Heavy to Light Meson Exclusive Semileptonic Decays in Effective Field Theory of Heavy Quarks

W.Y. Wang*, Y.L. Wu† and M. Zhong †

* Department of Physics, Tsinghua University, Beijing 100084, China
† Institute of Theoretical Physics, Academia Sinica, Beijing 100080, China

Abstract

We present a general study on exclusive semileptonic decays of heavy ($B$, $D$, $B_s$) to light ($\pi$, $\rho$, $K$, $K^*$) mesons in the framework of effective field theory of heavy quarks. The decays to pseudoscalar and vector light mesons can be systematically characterized by a set of wave functions which are independent of the heavy quark mass except for the implicit scale dependence. Form factors for these decays are calculated using the light cone sum rule (LCSR) method at the leading order of $1/m_Q$ expansion. The branching ratios of these decays and the extraction of the relevant CMK matrix elements are discussed. The light flavor SU(3) breaking effects are also explored.

Keywords: heavy to light, semileptonic decay, CKM matrix elements, effective field theory, light cone sum rule

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I. INTRODUCTION

Heavy meson (B and D) decays, both inclusive and exclusive, have long been an interesting subject in both experimental and theoretical studies. These decays play their special role in extracting the CKM matrix elements and probing new physics beyond the standard model (SM). The inclusive decays of heavy mesons are theoretically cleaner than exclusive decays, which makes them suitable to search for new physics effects deviating from the SM predictions. However, the inclusive decays are more difficult to measure in experiments. On the other hand, exclusive decays are cleaner in experimental measurements but more difficult in theoretical calculations as they require the knowledge of form factors, which contains long distance ingredients and has to be estimated via nonperturbative methods such as sum rules, lattice simulations or phenomenological models.

The heavy to light exclusive decays can be grouped as [1] semileptonic decays and rare decays. In this paper we focus on the exclusive semileptonic decays of $B$, $D$ and $B_s$ heavy mesons into light pseudoscalar and vector mesons $\pi$, $\rho$, $K$, $K^*$. The main difficulty in studying these decays is for evaluating the relevant form factors, which characterize the long distance effects in the hadronic transition matrix elements.

These decays have been analyzed by using sum rules in full QCD theory. Recently, the effective theory of heavy quarks has also been applied to discuss the heavy to light decays [2–6]. It has been shown that Heavy quark symmetry (HQS) greatly simplifies a lot of processes which involve heavy hadrons. It is expected that the effective field theory of heavy quarks may further help us to improve our understanding on heavy to light decays. For that purpose, we have investigated in Refs. [5,6] the $B \to \pi(\rho)l\nu$ decays within the framework of effective field theory of heavy quarks. The relevant form factors were calculated at the leading order of heavy quark expansion. The extracted values of $|V_{ub}|$ in those papers [5,6] were found to agree with each other and also agree with that obtained from sum rule calculations in full QCD. Our previous results have shown the reliability of the heavy quark expansion in the light cone sum rule calculations.

In this paper we shall apply the heavy quark expansion and light cone sum rule techniques to other heavy to light exclusive semileptonic decay channels and use the power of heavy quark flavor symmetry to reduce the number of independent form factors. For completeness, we will consider decays of both bottom and charm mesons with the final mesons including both nonstrange and strange ones. The later invokes the light flavor SU(3) symmetry and its breaking effects. Therefore, this paper will provide a more complete discussion on exclusive heavy to light meson decays. As a consequence, it may become useful for the extraction of relevant CKM matrix elements.

The paper is organized as follows: In section II, we first formulate the heavy to light transition matrix elements in the framework of effective field theory of heavy quarks, and then present the Light cone sum rules for the heavy flavor independent wave functions. Some of the formulae are found to have a more general meaning and similar to those presented in Refs. [5,6]. In reviewing them, we will pay our main attention to their relations and differences among different decay channels. Section III contains our numerical analysis on the heavy to light transition form factors. Based on the results of section III, we discuss in section IV the branching ratios and CKM matrix elements. A short summary is presented in section V.
II. WAVE FUNCTIONS AND LIGHT CONE SUM RULES

For convenience of discussions, we denote in this paper the light pseudoscalar and vector mesons as \( P \) and \( V \) respectively, and use \( M \) to represent the heavy mesons \( B, B_s \) and \( D \). Then the decay matrix elements and form factors can be written in a general form as follows

\[
\langle P(p) | \bar{q} \gamma^\mu Q | M(p + q) \rangle = 2 f_+(q^2) \rho^\mu + (f_+(q^2) + f_-(q^2)) q^\mu,
\]

\[
\langle V(p, \epsilon^*) | \bar{q} \gamma^\mu (1 - \gamma^5) Q | M(p + q) \rangle = -i (m_M + m_V) A_1(q^2) \epsilon^{\ast \mu} + i \frac{A_2(q^2)}{m_M + m_V} (\epsilon^* \cdot (p + q))
\]

\[
\times (2p + q)^\mu + i \frac{A_3(q^2)}{m_M + m_V} (\epsilon^* \cdot (p + q)) q^\mu - \frac{2V(q^2)}{m_M + m_V} \epsilon^{\mu \alpha \beta \gamma} \epsilon^*_\alpha (p + q)_\beta p_\gamma, \tag{2.1}
\]

where \( q \) in the currents (not to be confused with the momentum) represents light quarks (\( u, d \) or \( s \)), and \( Q \) denotes any heavy quark (\( b \) or \( c \)). In the effective theory of heavy quarks, these matrix elements can be expanded in powers of \( 1/m_Q \). The leading order matrix element in the \( 1/m_Q \) expansion can be simply expressed as the following trace formulae [5–8]

\[
\langle P(p) | \bar{q} \Gamma Q^+_v | M_v \rangle = -i \text{Tr} [\Omega(v, p) \Gamma M_v],
\]

\[
\langle V(p, \epsilon^*) | \bar{q} \Gamma Q^+_v | M_v \rangle = -i \text{Tr} [\Omega(v, p) \Gamma M_v] \tag{2.2}
\]

with

\[
\pi(v, p) = \gamma^5 [A(v \cdot p, \mu) + \hat{p} B(v \cdot p, \mu)],
\]

\[
\Omega(v, p) = L_1(v \cdot p) \epsilon^* + L_2(v \cdot p) \epsilon^* + [L_3(v \cdot p) \epsilon^* + L_4(v \cdot p) \epsilon^*] \hat{p} \tag{2.3}
\]

where \( \hat{p}^\mu = p^\mu / v \cdot p \). \( Q^+_v \) in eq.(2.2) is the effective heavy quark field variable introduced in the effective theory [9,10], which carries only the residual momentum \( k^\mu = p^\mu - m_Q v^\mu \) with \( v^\mu \) the heavy meson’s velocity. Correspondingly \( M_v \) in eq.(2.2) is the effective heavy meson state. It is related to the heavy meson state \( M \) in eq.(2.1) by the normalization of the hadronic matrix elements as follows [10]:

\[
\frac{1}{\sqrt{m_M}} \langle \pi(p) | \bar{q} \Gamma Q | M \rangle = \frac{1}{\sqrt{\tilde{\Lambda}_M}} \{ \langle \pi(p) | \bar{q} \Gamma Q^+_v | M_v \rangle + O(1/m_Q) \}. \tag{2.4}
\]

with \( \tilde{\Lambda}_M = m_M - m_Q \) the binding energy and

\[
\mathcal{M}_v = -\sqrt{\tilde{\Lambda}} \frac{1 + \hat{p}}{2} \gamma^5 \tag{2.5}
\]

the heavy pseudoscalar wave function in the heavy quark effective field theory [10], which exhibits a manifest heavy flavor symmetry. Here \( \tilde{\Lambda} = \lim_{M_Q \to \infty} \tilde{\Lambda}_M \) is the heavy flavor independent binding energy.

Note that the form factors introduced in (2.1) are heavy flavor dependent. But the functions \( A, B \) and \( L_i (i = 1, 2, 3, 4) \) in (2.3) are leading order wave functions in the \( 1/m_Q \) expansion, so they should be, at least, explicitly independent of the heavy quark mass. Therefore at certain order we can formulate all this kind of wave functions conveniently in a heavy quark mass independent way. This is an advantage to study the decays in the
effective theory. The relations between the form factors defined in (2.1) and the universal wave functions defined in eqs.(2.2) and (2.3) can be derived straightforwardly. One has

\[ f_{\pm}(q^2) = \frac{1}{m_M} \sqrt{\frac{m_M \Lambda}{\Lambda_M}} \{ A(v \cdot p) \pm B(v \cdot p) \frac{m_M}{v \cdot p} \} + \cdots; \]

\[ A_1(q^2) = \frac{2}{m_M + m_V} \sqrt{\frac{m_M \Lambda}{\Lambda_M}} \{ L_1(v \cdot p) + L_3(v \cdot p) \} + \cdots; \]

\[ A_2(q^2) = 2(m_M + m_V) \sqrt{\frac{m_M \Lambda}{\Lambda_M}} \left\{ \frac{L_2(v \cdot p)}{2m_M^2} + \frac{L_3(v \cdot p) - L_4(v \cdot p)}{2m_M(v \cdot p)} \right\} + \cdots; \]

\[ A_3(q^2) = 2(m_M + m_V) \sqrt{\frac{m_M \Lambda}{\Lambda_M}} \left\{ \frac{L_2(v \cdot p)}{2m_M^2} - \frac{L_3(v \cdot p) - L_4(v \cdot p)}{2m_M(v \cdot p)} \right\} + \cdots; \]

\[ V(q^2) = \sqrt{\frac{m_M \Lambda}{m_M} + \frac{m_V}{m_M(v \cdot p)}} L_3(v \cdot p) + \cdots. \]  

(2.6)

It is seen that if we can estimate the wave functions \( A, B \) and \( L_i(i = 1, 2, 3) \) to a relatively precise extent, then the form factors \( f_{\pm}, f_-, A_i(i = 1, 2, 3) \) and \( V \) for different decay channels can be easily obtained by adopting the relevant parameters such as masses and binding energies of the mesons. In other words, eq.(2.6) shows the relations among different decays. From this point of view, considering the whole group of heavy to light semileptonic decays, one can say that to certain order of the \( 1/m_Q \) expansion, the HQS and heavy quark expansion can greatly simplify the theoretical analysis and reduce the number of independent functions, though for a single channel the number of independent functions does not decrease.

The wave functions can be explored from the study of appropriate two point correlation functions. For example, for decays into pseudoscalar and vector light mesons, using the interpolating current \( \bar{Q}i\gamma^5q \) for the pseudoscalar heavy mesons, we consider the functions:

\[ P^\mu(p, q) = i \int d^4x e^{iq \cdot x} \langle P(p) | T \{ \bar{q}(x) \gamma^\mu Q(x), \bar{Q}(0)i\gamma^5q(0) \} | 0 \rangle, \]

\[ V^\mu(p, q) = i \int d^4x e^{-ip \cdot x} \langle V(p, \epsilon^*) | T \{ \bar{q}(0) \gamma^\mu (1 - \gamma^5) Q(0), \bar{Q}(x)i\gamma^5q(x) \} | 0 \rangle. \]  

(2.7)

In applying for the sum rule method, on the phenomenological considerations, a complete set of states with the heavy meson quantum numbers are inserted into the above two point functions, i.e., between the two currents. For the insertion of the ground states of heavy mesons, one obtains meson pole contributions. While for the insertion of higher resonances, the results are generally written in the form of integrals over physical densities \( \rho_p(v \cdot p, s) \) and \( \rho_v(v \cdot p, s) \). In the resulting formulae, the matrix elements can be expanded in powers of \( 1/m_Q \) in the effective theory. In this paper we consider only the leading order contributions in the heavy quark expansion. Similar to Refs. [5,6], we have

\[ P^\mu(p, q) = \frac{2F}{\Lambda_M} \frac{A_{\nu}^\mu}{2\Lambda_M - 2v \cdot k} + \int_{s_0}^{\infty} ds \frac{\rho_p(v \cdot p, s)}{s - 2v \cdot k} + \text{subtractions}, \]

\[ V^\mu(p, q) = \frac{m_M \Lambda}{m_Q \Lambda_M} \frac{2F}{2\Lambda_M - 2v \cdot k} \left\{ (L_1 + L_3) \epsilon^* \cdot v - L_2 v^\mu (\epsilon^* \cdot v) - (L_3 - L_4) p^\mu \frac{\epsilon^* \cdot v}{v \cdot p} \right\}. \]

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\[ + i \frac{L_3}{v \cdot p} \epsilon^{\mu \nu \alpha \beta} \epsilon'_\mu p_\alpha v_\beta \} + \int_{s_0}^{\infty} ds \frac{\rho V(v \cdot p, s)}{s - 2v \cdot k} + \text{subtractions.} \]  

(2.8)

where \( F \) is the scaled decay constant of heavy meson at the leading order of \( 1/m_Q \) [11].

The correlation functions in (2.7) can be reformulated in the framework of effective theory, i.e., the field variables and meson states can be replaced by the corresponding counterparts in effective theory, and at the same time each correlator turns into an expansion series in powers of \( 1/m_Q \). In the effective theory of heavy quark, (2.7) has the following form

\[
P^\mu(p, q) = i \int d^4xe^{i(q - mv)_\nu x} \langle P(p)|T\bar{q}(x)\gamma^\mu Q^+_v(x), \bar{Q}^+_v(0)i\gamma^5 q(0)|0 \rangle + O(1/m_Q),
\]

\[
V^\mu(p, q) = i \int d^4xe^{-ipM^\nu x + im_q v \cdot x} \langle V(p, \epsilon^*)|T\{\bar{q}(0)\gamma^\mu (1 - \gamma^5)Q^+_v(0), \bar{Q}^+_v(x)i\gamma^5 q(x)\}|0 \rangle + O(1/m_Q).
\]  

(2.9)

In deriving \( B \to \pi(\rho)\nu \) decays in effective theory of heavy quark, we have considered in Refs. [5,6] the \( \pi \) meson distribution amplitudes up to twist 4 and \( \rho \) meson distribution amplitudes up to twist 2. The purpose of this paper is to reach a relatively complete knowledge on the whole group of heavy to light exclusive semileptonic decays from effective theory of heavy quark. We include for light pseudoscalar and vector mesons the distribution amplitudes up to the same order as in Refs. [5,6]. The \( \pi, \rho \) distribution amplitudes are defined by

\[
\langle \pi(p)|\bar{u}(x)\gamma^\mu \gamma^5 d(0)|0 \rangle = -ip^\mu f_\pi \int_0^1 du \epsilon^{\mu p x} \phi_\pi(u) + x^2 g_1(u)
\]

\[+ f_\pi(x^\mu - \frac{x^2 p^\mu}{x \cdot p}) \int_0^1 du \epsilon^{\mu p x} g_2(u), \]

\[
\langle \pi(p)|\bar{u}(x)i\gamma^5 d(0)|0 \rangle = \frac{f_\pi m_\pi^2}{m_u + m_d} \int_0^1 du \epsilon^{\mu p x} \phi_\rho(u),
\]

\[
\langle \pi(p)|\bar{u}(x)\sigma_{\mu\nu} \gamma^5 d(0)|0 \rangle = i(p_\mu x_\nu - p_\nu x_\mu) \frac{f_\pi m_\pi^2}{6(m_u + m_d)} \int_0^1 du \epsilon^{\mu p x} \phi_{\sigma}(u),
\]

\[
\langle \rho(p, \epsilon^*)|\bar{u}(0)\sigma_{\mu\nu} d(x)|0 \rangle = -if_\rho (\epsilon^*_\mu p_\nu - \epsilon^*_\nu p_\mu) \int_0^1 du \epsilon^{\mu p x} \phi_{\rho}(u),
\]

\[
\langle \rho(p, \epsilon^*)|\bar{u}(0)\gamma_\mu d(x)|0 \rangle = f_\rho m_\rho p_\mu \frac{\epsilon^* \cdot x}{p \cdot x} \int_0^1 du \epsilon^{\mu p x} \phi_{\rho}(u)
\]

\[+ f_\rho m_\rho (\epsilon^*_\mu - \epsilon^*_\nu x_\mu - \epsilon^*_\nu x_\mu) \int_0^1 du \epsilon^{\mu p x} g_{\rho}(u),
\]

\[
\langle \rho(p, \epsilon^*)|\bar{u}(0)\gamma_\mu \gamma_5 d(x)|0 \rangle = \frac{1}{4} f_\rho m_\rho \epsilon_{\mu
u\alpha\beta} \epsilon^{\nu p x} x_\alpha x_\beta \int_0^1 du \epsilon^{\mu p x} g_{\rho}^{(a)}(u). \]  

(2.10)

For \( K \) and \( K^* \) mesons, we need only replace the \( \pi \) and \( \rho \) mesons and \( d \) quark in the lhs. of eq. (2.10) into \( K \) and \( K^* \) mesons and \( s \) quark, and at the same time change the quantities and distribution amplitudes related to \( \pi \) and \( \rho \) mesons to those related to \( K \) and \( K^* \) mesons in the rhs. of (2.10). For decays into \( K \) and \( K^* \), the light flavor SU(3) breaking effects may exhibit via both the light meson related quantities and the \( K \) and \( K^* \) meson distribution amplitudes.
The standard procedure of light cone sum rule method is to calculate the correlation functions in deep Euclidean region by using QCD or effective theories, then equate the results with the phenomenological representations. In searching for reasonable and stable results, the quark-hadron duality and Borel transformation are generally applied to both sides of the equations. The theoretical calculations can often become simpler in the framework of effective theory than in QCD, as one can see in Refs. [5,6]. Adopting procedures similar to Refs. [5,6], we have

\[ A(y) = -\frac{f_{\pi(K)}}{4F} \int_0^{s_0} ds e^{2s_0 - \frac{y}{2}} \left[ \frac{1}{y^2} \partial_u g_2(u) - \frac{\mu_{\pi(K)}}{y} \phi_{\pi}(u) - \frac{\mu_{\pi(K)}}{6y} \partial_u \phi_{\sigma}(u) \right]_{u=1-\frac{2s}{y}}, \]

\[ B(y) = -\frac{f_{\pi(K)}}{4F} \int_0^{s_0} ds e^{2s_0 - \frac{y}{2}} \left[ -\phi_{\pi(K)}(u) + \frac{1}{y^2} \partial^2 u g_1(u) - \frac{1}{y^2} \partial^2 u g_2(u) + \frac{\mu_{\pi(K)}}{6y} \partial^2 u \phi_{\sigma}(u) \right]_{u=1-\frac{2s}{y}}, \]

\[ L_1(y) = \frac{1}{4F} \int_0^{s_0} ds e^{2s_0 - \frac{y}{2}} \frac{1}{y} f_V m_V [g_1^{(a)}(u)] + \frac{1}{4} \left( \partial_u g_2^{(a)}(u) \right)_{u=1-\frac{2s}{y}}, \]

\[ L_3(y) = \frac{1}{4F} \int_0^{s_0} ds e^{2s_0 - \frac{y}{2}} \left[ -\frac{1}{4y} f_V m_V \left( \partial_u g_1^{(a)}(u) \right) + f_V \phi_\perp(u) \right]_{u=1-\frac{2s}{y}}, \]

\[ L_4(y) = \frac{1}{4F} \int_0^{s_0} ds e^{2s_0 - \frac{y}{2}} \frac{1}{y} f_V m_V [\phi mundane \perp(u) - g_1^{(a)}(u)] + \frac{1}{4} \left( \partial_u g_2^{(a)}(u) \right)_{u=1-\frac{2s}{y}}. \]  

(2.11)

\( L_2(y) \) equals zero in the present approximation since no twist 2 distribution functions contribute to it. As a consequence, one can see from (2.6) that \( A_2 \) and \( A_3 \) have the same absolute value but opposite signs at the leading order.

Before proceeding, we would like to address that though the formulae in (2.11) for wave functions are explicitly independent of the heavy quark mass, it does not mean that the values of these functions for decays of different heavy mesons are the same. This is because for decays of different heavy mesons one needs to take into account different energy scales for different heavy mesons in calculating the corresponding functions. It is this difference that makes the distribution amplitudes and other light meson parameters in (2.11) change their values for different heavy meson decays. There is also an exponent \( e^{2\Lambda_{BM}/T} \) in each formula, which indicates the dependence on the binding energy of the heavy meson. Since \( \Lambda_B - \Lambda_D \) is small, this exponent only introduces a slight difference among the wave functions for different heavy mesons. On the other hand, there are light meson distribution amplitudes, light meson masses and other light meson parameters in (2.11). Consequently, the resulting numerical values for the universal wave functions may be quite different for different light mesons.

III. NUMERICAL ANALYSIS OF THE WAVE FUNCTIONS

The light cone distribution amplitudes embody the nonperturbative contributions, and they are of crucial importance for the precision that light cone sum rules can reach. The study of these distribution amplitudes constitutes an important and difficult project. They have been studied by several groups. The asymptotic form and the scale dependence of these functions are given by perturbative QCD [12,13].

For light pseudoscalar mesons, the leading twist distribution amplitudes are generally written as an expansion in terms of the Gegenbauer polynomials \( C^{3/2}_n(x) \) as follows:
\[
\phi_{\pi(K)}(u, \mu) = 6u(1-u)[1 + \frac{4}{\mu^2} \sum_{n=1}^{4} a_n^{\pi(K)}(\mu) C_n^{3/2}(2u-1)].
\] (3.1)

One should perform the sum rule analysis at appropriate energy scale \( \mu \). In the processes of \( B_s \) and \( D \) decays, the scales can be set by the typical virtualities of the heavy quarks, for example \([14]\), \( \mu_b = \sqrt{m_B^2 - m_\pi^2} \approx 2.4\text{GeV} \) and \( \mu_c = \sqrt{m_B^2 - m_\pi^2} \approx 1.3\text{GeV} \), respectively.

For \( \pi \) meson, we use \([14]\)

\[
a_2^\pi(\mu_c) = 0.41, \quad a_4^\pi(\mu_c) = 0.23, \quad a_1^\pi = a_3^\pi = 0,
\] (3.2)

while for the kaon distribution amplitude \( \phi_K \), when the light flavor SU(3) breaking effects are considered, we may use \([4]\)

\[
a_1^K(\mu_c) = 0.17, \quad a_2^K(\mu_c) = 0.21, \quad a_3^K(\mu_c) = 0.07, \quad a_4^K(\mu_c) = 0.08,
\] (3.3)

where the nonvanishing values of the coefficients \( a_1, a_3 \) imply the asymmetric momentum distributions for the \( s \) and \( u, d \) quarks inside the \( K \) meson. Since the Gegenbauer moments \( a_i \) renormalize multiplicatively, the values of \( a_i(\mu_b) \) can be obtained from (3.2) and (3.3) through the renormalization group evolution.

We neglect the SU(3) breaking effects in the twist 3 and 4 distribution amplitudes included in this paper. This is justified by the analyses in Ref. \([15]\), which indicates that these breaking effects would influence the light cone sum rules very slightly. Therefore we take for both \( \pi \) and \( K \) the following twist 3 and 4 distribution amplitudes \([3,14,16]\):

\[
\phi_p(u) = 1 + \frac{1}{2}B_2[3(2u-1)^2 - 1] + \frac{1}{8}B_4[35(2u-1)^4 - 30(2u-1)^2 + 3],
\]

\[
\phi_\sigma(u) = 6u(1-u)[1 + \frac{3}{2}C_2[5(2u-1)^2 - 1] + \frac{15}{8}C_4[21(2u-1)^4 - 14(2u-1)^2 + 1]],
\]

\[
g_1(u) = \frac{5}{2} \delta^2 u^2(1-u)^2 + \frac{1}{2} \epsilon \delta^2[u(1-u)(2 + 13u(1-u) + 10u^3 \log u(2 - 3u + \frac{6}{5}u^2))
+ 10(1-u)^3 \log((1-u)(2 - 3(1-u) + \frac{6}{5}(1-u)^2))],
\]

\[
g_2(u) = \frac{10}{3} \delta^2 u(1-u)(2u-1),
\] (3.4)

where

\[
B_2(\mu_b) = 0.29, \quad B_4(\mu_b) = 0.58, \quad B_2(\mu_c) = 0.41, \quad B_4(\mu_c) = 0.925,
\]

\[
C_2(\mu_b) = 0.059, \quad C_4(\mu_b) = 0.034, \quad C_2(\mu_c) = 0.087, \quad C_4(\mu_c) = 0.054,
\]

\[
\delta^2(\mu_b) = 0.17\text{GeV}^2, \quad \delta^2(\mu_c) = 0.19\text{GeV}^2, \quad \epsilon(\mu_b) = 0.36, \quad \epsilon(\mu_c) = 0.45.
\] (3.5)

Besides in these distribution amplitudes, the SU(3) breaking effects also emerge in the light meson constants in the coefficients of distribution amplitudes. We take \( f_\pi = 0.132\text{GeV} \), \( f_K = 0.16\text{GeV} \) and \( \mu_\pi = m_\pi^2/(m_u + m_d) \) with \( \mu_\pi(1\text{GeV}) = 1.65\text{GeV} \) (as \( \mu_\pi(\mu_c) = 1.76\text{GeV} \), \( \mu_\pi(\mu_b) = 2.02\text{GeV} \)). For \( \mu_K = m_K^2/(m_s + m_u + m_d) \), we use the advocation in Ref. \([15]\) to rely on chiral perturbation theory in the SU(3) limit and so use \( \mu_K = \mu_\pi \). For the quantities relevant to heavy hadrons, here we use the data evaluated in the previous paper Ref. \([11]\). In particular, there we yielded \( \bar{\Lambda} = 0.53 \pm 0.08\text{GeV} \), \( F = 0.30 \pm 0.06\text{GeV}^{3/2} \).
For decays into light vector mesons, the leading twist distribution functions $\phi_{\perp}$ and $\phi_{\parallel}$ can also be expanded in Gegenbauer polynomials $C_n^{3/2}(x)$ with the coefficients running with the scale and described by the renormalization group method. Explicitly we have

$$
\phi_{\perp(\parallel)}(u, \mu) = 6u(1 - u)[1 + \sum_{n=2,4,\ldots} a_n^{\perp(\parallel)}(\mu) C_n^{3/2}(2u - 1)],
$$

$$
a_n^{\perp(\parallel)}(\mu) = a_n^{\perp(\parallel)}(\mu_0) \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{(\gamma_n^{\perp(\parallel)} - \gamma_0^{\perp(\parallel)})/(2\beta_0)}, \tag{3.6}
$$

where $\beta_0 = 11 - (2/3)n_f$, and $\gamma_n^{\perp}$, $\gamma_n^{\parallel}$ are the one loop anomalous dimensions [17,18]. The coefficients $a_n^{\perp}$, $a_n^{\parallel}$ have been studied extensively in [1]. Here we use the values for $\rho$ and $K^*$ mesons presented in that paper, where the SU(3) breaking effects are included for $K^*$ meson.

The functions $g_{\perp}^{(v)}$ and $g_{\perp}^{(a)}$ describe transverse polarizations of quarks in the longitudinally polarized mesons. They receive contributions of both twist 2 and twist 3. In this paper we will include only the twist 2 contributions, which are related to the longitudinal distribution $\phi_{\parallel}(u, \mu)$ by Wandzura-Wilczek type relations [19,20]:

$$
g_{\perp}^{(v),\text{twist }2}(u, \mu) = \frac{1}{2}\left[\int_0^u dv \frac{\phi_{\parallel}(v, \mu)}{1 - v} + \int_u^1 dv \frac{\phi_{\parallel}(v, \mu)}{v}\right],
$$

$$
g_{\perp}^{(a),\text{twist }2}(u, \mu) = 2[(1 - u)\int_0^u dv \frac{\phi_{\parallel}(v, \mu)}{1 - v} + u\int_u^1 dv \frac{\phi_{\parallel}(v, \mu)}{v}]. \tag{3.7}
$$

The quantities $f_\rho$ and $f_{K^*}$ are the decay constants of vector mesons, and $f_\rho^\perp$ and $f_{K^*}^\perp$ are couplings defined via

$$
\langle 0|\bar{u}\gamma_{\mu}q|V(p, \epsilon)\rangle = i(\epsilon_{\mu}p_\nu - \epsilon_{\nu}p_\mu)f_{V}^\perp \tag{3.8}
$$

with the light quark $q = d$ or $s$, which is corresponding to the vector meson $V = \rho$ or $K^*$. In the calculations, we use for these couplings [19,21,22,1]:

$$
f_\rho = 195 \pm 7\text{MeV}, \quad f_\rho^\perp = 160 \pm 10\text{MeV},
$$

$$
f_{K^*} = 226 \pm 28\text{MeV}, \quad f_{K^*}^\perp = 185 \pm 10\text{MeV}. \tag{3.9}
$$

From (2.6) and (2.11), the data for form factors can be obtained by using the distribution amplitudes and meson quantities presented above. As an example, Figs.1-2 show the variation of $A_{1}^{D-\rho}$ and $A_{1}^{D-\bar{K}^*}$ with respect to the Borel parameter $T$ at the fixed value of $s_0 = 1.5\text{GeV}$, $2\text{GeV}$, $2.5\text{GeV}$. According to the light cone sum rule criterion that both the higher resonance contributions and the contributions from higher twist distribution amplitudes should not be too large (say larger than 30%), the interested region of $T$ should be $1\text{GeV} < T < 2\text{GeV}$. In this region the curves become the most stable at the threshold energy $s_0 = 2 \pm 0.5\text{GeV}$. We have also studied respectively the variations of form factors in all interested decays with respect to the Borel parameter $T$. From our detailed study, for all form factors except $V$, there exist reliable regions of $s_0$ and $T$, that well satisfy the requirement of stability in the sum rule analysis. The stability of form factor $V$ in all the heavy to light vector decays is not as good as the one of other form factors. This introduces an uncertainty larger than those from other form factors. The threshold energies for
the form factors $f_\pm$ and $A_i(i = 1, 2, 3)$ in these decays are found to vary from 0.4GeV to 3.5GeV. Since there exist no very stable regions for $V$ respect to $T$ in our present analysis, for a simple consideration, we may calculate $V$ using the same values of $s_0$ obtained from the analysis of form factors $A_i$. With such determined ranges of $T$ and $s_0$, all the form factors as functions of the momentum transfer $q^2$ can be derived from eqs.(2.6) and (2.11).

It is known that for the final mesons being light pseudoscalars, the light cone expansion and the sum rule method will break down at large momentum transfer (numerically as $q^2$ approaches near $m_b^2$) [3]. As a result, the curves of wave functions calculated from light cone sum rules may become unstable at large $q^2$ region. Thus at this region we have to rely on other approximation such as single or double pole approximation. Here we use for $B \to \pi$ transition

$$f_+(q^2) = \frac{f_{B^*} g_{B^* B \pi}}{2m_{B^*}(1 - q^2/m_{B^*}^2)}$$  \hspace{1cm} (3.10)$$

for large $q^2$ (small $y$) regions of B decays into light pseudoscalar mesons. And similar monopole approximation formulae will be applied to other heavy mesons decaying into light pseudoscalar mesons. In our numerical calculations, we will take: $f_{B^*} g_{B^* B \pi} = 4.4 \pm 1.3$GeV, $f_{D^*} g_{D^* D \pi} = 2.7 \pm 0.8$GeV, $f_{D^*} g_{D^* D K} = 3.1 \pm 0.6$GeV [4], and $f_{B^*} g_{B^* B_s K} = 3.88 \pm 0.31$GeV [23].

As a good approximation, for the behavior of the form factors in the whole kinematically accessible region, we use the parametrization

$$F(q^2) = \frac{F(0)}{1 - a_Fq^2/m_B^2 + b_F(q^2/m_B^2)^2},$$ \hspace{1cm} (3.11)$$

where $F(q^2)$ can be any of the form factors $f_+, f_-, A_i(i = 1, 2, 3)$ and $V$. Thus each form factor will be parameterized by 3 parameters in eq.(3.11) that need to be fitted. For decays into pseudoscalar mesons, as the light cone sum rules are most suitable for describing the low $q^2$ region of the form factors and the very high $q^2$ region is hard to be reached by this approach, we shall use the light cone sum rule results at small $q^2$ region and the monopole approximation eq.(3.10) at large $q^2$ region to fit the 3 parameters in (3.11). For decays into vector mesons, we use only the light cone sum rule predictions in fitting the parameters in eq.(3.11). This is because the kinematically allowed ranges of $B$ to vector meson decays are small compared with that ranges of $B$ to pseudoscalar meson decays, and therefore the sum rules are expected to yield reasonable values for most allowed regions of $q^2$ in the vector meson cases.

At the central values of suitable thresholds $s_0$, we can fix the parameters for each form factor. Our numerical results for those parameters are presented in Tables.1-2. And the form factors as functions of $q^2$ in the three-parameter space are shown in Figs.3-7. The data for $f_-$ are not presented because they are irrelevant to the decay widths when the lepton masses are neglected.
 variation of thresholds they could arise from the meson constants, the light cone distribution amplitudes and the parameters. It is shown in Figs.3-6 that the SU(3) symmetry breaking effect is relatively processes. Besides this, the SU(3) breaking effects also arise from relevant light meson

In general, the SU(3) breaking effects should not be neglected in the considered transition

The values of $F(0)$, $a_F$, $b_F$ presented in tables.1-2 are slightly different from those given in Refs. [5,6]. The reason is that the input data of hadron quantities such as $f_{B^*} g_{B^*B^*}$ differ from those in Refs. [5,6]. There actually exist some uncertainties to the form factors, they could arise from the meson constants, the light cone distribution amplitudes and the variation of thresholds $s_0$. We notice that the latter comprises the largest uncertainty, which may be 15% or so. Variations of other input parameters within their allowed ranges which we have discussed would increase the uncertainties to about 20%. So, including uncertainties from higher twist amplitudes and other systematic uncertainties in light cone sum rule method, we quote an uncertainty of about 25%.

It is also found in our investigations that SU(3) symmetry breaking effects considered in the Gegenbauer polynomial moments $a_i (i = 1, 2, 3, 4)$ invoke the changes of $F(0)$ by 5-15%. In general, the SU(3) breaking effects should not be neglected in the considered transition processes. Besides this, the SU(3) breaking effects also arise from relevant light meson parameters. It is shown in Figs.3-6 that the SU(3) symmetry breaking effect is relatively

| $B \to \pi l \nu$ | $f_+$ | 0.37 ± 0.10 | 1.28 ± 0.13 | 0.30 ± 0.20 | 1.0 ± 0.4 |
| $B_s \to K l \nu$ | $f_+$ | 0.47 ± 0.10 | 1.12 ± 0.25 | 0.34 ± 0.17 | 2.7 ± 0.8 |
| $D \to \pi l \nu$ | $f_+$ | 0.67 ± 0.19 | 1.30 ± 0.30 | 0.68 ± 0.38 | 0.8 ± 0.4 |
| $D \to K l \nu$ | $f_+$ | 0.67 ± 0.20 | 1.60 ± 0.43 | 1.50 ± 0.82 | 0.8 ± 0.4 |

Table 1. Results for the form factor $f_+$ of heavy to light pseudoscalar meson decays. $s_0$ is the threshold at which the parameters $F(0)$, $a_F$, $b_F$ are fitted.

| $B \to \rho l \nu$ | $A_1$ | 0.29 ± 0.05 | 0.35 ± 0.15 | −0.24 ± 0.12 | 2.1 ± 0.6 |
| $B_s \to K^* l \nu$ | $A_1$ | 0.28 ± 0.04 | 1.09 ± 0.13 | 0.20 ± 0.18 | 2.5 ± 0.5 |
| $A_2$ | −0.28 ± 0.04 | 1.09 ± 0.13 | 0.20 ± 0.18 | 2.5 ± 0.5 |
| $V$ | 0.15 ± 0.04 | 1.03 ± 0.13 | −0.22 ± 0.31 | 2.5 ± 0.5 |
| $D \to \rho l \nu$ | $A_1$ | 0.57 ± 0.08 | 0.60 ± 0.20 | 1.07 ± 1.32 | 2.0 ± 0.5 |
| $A_2$ | 0.52 ± 0.06 | 0.66 ± 0.13 | −2.03 ± 1.52 | 2.0 ± 0.5 |
| $A_3$ | −0.52 ± 0.06 | 0.66 ± 0.13 | −2.03 ± 1.52 | 2.0 ± 0.5 |
| $V$ | 0.31 ± 0.12 | 2.07 ± 0.51 | 3.47 ± 2.76 | 2.0 ± 0.5 |
| $D \to K^* l \nu$ | $A_1$ | 0.59 ± 0.10 | 0.58 ± 0.10 | 0.11 ± 0.28 | 2.0 ± 0.5 |
| $A_2$ | 0.55 ± 0.08 | 0.84 ± 0.31 | −1.29 ± 1.12 | 2.0 ± 0.5 |
| $A_3$ | −0.55 ± 0.08 | 0.84 ± 0.31 | −1.29 ± 1.12 | 2.0 ± 0.5 |
| $V$ | 0.22 ± 0.12 | 2.33 ± 1.16 | 2.07 ± 2.25 | 2.0 ± 0.5 |

Table 2. Results for the form factors of heavy to light vector meson decays. $s_0$ is the threshold at which the parameters $F(0)$, $a_F$, $b_F$ are fitted.
small for \( f_+ \) and \( A_i (i = 1, 2, 3) \). However for form factor \( V \) as shown in Fig.7, there is a large difference between decays to \( \rho \) and \( K^* \). This may be due to the unstability of the curves for \( V \) in sum rule analyses. According to the amount of SU(3) breaking effects for \( f_+ \) and \( A_i \), a difference smaller than that shown in Fig.7 is expected in a more reliable calculation.

## IV. BRANCHING RATIOS AND CKM MATRIX ELEMENTS

With the form factors derived in the previous sections, the decay widths and branching ratios of exclusive semileptonic decays can be predicted if we know the values of relevant CKM matrix elements. When the lepton masses are neglected, we have for decays to pseudoscalar mesons:

\[
\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{qQ}|^2}{24\pi^3} \left( (m_M^2 + m_V^2 - q^2)^2 - m_V^2 \right)^{3/2} [f_+(q^2)]^2, \tag{4.1}
\]

and for decays to vector mesons:

\[
\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{qQ}|^2}{192\pi^3 m_M^3} \lambda^{1/2} q^2 (H_0^2 + H_+^2 + H_0^2) \tag{4.2}
\]

with \( \lambda \equiv (m_M^2 + m_V^2 - q^2)^2 - 4m_M^2m_V^2 \) and the helicity amplitudes defined as

\[
H_\pm = (m_M + m_V)A_1(q^2) \mp \frac{\lambda^{1/2}}{m_M + m_V} V(q^2),
\]

\[
H_0 = \frac{1}{2m_V\sqrt{q^2}} \{(m_M^2 - m_V^2 - q^2)(m_M + m_V)A_1(q^2) - \frac{\lambda}{m_M + m_V} A_2(q^2)\}. \tag{4.3}
\]

On the other hand, one can extract the values of CKM matrix elements from the experimental measurements of branching ratios. In Refs. [5] and [6], \( |V_{ub}| \) is extracted from \( B \to \pi(\rho)l\nu \) decays with using the branching ratio measurements. The results obtained there were \( |V_{ub}| = (3.4 \pm 0.5 \pm 0.5) \times 10^{-3} \) and \( |V_{ub}| = (3.9 \pm 0.6 \pm 0.7) \times 10^{-3} \) via the two decays, respectively. Here the first (second) error corresponds to the experimental (theoretical) uncertainty.

In this paper we would like to predict the branching ratios from the values of CKM matrix elements: \( |V_{ub}| = 0.0037, \ |V_{cd}| = 0.22, \ |V_{cs}| = 0.97 \). Finishing the integration over \( q^2 \), we obtain the corresponding branch ratios given in table.3.

| \( B \to \pi l\nu \) | \( \Gamma/(|V_{qQ}|^2)(\text{ps}^{-1}) \) | \( \text{Br} \) |
|---|---|---|
| \( B_s \to K l\nu \) | 11.67 ± 1.5 | (2.492 ± 0.416) \times 10^{-4} |
| \( B \to \rho l\nu \) | 14.26 ± 2.78 | (2.915 ± 0.688) \times 10^{-4} |
| \( B_s \to K^* l\nu \) | 13.02 ± 4.0 | (2.781 ± 0.961) \times 10^{-3} |
| \( \bar{D} \to \pi l\nu \) | 17.18 ± 4.0 | (3.512 ± 0.962) \times 10^{-4} |
| \( \bar{D} \to K l\nu \) | 0.152 ± 0.06 | (3.035 ± 1.216) \times 10^{-3} |
| \( \bar{D} \to \rho l\nu \) | 0.105 ± 0.04 | (4.074 ± 1.580) \times 10^{-2} |
| \( \bar{D} \to K^* l\nu \) | 0.0686 ± 0.0154 | (1.369 ± 0.316) \times 10^{-3} |
| \( \bar{D} \to K l\nu \) | 0.0486 ± 0.01 | (1.882 ± 0.401) \times 10^{-2} |
Table 3. Decay widths ($\Gamma/(|V_{QQ}|^2)$) and Branching ratios ($Br$) for heavy to light meson decays. In deriving the branching ratios we used $|V_{ub}| = 0.0037$, $|V_{cd}| = 0.22$, $|V_{cs}| = 0.97$ and the lifetimes of heavy mesons: $	au_{B^0} = 1.56 \pm 0.06\text{ps}$, $	au_{D^0} = 0.4126 \pm 0.0028\text{ps}$, $	au_{B_s} = 1.493 \pm 0.062\text{ps}$.

The largest uncertainty for the branching ratios in Table 3 is about 40%, which is the same as that quoted in Ref. [4]. So, the precision of heavy meson decay measurements expected in the new B factories can not be matched well unless these theoretical uncertainties could be reduced by a factor of about 2. This reduction may be obtained from consideration of both higher twist contributions, better determination of the meson constants and the higher order contributions in the heavy quark expansion.

V. SUMMARY

In summary, we have studied the exclusive semileptonic decays of heavy to light mesons by applying for the effective field theory of heavy quarks and light cone sum rule approach. The form factors for the decays $B(D, B_s) \rightarrow \pi(\rho, K, K^*) l \nu$ have been calculated in detail by using the light cone sum rule method in the effective theory of heavy quark. It has been seen that the heavy quark symmetry leads to a great simplification for heavy to light transitions as the decays of different heavy hadrons (such as $B$ and $D$) can be approximately characterized by the same set of wave functions ($L_i(i = 1, 2, 3, 4)$) at the leading order of $1/m_Q$ expansion. These wave functions are explicitly independent of the heavy quark mass, but only have a slight dependence on the binding energy of heavy hadrons (in the $\bar{\Lambda}_M$). In such calculations, the uncertainties for the form factors are generally about 25%, which, together with the meson constants, may give the branching ratios with a total uncertainty up to 40%. We have also estimated the light flavor SU(3) symmetry breaking effects in these semileptonic decays and found that the SU(3) symmetry breaking effects may influence the form factors up to a total amount of 20%. We may conclude that the branching ratios of those heavy to light meson semileptonic decays can be reasonably calculated based on the light cone sum rule approach within the framework of heavy quark effective field theory. Nevertheless, in order to match the expected more precise experimental measurements at B factories in the near future, more accurate calculations for the exclusive heavy to light meson semileptonic decays are needed.

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Fig. 1-2. The form factors $A_1^{D\to \rho}$ and $A_1^{D\to K^*}$ as functions of the Borel parameter $T$ for different values of the continuum threshold $s_0$. The dashed, solid and dotted curves correspond to $s_0 = 1.5$, 2 and 2.5 GeV respectively. Considered here is at the momentum transfer $q^2 = 0 GeV^2$. 

Fig. 3-4.
Fig. 3-7. Results for the heavy-to-light decay form factors from light cone sum rule study. The solid, dashed, dot-dashed and dotted curves correspond to $B \to \pi(\rho)$, $B_s \to K(K^*)$, $D \to \pi(\rho)$ and $D \to K(K^*)$ decays, respectively. The dot-dashed and dotted curves in Fig.3 almost coincide with each other, and so do the solid and dashed curves in Fig.4-6 almost.