EFFECTIVE SCREENED POTENTIALS OF STRONGLY COUPLED SEMICLASSICAL PLASMA

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Abstract. The pseudopotentials of particle interaction of a strongly coupled semiclassical plasma, taking into account both quantum-mechanical effects of diffraction at short distances and also screening field effects at large distances are obtained. The limiting cases of potentials are considered.

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I. Introduction.

At the present time the study of strongly coupled plasmas properties is of great interest, it caused by fundamental investigations of astrophysical objects and inertial confinement fusion plasmas [1]-[6]. In such strongly coupled systems the collective (screening) and quantum-mechanical effects play an important role in studies of thermodynamic and kinetic properties of strongly coupled plasmas. Furthermore, in kinetic theory which is based on kinetic equations, there are problems with divergence of collision integrals at
big and small scattering angles. The one way to eliminate them is the using of the bare Coulomb potential, being cutted at large and short distances. On the other hand, it is known that the divergence at small angles appears as a result of not taking into account the plasma’s screening effects on interparticles scattering in the system. The other method of eliminating divergence at long ranges and correctly describing the properties of strongly coupled plasmas is to use the effective potentials of interaction of charged particles. These potentials take into account the influence of surrounding particles, it leads to the screening of external potential \( \varphi(r) \). Therefore, the strongly screened effective potential that takes into account three-particle correlations is obtained in [7]. Notice that this potential does not take into account quantum mechanical effects and describes the interaction between particles of classical dense plasma only.

In this paper the effective potential for semiclassical plasma is obtained. This potential contains quantum diffraction effects at short distances as well as screening effects for large distances.

II. The system parameters.

The fully ionized plasma consisting of electrons and ions is considered. Number density is considered in the range of \( n = n_e + n_i = 10^{20} - 10^{24} \text{cm}^{-3} \), and the temperature domain is \( 10^5 - 10^7 \text{K} \). The average distance between particles (or the Wigner-Seitz radius) is :

\[
a = \left( \frac{3}{4\pi n} \right)^{1/3},
\]

where \( n_i = Zn_e \) are number densities of ions and electrons. Other parameter characterizing the state of system is coupling parameter:

\[
\Gamma = \frac{(Ze)^2}{(ak_BT)},
\]

where \( T \) is plasma temperature, \( k_B \) denotes the Boltzmann constant, \( \Gamma \) is the ratio between average Coulomb interaction energy and thermal energy. For strongly coupled plasma
Density parameter is defined as

\[ r_s = a/a_B, \]

where \( a_B = \hbar^2/(m_e e^2) \) is Bohr radius, \( r_s \) increases with decreasing of density.

The degree of Fermi degeneracy for the electrons is measured by the ratio:

\[ \Theta = \frac{k_B T}{E_F} = 2\left(\frac{4}{9\pi}\right)^{2/3} \frac{2^{5/3} Z^{5/3} r_s}{\Gamma}, \]

where \( E_F \) is the Fermi energy of electrons. The condition \( \Theta \geq 1 \) corresponds to the state of weakly and intermediate degeneracy.

III. Pseudopotentials.

It is well-known that the electrical field of particles in plasma induces the density fluctuations of charges, i.e. the polarization effect is evolved. As a consequence, the opposite charged "cloud" around test charged particle is formed, it leads to screening of initial potential [8]. The relationship between the total (screened) potential and externally applied one is as follows:

\[ \Phi(q) = \frac{\varphi(q)}{\varepsilon(q)}, \]

(1)

where \( \Phi(q), \varphi(q) \) are Fourier transforms of the screened and external potentials, respectively. Fourier transformation is:

\[ \varphi(q) = \frac{4\pi}{q} \int_0^\infty r \varphi(r) \sin(qr) dr, \]

(2)

\( \varepsilon(q) \) is the dielectric function of static screening. In the random-phase approach for weakly degenerate plasma \( \varepsilon(q) \) can be derived from following relation [9]:

\[ \varepsilon(q) = 1 + \sum_\alpha \frac{4\pi n_\alpha \varphi(q)}{k_B T}, \]

(3)
In case of the Coulomb potential, we have:

\[ \varphi_c(q) = \frac{4\pi e^2}{q^2} \quad (4) \]

\[ \varepsilon_c(q) = \frac{q^2 + \kappa^2}{q^2} \quad (5) \]

where \( \kappa^2 = (4\pi n e^2)/(k_B T) \equiv 1/r_D^2 \) is the value that is inverse proportional to the square of Debye radius. Using Eqs. (1),(4),(5), Fourier transform of effective potential \( \Phi(q) \) can be obtained. The inverse Fourier transformation

\[ \Phi(r) = \frac{1}{2\pi^2 r} \int_{0}^{\infty} q \Phi(q) \sin(qr) dq, \quad (6) \]

gives well-known Debye-Hückel potential:

\[ \Phi_{DH}(r) = \frac{e^2}{r} e^{-\kappa r}. \quad (7) \]

Hence the divergence at the large distances is avoided by taking into account screening effects, but it still remains at short distances. However, it is known that taking into consideration the quantum-mechanical effects leads to the finite values of pseudopotential at short distance. This method is called the method of thermodynamic pseudopotential, and the effective temperature dependent pseudopotential is obtained as a result of correlation between the classical Boltzmann factor and the quantum-mechanical Slater sum [10]. In this framework the potential taking into consideration effects of diffraction has been derived for high densities and temperatures [11], [13],[12]

\[ \varphi_{\alpha\beta}(r) = \frac{Z_\alpha Z_\beta e^2}{r} (1 - e^{-r/\lambda_{\alpha\beta}}) \quad (8) \]

where \( \lambda = \hbar/\sqrt{2\pi \mu_{\alpha\beta} k_B T} \) is the thermal de Broglie wavelength of \( \alpha - \beta \) pair, \( \mu_{\alpha\beta} \) is the reduced mass of \( \alpha - \beta \) pair. Potential (8) is not screened at large distances. It raises the idea to obtain the effective potential on the basis of the dielectric response function as a result of screening the potential (8). Applying Eqs.(2),(3),(1) to (8) we have obtained:

\[ \varepsilon(q) = \frac{q^4 + q^2/\lambda_{\alpha\beta}^2 + \kappa^2/\lambda_{\alpha\beta}^2}{q^2(q^2 + 1/\lambda_{\alpha\beta}^2)}, \quad (9) \]
The pseudopotential $\Phi_{\alpha\beta}(r)$ can be restored from (6),(10):

$$
\Phi_{\alpha\beta}(r) = \frac{Z_\alpha Z_\beta e^2}{\sqrt{1 - 4\lambda_{\alpha\beta}^2\kappa^2}} \left( e^{\frac{-Ar}{r}} - e^{\frac{-Br}{r}} \right)
$$

where

$$
B^2 = \frac{1 + \sqrt{1 - 4\lambda_{\alpha\beta}^2\kappa^2}}{2\lambda_{\alpha\beta}^2},
$$

$$
A^2 = \frac{1 - \sqrt{1 - 4\lambda_{\alpha\beta}^2\kappa^2}}{2\lambda_{\alpha\beta}^2}.
$$

It is obvious that pseudopotential (11) is valid when $4\lambda_{\alpha\beta}^2\kappa^2 < 1$ (or $\lambda_{\alpha\beta} < \frac{r_D}{2}$), i.e. in the region of weakly degenerate plasmas.

Since potential (8) behaves as the Coulomb potential at large distances it is natural to suppose that dielectric function is defined by Coulomb’s relation (5). In this approach, using expression (5) for $\varepsilon(q)$ instead of (9), one can obtain the expression for effective potential:

$$
\Phi_{\alpha\beta}(r) = \frac{Z_\alpha Z_\beta e^2}{1 - \lambda_{\alpha\beta}^2\kappa^2} \left( e^{\frac{-\kappa r}{r}} - e^{\frac{-r/\lambda_{\alpha\beta}}{r}} \right)
$$

Let us consider the limiting cases of the expressions (11),(12).

Case 1. $\kappa \to 0 (r_D \to \infty)$. From the physics point of view it means that there are no screening effects. Then potentials (11) and (12) coincide with (8).

Case 2. In the absence of diffraction effects ($\lambda_{\alpha\beta} \to 0$), from (11) and (12) we have the potential (7).

Case 3. When both screening and diffraction effects are absent ($\lambda_{\alpha\beta} \to 0, \kappa \to 0$), pseudopotentials (11) and (12) coincide with the Coulomb potential.

Case 4. $\lambda_{\alpha\beta}\kappa \ll 1(\lambda_{\alpha\beta} \ll r_D)$. In this case pseudopotential (11) coincides with
potential (8) and limiting formula of pseudopotential (12) is as follows:

$$\Phi_{\alpha\beta}(r) = \frac{Z_{\alpha}Z_{\beta}e^2}{r}(e^{-\kappa r} - e^{-r/\lambda_{\alpha\beta}}).$$

(13)

Similar limiting expression have been derived in [14] on the basis of the linear chain of Bogolyubov’s equations in pair correlations approach by the micropotential taking into consideration diffraction and symmetry effects. It should be noted here that results of [14] are valid for weak coupled plasmas ( $\Gamma < 1$).

The pseudopotentials (11) and (12) as a function of a distance between particles for the different values of parameters $r_s$ and $\Gamma$ are presented on the figures. It is shown that both potentials coincide with the Debye-Hückel potential at $r \to \infty$ and they have finite value at $r \to 0$. Discrepancy between (11) and (12) grows with increasing of coupling parameter at fixed $r_s$ (Figs.1, 2 and 3) and also with increasing of density at fixed $\Gamma$ (Figs.3,4). Under the same conditions the discrepancy between limiting expression (13) and pseudopotentials (11), (12) grows too (Figs.5,6 and 7).

Consequently, pseudopotentials derived in this work take into account both quantum-mechanical effects of diffraction at short distances and also screening field effects at large distances between particles. Last time some authors have calculated potentials which take into consideration quantum-mechanical effects and collective events [14],[15],[16] by different methods. However, analytical expression for pseudopotential which is valid in wide region of $r_s$ and $\Gamma$ has not been yet obtained.

In conclusion it should be noted here that the expressions for pseudopotentials derived in this work are simple enough and can be easily used in analytical calculations and computer simulations in investigations of strongly coupled semiclassical plasma properties.
References

Figure captions

Figure 1: Effective potentials of the interaction between particles of a semiclassical hydrogen plasma. Solid line: formula (12); circles denote the formula (11); dashed line: potential (8); dashed-dotted line represents the Debye-Hückel potential.

Figure 2: The same as on Fig.1.

Figure 3: The same as on Fig.1.

Figure 4: The same as on Fig.1.

Figure 5: Effective potentials of the interaction between particles of a semiclassical hydrogen plasma. Solid line: formula (12); circles denote the formula (11); triangles represent the potential (13).

Figure 6: The same as on Fig.5.

Figure 7: The same as on Fig.5.
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Figure 5: Effective potentials of the interaction between particles of a semiclassical hydrogen plasma. Solid line: formula (12); circles denote the formula (11); triangles represent the potential (13).
Figure 6: The same as on Fig.5.
Figure 7: The same as on Fig.5.