled inflation, describing specifically the thermodynamic state of the universe during inflation. In this picture, the supercooled inflation phase is separated from the radiation–dominated regime by a brief reheating period, in which vast amounts of radiation are rapidly produced.

In supercooled inflation the initial seeds of density perturbations result from quantum fluctuations of the inflaton field. To a good approximation, the fluctuations have a Gaussian distribution and produce a nearly scale–invariant spectrum. A perfect Gaussian distribution would imply that the density perturbations have no connected correlations higher than the 2–point correlation function in real and Fourier space. However, the self–interaction of the inflaton field is known to produce non-zero, but extremely small, non–Gaussian effects, and there have been predictions calculated of these effects [1] and their detectability in the distribution of the cosmic microwave background (CMB) fluctuations [2,3]. Other structure formation scenarios, such as the class of defect models, and multiple–field inflation models, generally give larger deviations from Gaussianity.
There are many ways of testing the Gaussian hypothesis, such as the genus and Euler-Poincaré statistic [4–7], studies of tensor modes in the CMB [8], excursion set properties [9,10], peak statistics [11–14] and wavelet analyses (e.g. [15–17]). Then there are the set of higher-order correlation functions, such as the three-point function (e.g. [18–20,1]), the bispectrum [21–23], and the trispectrum [24]. A significantly non-Gaussian signal in the CMB sky, measured from the data of the Cosmic Background Explorer (COBE) Differential Microwave Radiometer (DMR) satellite instrument, launched in 1989, has been claimed [23], but doubts have been cast on this as a significant primordial signal, and more recent papers using this data [24–26] and, on a smaller angular scale, the MAXIMA balloon experiment data of 1998 [27,28] find results largely consistent with Gaussianity.

Gangui et al. [1] calculated predictions of the bispectrum for several variants of the supercooled inflationary scenario. In the limit of no instrument or sampling noise, the minimum variance on CMB data is the cosmic variance [29] – arising from fact that our Universe could only be one of a Gaussian ensemble. Gangui et al. [1] found the values for skewness for the bulk of their single field models to be considerably smaller in magnitude than that resulting from cosmic variance. The skewness is, however, only a single statistic which is part of a much wider class – the three-point correlation function. There is the prospect of doing much better by using the whole class or, equivalently, its harmonic counterpart, the bispectrum.

COBE had an angular resolution of 7°. For COBE it has been possible to analyse all the modes of the bispectrum up to the resolution limit [30,23]. The Planck Surveyor satellite is planned for launch in 2008. It will have an angular resolution of down to 5 arcminutes. The Microwave Anisotropy Probe (MAP) is operating now, with a resolution of 12.6’. Both of these experiments allow vast numbers of bispectrum modes to be analysed in principle, and it is interesting to see if either could distinguish warm inflation from supercooled inflation on the basis of the bispectrum.

In this paper we predict the form of the fluctuations and quantify the non-Gaussianity, using the bispectrum as a measure, for warm inflation dynamics. Warm inflation [31] differs
from the standard, supercooled picture of inflationary cosmology in that the process of radiation production becomes an important constituent of the theory. In particular, radiation production occurs concurrently with inflationary expansion and the presence of this radiation influences the seeds of density perturbations.

The warm inflation picture of inflationary dynamics is a comprehensive set of possible interactions between fields during inflation. In this picture no a priori assumptions are made about multi-field interactions, thus particle production, during the inflationary epoch. As such, the warm inflation picture makes explicit that the thermodynamic state of the universe during inflation is a dynamical question. In particular, the supercooled inflation emerges as one limiting case in which the interactions are negligible. More commonly dissipation is possible in warm inflation with a resulting density of radiation present during inflation [31–33]. Here both strong [31–33] and weak [32,34,35] dissipative regimes have been examined which offer several variants to the basic picture.

A variety of warm inflation models have been developed at a phenomenological level [33,34,36,37]. From this it has generally been understood that warm inflation can solve the basic cosmological puzzles of horizon, flatness and density fluctuations. However, up to now no study has been made on the degree of non–Gaussianity typically emerging in warm inflation models and it is important to quantify such effects.

In this paper, a general methodology is developed for computing non–Gaussian effects in warm inflation scalar field models. The statistic we use to quantify the resulting non–Gaussianity in the cosmic microwave background predicted for the case of warm inflation models is the bispectrum. This formalism is then applied to the $\lambda\phi^4$, $\lambda\phi^3$ and $m^2\phi^2$ models for the strong dissipative regime of warm inflation in the interest of discovering whether these effects are comparable, or indeed distinguishable, from the predictions of supercooled inflation.
II. WARM INFLATION DYNAMICS

The equation of motion for the zero mode of the scalar inflaton field, $\phi$, in general has the form

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma \dot{\phi} + V'(\phi) = 0,$$

where the overdots represent time derivatives, $H = \frac{\dot{R}}{R}$ is the Hubble parameter, and $R(t)$ is the cosmic expansion factor. The necessary condition for inflation is domination of the inflaton potential energy over all other energy components in the universe. This is achieved by requiring the inflaton potential to have sizable magnitude and be very flat. The flatness of the potential allows slow roll motion of $\phi$, so that inflaton kinetic energy becomes negligible with the effect that the $\ddot{\phi}$ term can be dropped from the equation of motion. In supercooled inflation dissipative effects, which in Eq.(1) are symbolically expressed through the term $\Gamma \dot{\phi}$, are assumed negligible during the inflation period and only emerge during the subsequent reheating period. On the other hand, the basic observation of warm inflation is that inflationary conditions remain energetically possible even in the presence of a sizable radiation component. Thus in warm inflation dissipation remains active during inflation, with the simplest representation of these effects being the form in Eq.(1). The resulting evolution equation for the inflaton in warm inflation therefore is

$$\frac{d\phi}{dt} = -\frac{1}{3H + \Gamma} \frac{dV(\phi)}{d\phi}$$

where similar to supercooled inflation, the slow-roll condition is required, $(3H + \Gamma)|\dot{\phi}| \gg |\ddot{\phi}|$.

The presence of dissipation implies throughout the inflation period radiation is produced from conversion of vacuum energy. In the context of Friedmann cosmology, the stress energy conservation equation consisting of a radiation, $\rho_r$, and a vacuum energy component, $\rho_v$, is

$$\dot{\rho}_r(t) = -4\rho_r(t)H - \dot{\rho}_v(t).$$

If there were there no dissipation, then $\dot{\rho}_v = 0$ and the radiation component would be rapidly red–shifted away as $\rho_r \sim e^{-4Ht}$. However with dissipation, radiation is being produced
continuously from conversion of scalar field vacuum energy. For dissipation of the form in Eq. (1), \(-\dot{\rho}_v = \Gamma \dot{\phi}^2\).

Note that in general the conversion of vacuum energy into radiation will result in some type of reaction back upon the scalar field, with the \(\Gamma \dot{\phi}\) term being the simplest phenomenological possibility. Attempts to obtain first principles scalar field evolution equations for warm inflation have obtained dissipative effects which in general are temporally nonlocal although limits also have been obtained in which the effects are of the local form in Eq. (1) [31,38,42].

The production of radiation during inflation in general will influence the seeds of density fluctuations. In particular, if the temperature during inflation is bigger than the Hubble parameter, \(T > H\), then structure formation can be significantly affected by the thermal component. In a preliminary work leading to the development of the warm inflation scenario [32], it was shown that a tiny dissipative component \(\Gamma \gtrsim 10^{-5}H\) already is adequate to realize \(T > H\). Subsequently the phenomenology of both weak \(\Gamma < H\) [32,34] and strong \(\Gamma \geq H\) [33,38–40] dissipative regimes have been studied for warm inflation. More attention has been given to the strong dissipative regime. This primarily is because the main focus of first principles quantum field theory studies of warm inflation [38,39,41,40,42–44] have been in this regime, since it is the more difficult of the two, and once this regime is understood the weak dissipative regime easily would follow. In this paper, expressions will be obtained which apply to the strong dissipative regime, such that Eq. (2) reduces to \(\frac{d\phi}{dt} = -V' \Gamma\), and during inflation \(\dot{\rho}_v \sim 0, 4\rho_v H \sim -\dot{\rho}_v\).

In order to treat the fluctuations of the inflaton field \(\delta\phi(x, t)\), it is assumed that they are small and the full inflaton field is expressed as \(\phi(x, t) = \phi_0(t) + \delta\phi(x, t)\) with \(\phi_0\) being the homogeneous ‘background’ field, \(\delta\phi(x, t) \ll \phi_0(t)\). The equation of motion for the fluctuations of the inflaton field can be obtained by imposing a near–thermal–equilibrium, Markovian approximation, which therefore implies the fluctuation–dissipation theorem is applicable. From this the equation of motion for the full inflaton field emerges as
\[
\frac{d\phi(x, t)}{dt} = \frac{1}{\Gamma} \left[ e^{-2Ht} \nabla^2 \phi(x, t) - V'(\phi(x, t)) + \eta(x, t) \right].
\] (4)

Implementing the fluctuation–dissipation theorem immediately determines the properties of the noise. With respect to physical coordinates and in momentum space, these properties are

\[
\langle \eta \rangle = 0
\] (5)

\[
\langle \eta(k, t)\eta(k', t') \rangle = 2\Gamma T(2\pi)^3 \delta^{(3)}(k - k')\delta(t - t')
\] (6)

III. THE STATISTICS OF WARM–INFLATION PERTURBATIONS

A. The Predictions and Properties of Gaussian Fields

Single field inflation models broadly predict Gaussian primordial density fluctuations. Multiple field inflation models may lead to a non–Gaussian distribution (e.g. chi–squared). When second–order effects are taken into account, however, there are corrections to these general predictions. There is a resulting non–Gaussian signal in the CMB, the magnitude of which varies depending upon the self-interaction of the inflaton field [19,46,1].

The statistical properties of a Gaussian field with mean zero are fully contained in its power spectrum or its two–point correlation function in real space. Higher order correlations can be expanded in terms of these quantities, so correlation functions of even order of the distribution can be written as products of two–point functions, and correlations of odd–order can be written in terms of products of two–point functions and the expectation value of the field. As there are no connected correlations over the two–point, all odd order correlation functions of a multivariate Gaussian distribution with zero mean are equal to zero. For a non–Gaussian field, the higher–order connected correlation function can be non–zero.

The three–point correlation function of the density perturbation distribution in Fourier space, otherwise known as the bispectrum [21–23,1] is the quantity we have chosen to evaluate
as our measure of the non–Gaussianity generated by warm inflation. This quantity translates to the harmonic bispectrum of the CMB.

B. The Warm Inflation Bispectrum

We will begin by expanding Eq.(4) in order to obtain the evolution equations up to second order in the fluctuations \( \delta \phi(x, t) = \delta \phi_1(x, t) + \delta \phi_2(x, t) \), where \( \delta \phi_1 = \mathcal{O}(\delta \phi) \) and \( \delta \phi_2 = \mathcal{O}(\delta \phi^2) \). All calculations that follow will be in momentum space, and with respect to the physical versus comoving momenta, where recall

\[
k_{\text{phys}} = k_{\text{com}} e^{-Ht}.
\] (7)

Hereafter for physical momenta, the notation will have no subscripts \( k_{\text{phys}} \equiv k \), and magnitudes will be denoted without boldfacing as \( k \equiv |k| \). The choice of physical coordinates arises since we are interested in the evolution of the inflaton mode while they are sub–horizon scale. During this time, the dominant effect on the modes is from the high temperature heat bath, and these effects are more conveniently analyzed in physical coordinates. When evaluating the equation of motion for the fluctuations, the time dependence of the physical modes will be treated adiabatically with respect to the characteristic macroscopic time scale, the Hubble time \( \sim 1/H \). Thus the evolution equations to be written for the inflaton modes will only be valid over a time interval \( \sim 1/H \) and a complete solution over longer time for a given mode can be obtained by piecewise construction. Thus the equations of motion of the first and second order fluctuations over a time period \( \sim 1/H \) are

\[
\frac{d}{dt}(\delta \phi_1(k, t)) = \frac{1}{\Gamma} \left[ -k^2 \delta \phi_1(k, t) - V''(\phi_o(t)) \delta \phi_1(k, t) \right] + \eta(k, t)
\] (8)

\[
\frac{d}{dt}(\delta \phi_2(k, t)) = \frac{1}{\Gamma} \left[ -k^2 \delta \phi_2(k, t) - V''(\phi_o(t)) \delta \phi_2(k, t) \right] - \frac{1}{2} V'''(\phi_o(t)) \delta \phi_1(k, t)^2.
\] (9)
where \( \delta \phi (k, t) = \delta \phi_1 (k, t) + \delta \phi_2 (k, t) \) is the inflaton mode with physical momentum \( k \) at cosmological – corresponding to the homogeneous background – time \( t \). The properties of the noise are given in Eqs.(5,6). We also assume here that the evolution of scalar field fluctuations can be studied in a particular gauge where metric perturbations can be neglected compared with those of the scalar field itself. The latter assumption, originally made in Ref. \([19]\), allows one to focus on the computation of those non–Gaussian features which are directly produced by non–linearities (i.e. self–interactions) of the scalar field itself, rather than by their backreaction on the underlying geometry. As we will see, this approach will give rise to a level of non-Gaussianity which is comparable to that obtained in Ref. \([1]\), where backreaction effects were simply modelled through a local modification of the Hubble expansion rate during inflation.

Dividing cosmic time into successive time intervals of order \( 1/H, t_n - t_{n-1} = 1/H \), the solutions of Eqs. (8) and (9) for \( t_{n-1} < t < t_n \) are respectively

\[
\delta \phi_1 (k, t) = A(k, t - t_{n-1}) \int_{t_{n-1}}^t dt' \eta(k, t') A(k, t' - t_{n-1})^{-1}
\]

\[
+ A(k, t - t_{n-1}) \delta \phi_1 (k e^{-H(t_n-t_{n-1})}, t_{n-1}) \tag{10}
\]

\[
\delta \phi_2 (k, t) = A(k, t - t_{n-1}) \int_{t_{n-1}}^t dt'
\]

\[
B(t) \left[ \int \frac{d^3 p}{(2\pi)^3} \delta \phi_1 (p, t') \delta \phi_1 (k - p, t') \right] A(k, t' - t_{n-1})^{-1}
\]

\[
+ A(k, t - t_{n-1}) \delta \phi_2 (k e^{-H(t_n-t_{n-1})}, t_{n-1}), \tag{11}
\]

where

\[
A(k, t) = \exp \left[ - \int_{t_0}^t \left( \frac{k^2}{\Gamma} + \frac{V''(\phi_o(t'))}{\Gamma} \right) dt' \right] \tag{12}
\]

\[
B(t) = - \frac{V'''(\phi_o(t))}{\Gamma}. \tag{13}
\]
In both solutions Eqs. (10) and (11), the second term on the RHS are “memory” terms that reflect the state of the given mode at the beginning of the given time interval. The relevance of these memory terms leads to the important concept of freeze–out [40]. By definition of freeze–out, for $|k| \gtrsim k_F$ the memory terms damp away within a Hubble time and for $|k| \lesssim k_F$ they do not. To quantify this criterion, the freeze–out momentum $k_F$ is defined by the condition

$$\frac{k^2 + V''(\phi_o)}{H \Gamma} > 1$$

In general for warm inflation $V''(\phi_o) < \Gamma H$, so the above condition can be simplified to $k_F = \sqrt{\Gamma H}$. In supercooled inflation the freeze–out wavenumber would correspond to the Hubble scale, as the quantum fluctuation becomes classical on horizon exit. For warm inflation the fluctuations freeze in before horizon exit.

Freeze–out implies noteworthy features about the solutions. When $k > k_F$, since the memory terms are negligible, $\delta \phi_1, \delta \phi_2$ primarily are determined by the state of the environment nearby in time. Then, when $k < k_F$, $\delta \phi_1, \delta \phi_2$ are determined dominantly by their state at time of freeze–out. These two facts imply the time–slicing approach we used to solve for $\delta \phi_1, \delta \phi_2$ is well justified.

From the solutions Eqs. (10) and (11), we are interested in computing the three-point correlation function of the inflaton fluctuations at the largest observable scales, which in particular cross the horizon in the interval $\sim 50 - 60$ e-folds before the end of inflation. During this time interval, we will compute the defining parameters of the three–point correlation function, which as will be seen can be expressed in terms of a bispectrum that well approximates the Sachs–Wolfe regime, $l \approx 50$. Thus from the three-point function

$$\langle \delta \phi(\mathbf{k}_1, t_{60}) \delta \phi(\mathbf{k}_2, t_{60}) \delta \phi(\mathbf{k}_3, t_{60}) \rangle,$$

the amplitude and slope of the bispectrum are determined at time $t \approx t_{60}, 60$ e-folds before the end of inflation and for $\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3$ all all within a few e–folds of exiting the horizon.

The leading order contribution to this three-point correlation function comes from two first-order and one second-order fluctuation as
\[
\langle \delta \phi(k_1, t) \delta \phi(k_2, t) \delta \phi(k_3, t) \rangle = 
A(k_3, t - t_{60} - 1/H) \int_{t_{60} - 1/H}^{t_{60}} A^{-1}(k_3, t' - t_{60} - 1/H)B(t') 
\left[ \int \frac{dp^3}{(2\pi)^3} \langle \delta \phi_1(k_1, t_1) \delta \phi_1(p, t') \rangle \langle \delta \phi_1(k_2, t_2) \delta \phi_1(k_3 - p, t') \rangle \right] 
+ A(k_3, t - t_{60} - 1/H)\langle \delta \phi_1(k_1, t_{60}) \delta \phi_1(k_2, t_{60}) \rangle 
\delta \phi_2(k_3 e^{-1}, t_{60} - 1/H) 
+ (k_1 \leftrightarrow k_3) + (k_2 \leftrightarrow k_3) 
\] (15)

In this expression, since \(B(t')\) is slowly varying, it can be approximated as a constant. Similarly \(\delta \phi(k, t)\) can be fixed at its freeze–out value. The three-point function on the RHS arises from the memory term of \(\delta \phi_2\) in Eq. (11), since \(k < k_F\). In evaluating this quantity, first note the coefficient in front can be approximated as unity, \(A(k, t_{n-1} - t_{n-2}) \approx 1\). Furthermore, since all three momenta will overlap in the freeze–out region, the three-point correlation function at \(t_{60} - 1/H\) is approximately the same as at \(t_{60}\), and this property will repeat itself for the time interval

\[
\Delta t_F \equiv t_H - t_F \approx \frac{1}{H} \ln \left( \frac{k_F}{H} \right), 
\] (16)

where \(t_H\) represents the time at Hubble crossing of the smallest of the three inflation perturbation modes, and \(t_F\) represents the time when the last of the three wavevectors thermalizes. Thus Eq. (15) becomes

\[
\langle \delta \phi(k_1, t) \delta \phi(k_2, t) \delta \phi(k_3, t) \rangle \approx B(t_{60}) \Delta t_F 
\left[ \int \frac{dp^3}{(2\pi)^3} \langle \delta \phi_1(k_1, t_1) \delta \phi_1(p, t') \rangle \langle \delta \phi_1(k_2, t_2) \delta \phi_1(k_3 - p, t') \rangle \right] 
+ (k_1 \leftrightarrow k_3) + (k_2 \leftrightarrow k_3) \] . (17)

### C. Estimating the Magnitude of the Non–Gaussianity

It is possible to write a general expression for the bispectrum for slow roll, single field, supercooled inflation models as well as for the set of warm inflation models.
\begin{equation}
\langle \Phi(k_1)\Phi(k_2)\Phi(k_3) \rangle =
A_{\text{inf}} (2\pi)^3 \delta^3(k_1 + k_2 + k_3) \left[ P_\phi(k_1)P_\phi(k_2) + \text{perms} \right] \tag{18}
\end{equation}

The relation between the scalar field fluctuation and the gravitational field has the simple form [47]

\begin{equation}
\Phi(k) = -\frac{3}{5} \frac{H}{\dot{\phi}} \delta \phi(k), \tag{19}
\end{equation}

thus \( A_{\text{inf}} \) for a strongly dissipative warm inflation regime is

\begin{equation}
A_{\text{inf}}^{\text{warm}} = -\frac{10}{3} \left( \frac{\dot{\phi}}{H} \right) \left[ \frac{1}{H} \ln \left( \frac{k_F H}{\Gamma} \right) \right]. \tag{20}
\end{equation}

Comparative estimates for the non–Gaussianity of the cosmic microwave background can then be calculated using \( A_{\text{inf}} \) and the shape of the inflationary potential. To estimate the magnitude of \( A_{\text{inf}}^{\text{warm}} \), consider the model

\begin{equation}
V(\phi) = \frac{\lambda}{4!} \phi^4, \tag{21}
\end{equation}

in the region \( 0 < \phi < M \), where, fixing the origin of time at \( t = 0 \), initially \( \phi(0) = M \). Here the self–coupling constant \( \lambda \) is dimensionless. \( M \) sets a basic scale such as the Grand Unified scale \( \sim 10^{14}\text{GeV} \), although the final answer for \( A_{\text{inf}}^{\text{warm}} \) is independent of this scale.

The solution of the zero-mode evolution equation Eq. (2) for this case is

\begin{equation}
\phi_0(t) = \frac{M}{\left( \frac{\lambda M^2}{3!} t + 1 \right)^{1/2}}. \tag{22}
\end{equation}

In [33] it is shown the number of e-folds of inflation \( N_e \) for this quartic potential is

\begin{equation}
N_e \approx \frac{1}{2} \left( 1 + \frac{48\pi \Gamma^2}{3m_{\text{pl}}^2 \lambda} \right)^{1/2}. \tag{23}
\end{equation}

The fluctuations in the scalar field caused by thermal interactions with the radiation field are [40]

\begin{equation}
\delta \phi^2 = \frac{k_F T}{2\pi^2}. \tag{24}
\end{equation}
The temperature, \( T \), can be calculated using the relation \( \rho_r = (g_\ast \pi^2/30) T^4 \), where \( g_\ast \) is the number of relativistic fields \( \sim 150 \), taking into account the relation between the radiation energy density and the scalar field potential presented in Eq. (3). The CMB amplitude is given by

\[
\delta_H = \frac{2 H}{5 \phi} \delta \phi , \tag{25}
\]

where from COBE data it is measured to be \( \delta_H = 1.94 \times 10^{-5} \) [29,48,49].

Setting \( N_e = 60 \) and using Eqs. (22), (23) and (25), the value of \( \lambda \) is found to be

\[
\lambda = 7.2 \times 10^{-15} \tag{26}
\]

and this gives us all the quantities needed to evaluate \( A_{\text{inf}} \) for warm inflation, Eq. (20)

\[
A_{\text{warm inf}} = 7.44 \times 10^{-2} . \tag{27}
\]

The equivalent quantity to \( A_{\text{inf}} \) for supercooled inflation appears in Gangui et al. [1] as \( \Phi_3 \equiv A_{\text{inf}}^{\text{supercooled}} = 5.56 \times 10^{-2} \) for a quartic potential. A similar quantity, \( f_{NL} \), appears in [30] and a related quantity in [3], \( A_{\text{infl}} \). They are related as

\[
\Phi_3 \equiv A_{\text{inf}} \equiv 2 f_{NL} \equiv A_{\text{infl}} (2\pi)^6 . \tag{28}
\]

For this potential we see that the non–Gaussianity in the curvature in warm inflation models is comparable to that in supercooled inflation. Furthermore due to the slow-roll behavior, for general models with local behavior of the form \( \phi^q \) with \( 2 < q \leq 4 \), the value of \( A_{\text{warm inf}} \) will be within the same order of magnitude as Eq. (27). Note, however, that this is only the contribution arising from non–Gaussianity in the inflaton field. In addition, there is a contribution at the last-scattering surface arising from second-order gravitational perturbation theory. Even at this relatively early time, the nonlinear gravitational equations induce a contribution \( A_{\text{inf}}^{2\text{nd order}} = \mathcal{O}(1) \), [20,50–52,30,53,54], which is therefore the dominant source of non–Gaussianity in both warm inflation and supercooled inflation models. The prospects of measuring this are poor; the bispectrum of the microwave background radiation is the
most obvious possibility, and this has been investigated by [30]. Note that this assumes that the Hubble parameter is unchanged at horizon exit for all modes considered. The prospects are best for the Planck satellite [55], as this measures modes up to $\ell \sim 2000$, but even with the optimistic assumption that all foreground contaminants can be removed perfectly, the expected error on $f_{NL}$ is still 5. Since Silk damping causes the power to decline rapidly on scales smaller than the Planck beam, there is little prospect of a future, higher-resolution experiment being able to improve significantly on this. Similarly, polarisation measurements are unlikely to help, since most of the power is expected at $\ell \sim 100$, for which there are relatively few bispectrum coefficients.

**IV. CONCLUSIONS**

We have explored the evolutionary behaviour of gravitational field fluctuations generated by warm inflation, up to second order in the scalar field perturbations, for the particular case of strong dissipation. We have obtained predictions for the non–zero bispectrum of the gravitational perturbations due to the self–interaction of the inflaton field, and the resulting harmonic bispectrum for the Sachs–Wolfe region on the cosmic microwave background. Eq.(18) represents the gravitational bispectrum in a generic form, with the value of $A_{\text{inf}}$ resulting from the theory of the fluctuation generation mechanism. The CMB bispectrum can also be related to $A_{\text{inf}}$, provided that the potential is sufficiently flat that the Hubble parameter etc are approximately constant on horizon exit for all modes considered. We find that the inherently classical mechanism for the generation of fluctuations in warm inflation, which is palpably different from the corresponding mechanism for supercooled inflation, produce a level of non–Gaussianity of approximately the same magnitude, $A_{\text{inf}} \sim 10^{-2}$. This arises from non–Gaussianity in the inflaton field itself. The dominant contribution to the curvature bispectrum, however, comes from general relativistic second–order perturbation theory, which contributes an $A_{\text{inf}} \sim 1$, in both warm and supercooled inflation. Unfortunately this level of non–Gaussianity in the CMB bispectrum appears to be unobservable,
even with the Planck satellite. Conversely, any measure of primordial non-Gaussianity significantly in excess of $f_{NL} = 1$ would rule out both warm inflation and supercooled inflation, although some alternatives such as the curvaton model [56] or multi-field inflation models (e.g. [57] and refs. therein) could survive.

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