Event-independence, collective-independence, EPR-Bohm experiment and incompleteness of quantum mechanics

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Abstract

We analyse notion of independence in the EPR-Bohm framework by using comparative analysis of independence in conventional and frequency probability theories. Such an analysis is important to demonstrate that Bell’s inequality was obtained by using totally unjustified assumptions (e.g., the Bell-Clauser-Horne factorability condition). Our frequency analysis also demonstrated that Gill-Weihs-Zeilinger-Zukowski’s arguments based on ”the experimenter’s freedom to choose settings” to support the standard Bell approach are neither justified by the structure of the EPR-Bohm experiment. Finally, our analysis supports the original Einstein’s viewpoint that quantum mechanics is simply not complete.

Preprint of R. Gill, G. Weihs, A. Zeilinger, M. Zukowski [1] stimulated the interest to the role of independence conditions in the EPR-Bohm framework. I recall that preprint [1] was published as the rigid critical reply to works of K. Hess and W. Philipp, see e.g. [2]. Here I do not consider Hess-Philipp arguments, but only their conclusion: in general we do not have Bell’s inequality, since the Bell-Clauser-Horne factorability condition (sometimes called locality condition):

\[
P(A, B'/a, b', \lambda) = P(A/a, \lambda)P(B'/b', \lambda)
\]  

(1)

can be violated in very natural models.

The main Gill-Weihs-Zeilinger-Zukowski’s counter-argument against Hess-Philipp’s model is the following one: ”However, they themselves have neglected the experimenter’s freedom to choose settings...” In fact, this is independence condition. Therefore it would be useful to provide general analysis of the role of independence conditions in the EPR-Bohm experiment.

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This analysis is deeply related to the very foundations of probability theory, namely Kolmogorov (measure-theoretical model, i.e. conventional model) and von Mises (frequency model, nonconventional, but very experimental). Some parts of such an analysis can be found in my preprints [3], [4] (the last one also contains critical analysis of Gill-Weihs-Zeilinger-Zukowski’s as well as Hess-Philipp’s arguments). Here we concentrate us to the role of independence condition.

Our frequency analysis demonstrates that the Bell-Clauser-Horne factorability condition should be violated simply due to dependence of corresponding collectives (random sequences) of hidden variables, HV. So there is no Bell inequality. Collective dependence is a consequence of original EPR-correlations in pairs of particles. Therefore, as it was originally claimed by Einstein, Podolsky and Rosen, [5], quantum mechanics is not complete. ”The experimenter’s freedom to choose settings” that was the cornerstone of Gill-Weihs-Zeilinger-Zukowski’s considerations does not change anything: corresponding collectives are still dependent.

In the frequency approach (von Mises, [6]) probability is defined as the limit of relative frequencies in a collective (random sequence) produced by some experimental device. Two collective are said to be independent if corresponding experimental devices works independently. This implies factorization of probabilities in the multi-collective created by combination of two independent collectives, see [6], [7] for the details.

In the measure-theoretical approach (Kolmogorov, [8]) probability is defined as an abstract (normalized) measure. Two events are said to be independent if the probability of their intersection is factorized.

Gill, Weihs, Zeilinger, Zukowski (as well as all others) use Kolmogorov independence - event independence and I shall use von Mises independence: collective independence. These are two very different notions. The reader can find in von Mises book [6] as well as in my book [7] examples in that factorization of probability can occur simply as the result of the play with numbers. This is event independence. In the opposite to event independence, collective independence is the physical notion. And the EPR experiment evidently demonstrated that collective independence is the right notion to describe real physical experiments.

We consider the general probabilistic scheme of the EPR-Bohm experiment.

There are two physical systems, \( U_{\text{left}} \) and \( U_{\text{right}} \), producing the settings of measurement devices, \( a \) and \( b' \), and there is third system, \( U \), producing corre-
lated particles. I want to underline (and the frequency analysis immediately shows this) that, despite the independence of $U_{\text{left}}$, $U_{\text{right}}$, and $U$, collectives produced by pairs $(U_{\text{left}}, U)$ and $(U_{\text{right}}, U)$ need not be independent. Moreover, they should be dependent due to the presence of the common physical system, namely $U$. So we can do what ever we want with $U_{\text{left}}$, $U_{\text{right}}$ and even with $U$. But the whole statistical structure of the experiment induces dependence of collectives.

In our private Email discussion R. Gill noticed: "But note, I am talking about independence between the physical system generating the settings (e.g., tossing coins), and the physical system which could accept either setting and then output the outcomes.

This is a different independence from the one you were talking about. I would say, that it is a reasonable physical assumption that physically independent subsystems of the world (a coin toss here, some photons there) exist. If you deny this then of course anything is possible. Do you deny the possibility of tossing a coin independently of sending a photon through a polarizer?"

I do not deny this possibility. Frequency analysis demonstrated that the main problem is the presence of the common physical system $U$ (producing correlated particles) in the left-hand side as well as the right-hand side collectives. Thus the freedom of experimenters playing with devices $U_{\text{left}}$ and $U_{\text{right}}$ does not destroy $U$-dependence.

We present the formal collective description of the model.

Let $\lambda_j, j = 1, 2, \ldots$ be the value of the HV for the $j$th pair of correlated particles $(\pi^1_j, \pi^2_j)$ produced at the instance of time $t_j = j$. We consider two sequences of pairs and a sequence of triples (three collectives):

\[
x_{\omega_{\text{left}}, \lambda} = \{ (\omega_{\text{left} 1}, \lambda_1), \ldots, (\omega_{\text{left} N}, \lambda_N), \ldots \},
\]

\[
x_{\omega_{\text{right}}, \lambda} = \{ (\omega_{\text{right} 1}, \lambda_1), \ldots, (\omega_{\text{right} N}, \lambda_N), \ldots \},
\]

and

\[
x_{\omega_{\text{left}}, \lambda, \omega_{\text{right}}} = \{ (\omega_{\text{left} 1}, \lambda_1, \omega_{\text{right} 1}), \ldots, (\omega_{\text{left} N}, \lambda_N, \omega_{\text{right} N}), \ldots \},
\]

where $\omega_{\text{left} j}$ and $\omega_{\text{right} j}$ are internal states of apparatuses $U_{\text{left}}$ and $U_{\text{right}}$, respectively.\(^2\) Due to the presence of the common parameter $\lambda$ in both collectives $x_{\omega_{\text{left}}, \lambda}$ and $x_{\omega_{\text{right}}, \lambda}$, they are not independent. Therefore the probability distribution of the collective $x_{\omega_{\text{left}}, \lambda, \omega_{\text{right}}}$ could not be factorized.

\(^2\)In this framework it is not important that the experimenters have the freedom of choice of experimental setting for devices $U_{\text{left}}$ and $U_{\text{right}}$. 
As we have already remarked, in fact, our frequency analysis gives strong probabilistic support to the original EPR-arguments. It seems that A. Einstein rightly pointed to incompleteness of quantum mechanics. This incompleteness is a consequence of the impossibility to describe the correlation \(HV\)\(\lambda\) in quantum formalism. Nevertheless, quantum formalism gives the right answer in the EPR-framework, since dependence of collectives is coded into corresponding quantum state. However, even by having the right answer in quantum formalism we could not explain the origin of this answer. This induces unusual explanations such as e.g. nonlocality.\(^3\) We remark that our paper could not be used as an argument against nonlocality. Quantum mechanics could be nonlocal. However, Bell’s inequality definitely could not be used as an argument in nonlocality story.

Finally, we remark that if there is no independence, in particular, the Bell-Clauenser-Horne factorability condition, then we should get modified Bell’s inequalities, see [9]-[11], instead of the original Bell-type inequalities. Inequalities that I have obtained in [9], [10] need not be violated in quantum theory.

I would like to thank R. Gill for intensive discussions that clarified my own views to the EPR-experiments.

REFERENCES


7. A. Khrennikov, Interpretations of Probability. VSP Int. Sc. Publish-

\(^3\)We also notice that in our frequency analysis it is really impossible to find even a trace of some nonlocality that is typically related to the EPR-experiment.

