DIFFERENCE RESONANCE STUDY ON THE ELECTRON STORAGE RING ALADDIN AT SRC*

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We have studied the third-order betatron difference resonance on the electron storage ring Aladdin at SRC. The spare chromaticity correcting sextupoles were used to drive the desired harmonics, and a pulsed kicker magnet was fired to produce the coherent oscillation of a single beam bunch. We took beam position measurements from four pairs of stripline electrodes and data acquisition devices. We observed the coupling resonance. Several typical measurements are shown. A computer program was written using multi-particle tracking to simulate the bunch motion. The simulations agree well both with analytic predictions and with the experimental results.

KEY WORDS: Aladdin; difference resonance; storage ring; nonlinear dynamics.

1 INTRODUCTION

We are conducting a series of nonlinear dynamics studies on the electron storage ring Aladdin at the Synchrotron Radiation Center, University of Wisconsin-Madison. We have initially concentrated on the third-order resonances driven by sextupole errors in the magnetic field. We have presented several papers on our theoretical and experimental studies on the horizontal third-integral resonance.^{1,2}

The difference resonance

$$v_x - 2v_y = m \quad (1)$$

couples the two transverse degrees of freedom. This resonance does not lead to instabilities since particle motion is bounded in both the x and y directions. Particle loss occurs only at the vacuum chamber walls and other physical boundaries. Near the resonance, the coupling transfers 'energy' from x to y motion so that the y amplitude increases

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periodically. The degree of coupling depends on the separation of the betatron oscillation frequencies (tunes) ν_x, ν_y from the resonance values and on the strength of the sextupole field which drives the resonance. The details of the coupling also depend on the amplitude dependence of the tunes.

In a machine such as Aladdin with four superperiods, the chromaticity correcting sextupoles can only drive resonances with $m = 4n$, where n is an integer, since they are symmetrically distributed around the ring. However, magnet misalignments and other imperfections always exist and can drive nonlinear resonances with other harmonics m . Furthermore, the second-order sextupole effect and the fringe fields at quadrupole ends all contribute to the amplitude-dependent tune shifts. These tune shifts can alter the character of the resonance; in the case of unstable resonances, like the third-integral resonance, they could confine the beam within the physical aperture and make the experiments possible. In this paper, we first review the theoretical analysis of the difference resonance, then we discuss the experimental results, and finally we compare the experimental results with the results of numerical simulations. Further details can be found in Ref. 3.

2 THEORETICAL ANALYSIS

For the difference resonance $\nu_x - 2\nu_y = m$, we start with the Hamiltonian in terms of action-angle variables γ_x, J_x and γ_y, J_y ,

$$H = \nu_x J_x + \nu_y J_y - (2J_x)^{1/2} (2J_y) B_m \cos(\gamma_x - 2\gamma_y - m\theta + \zeta_m) + aJ_x^2 + 2bJ_x J_y + cJ_y^2, \quad (2)$$

where $\theta = 2\pi \frac{\text{distance along closed orbit}}{\text{total length of closed orbit}}$, m = circumferential harmonic number, B_m and ζ_m are the amplitude and phase of the sextupole parameter defined by

$$B_m e^{i\zeta_m} = \sum \frac{B''l}{16\pi B\rho} \beta_x^{1/2} \beta_y e^{i(\psi_x - 2\psi_y + m\theta)}. \quad (3)$$

In this expression, B'' and l are the second field gradient and the length of the sextupole; $B\rho$ is the magnetic rigidity of the particle; β_x, ψ_x and β_y, ψ_y are the betatron amplitude and phase functions; and the summation is over all sextupoles. In Eq. 2 the first two terms correspond to the linear motions, the third is the resonance term, and the last three give the general dependence of the tunes on the fourth power of the betatron amplitudes as specified by the parameters a, b , and c .

In order to transform away the θ dependence, we take the generating function

$$W(J_1, J_2, \gamma_x, \gamma_y; \theta) = J_1(\gamma_x - 2\gamma_y - m\theta + \zeta_m) + J_2\gamma_y. \quad (4)$$

With this function, the action-angle variables are transformed according to

$$J_x = \frac{\partial W}{\partial \gamma_x} = J_1, \quad (5)$$

$$J_y = \frac{\partial W}{\partial \gamma_y} = J_2 - 2J_1, \quad (6)$$

$$\gamma_1 = \frac{\partial W}{\partial J_1} = \gamma_x - 2\gamma_y - m\theta + \zeta_m \quad , \quad (7)$$

$$\gamma_2 = \frac{\partial W}{\partial J_2} = \gamma_y \quad , \quad (8)$$

and we get a new Hamiltonian in the form

$$\begin{aligned} \mathcal{H} = & \epsilon J_1 + v_y J_2 - 2(2J_1)^{1/2}(J_2 - 2J_1)B_m \cos \gamma_1 \\ & + aJ_1^2 + 2bJ_1(J_2 - 2J_1) + c(J_2 - 2J_1)^2 \quad , \end{aligned} \quad (9)$$

where

$$\epsilon = v_x - 2v_y - m \quad (10)$$

is the separation of the tunes from the resonance values. We have two constants of motion, $\mathcal{H}$ and J_2 , resulting from the fact that neither θ nor γ_2 appears in $\mathcal{H}$. According to Eqs. 5 and 6, the constant J_2 is given by

$$J_2 = J_y + 2J_x \quad . \quad (11)$$

For a given value of J_2 , two unstable fixed points with coordinates γ_1, J_1 given by

$$\begin{aligned} 2J_1 &= J_2 \quad , \\ 4B_m \cos \gamma_1 &= (2b - a)J_2^{1/2} - \epsilon J_2^{-1/2} \end{aligned} \quad (12)$$

are obtained from

$$\dot{\gamma}_1 = \frac{\partial \mathcal{H}}{\partial J_1} = 0, \quad \text{and} \quad \dot{J}_1 = -\frac{\partial \mathcal{H}}{\partial \gamma_1} = 0 \quad (13)$$

at

$$\mathcal{H} = \left(\frac{\epsilon}{2} + v_y\right) J_2 + \frac{a}{4} J_2^2 \quad . \quad (14)$$

The separatrices are mapping curves passing through the unstable fixed points and are obtained by substituting the value of $\mathcal{H}$ from Eq. 14 in Eq. 9. This gives

$$[2J_1 - J_2] \left[A(2J_1) + 2B(2J_1)^{1/2} \cos \gamma_1 + C \right] = 0 \quad , \quad (15)$$

where

$$A = a/4 - b + c, \quad B = B_m, \quad C = \epsilon/2 + (a/4 - c)J_2 \quad . \quad (16)$$

In terms of the rectangular coordinates

$$Q = (2J_1)^{1/2} \cos \gamma_1, \quad P = (2J_1)^{1/2} \sin \gamma_1 \quad , \quad (17)$$

Eq. 15 becomes

$$[P^2 + Q^2 - J_2] [A(P^2 + Q^2) + 2BQ + C] = 0 \quad . \quad (18)$$

This represents two circles. The circle obtained by setting the first factor to zero is centered at the origin and has the radius $J_2^{1/2}$. This is called the ‘limiting circle’. Setting the second factor to zero we get the ‘dividing circle’ which is centered on the Q -axis at

$$Q_c = -B/A \quad (19)$$

and has the radius (if real)

$$R = \frac{1}{A} \sqrt{B^2 - AC} \quad , \quad (20)$$

The dividing circle crosses the Q -axis at the points

$$Q_1 = Q_c \mp R \quad , \quad \text{and} \quad Q_2 = Q_c \pm R \quad , \quad (21)$$

where the upper sign is used if Q_c is positive, and the lower if it is negative. When a, b and c are small, the radius R becomes large and

$$Q_1 \simeq \pm \frac{C}{2B} \quad , \quad (22)$$

while Q_2 becomes large. The character of the *Poincaré* map depends on the relative sizes and positions of the two circles. In Fig. 1 we show three typical cases with different radii of the limiting circle. For all three cases we have taken positive values of ϵ, A, B , and C . The difference resonance is most pronounced for motions in which initially the y -amplitude is very small and the beam is kicked horizontally to a large $J_x = J_{x0}$. In the phase plane the initial motion is then very close to the limiting circle, namely

$$J_2 = J_y + 2J_x \simeq 2J_{x0} \quad . \quad (23)$$

In cases (a) and (c) of Fig. 1, the motion remains close to the limiting circle, and the y amplitude remains small, so there is no resonant coupling. In case (b) the two circles intersect, and the motion cannot remain near the limiting circle, but moves past the fixed point where the circles intersect and along the dividing circle. This is the case of large resonant coupling. The y amplitude increases to a maximum near the intersection $Q = Q_1$ of the dividing circle with the Q -axis. The maximum y amplitude is given approximately by

$$\sqrt{2J_{y \max}} \simeq \sqrt{2(2J_{x0} - Q_1^2)} \quad . \quad (24)$$

Since the phase point passes by an unstable fixed point (where the circles intersect) very slowly, we expect the y motion to consist of long periods with small amplitude and rises to large amplitude through coupling only for relatively brief durations. At the coupling peak, the x amplitude drops to a minimum given by $|Q_1|$. When $\epsilon = -2(a/4 - c)J_2$, we have complete coupling; the x amplitude drops to zero while the y amplitude reaches a maximum with $J_{y \max} \simeq J_2$.

The threshold of x amplitude for resonant coupling is given by $\sqrt{2J_{x0}} = |Q_1|$. If the initial value of J_x is below this threshold, given approximately by Eq. 22, then no resonant coupling occurs. Since Q_1 depends also on J_{x0} because of the nonlinear term in the parameter C [Eq. (16)], there is also an upper limit for the x amplitude, above which the nonlinearities move the tunes away from the resonance.

The ‘energy exchange’ time, namely, the time it takes to complete a cycle, can be shown to be much less than the synchrotron radiation damping time in Aladdin, so we expect to be able to see more than one full coupling cycle.

FIGURE 1: Resonance phase plane maps

FIGURE 2: Aladdin configuration

TABLE 1: Aladdin parameters at 800 MeV

Energy	800 MeV
Circumference	88.915 m
Revolution period	300 nsec
Betatron tunes	7.14, 7.23 (ν_x, ν_y)
Emittance	0.11π mm-mrad
RF harmonic number	15
RMS beam bunch length	144 mm (0.48 nsec)
Synchrotron radiation energy	17.4 keV/turn
Synchrotron radiation damping time	26.6 msec (89000 rev.)
Beam current	10 mA /bunch

3 EXPERIMENTAL ARRANGEMENT AND MEASUREMENT

Aladdin is a 1-GeV electron storage ring which is composed of four sectors as shown in Fig. 2. In normal operation it stores 15 beam bunches with transverse beam sizes $\sigma_x \simeq 0.48\text{mm}$, $\sigma_y \simeq 0.09\text{mm}$. Table 1 shows its operating parameters at the energy of 800 MeV.

For this experiment, only one beam bunch is used and the other 14 bunches are knocked out by the rf knockout technique.

3.1 Hardware

In the Aladdin ring, there are 4 pairs of unused chromaticity correcting sextupoles labeled SF1/SD1, SF4/SD4, SF7/SD7, and SF10/SD10, all at high β locations, the phase advances between SF and SD are about 22° in the horizontal and 11° in the vertical. Thus these sextupoles can be powered to give any desired harmonic at any phase.

A fast kicker magnet is used to kick the beam bunch horizontally and drive a coherent x oscillation. The kicker is mounted on the ring in such a way that it also excites a very small but detectable vertical oscillation. The kicker is made of ferrite with a one-turn coil. It can be pulsed for a total pulse length of 350 ns with a flat top of 150 ns.

Four stripline-electrode beam position monitors (BPMs) are located at the quadrupoles QF9, QD9 and Q1, Q2. They measure horizontal and vertical displacements of the beam centroid, X_1, Y_1 and X_2, Y_2 , respectively. Knowing the amplitude functions β_x, β_y and the phase advance between locations 1 and 2, one can then calculate the displacements and slopes at location 1.

Four two-channel ADC digitizers collect the BPM signals from the stripline electrodes. The ADCs are clocked by the rf synthesizer and triggered by the kicker magnet trigger. The data acquisition circuit must be carefully timed so that the maximum beam signal from the single circulating beam bunch can be detected. Fig. 3 shows a block diagram of the data acquisition circuit.

FIGURE 3: Block diagram of the data acquisition circuit

3.2 Data acquisition

Before starting a set of measurements, the closed orbit distortions are corrected to better than $20 \mu\text{m}$ by adjusting the steering magnets and the chromaticity is set to a small positive value to avoid the head-tail instability. We use as small a chromaticity as possible, to minimize the tune spread in the beam bunch.

The most easily accessible difference resonance is $2\nu_y - \nu_x = 7$. The operating tunes are moved close to the resonance by adjusting the quadrupoles manually. The 7th harmonic resonance should not be driven if the 4-fold symmetry of the Aladdin lattice were perfect. However, we found the resonance to be rather strong, indicating that there are sizable sextupole errors in the Aladdin lattice. As the coupling resonance is approached, it can be seen as a sharp increase in vertical amplitude from a TV monitor connected to a camera which is focussed on the synchrotron radiation from the beam. Since we need to control the magnitude of the sextupole driving term, we have first to compensate the sextupole errors. It was relatively easy to adjust the spare sextupoles SD7 and SD10 to eliminate the coupling. The compensation was made with SD7 and SD10 excited by currents of 7.5 A, and -22.5A , respectively. These large currents indicate a relatively large sextupole error somewhere in the Aladdin lattice. In the measurements, we changed either SD7 or SD10 or both, starting from these reference values, to drive the desired sextupole harmonic. The fast kicker magnet is then fired to give the beam bunch an initial controllable amplitude in the horizontal plane. The digitizers are turned on 25 turns before the kick and take data for a total of 4096 turns.

With the measured z_1 and z_2 , one can calculate the slope of the beam centroid at location 1 using the formula

$$p_{z1} = \left[\sqrt{\beta_{z1}/\beta_{z2}} z_2 - z_1 \cos \phi_{21} \right] / \sin \phi_{21} \quad , \quad (25)$$

where $z = x$ or y , and $\phi_{21} = \phi_2 = \phi_1$ is the phase difference between the BPMs which measure z_1 and z_2 . The displacement of the beam centroid is given by

$$z(mm) = 18.9 \cdot \frac{V_a - V_b}{V_a + V_b}, \quad (26)$$

where the V s are signals derived from the stripline electrode pairs through a diode-LC stretch filter. This expression is only approximate with a precision of $\pm 10\%$ over a range of 12 mm.

Figure 4 shows a typical set of measured data. We plot x and y versus turn number and give the frequency spectra of x and y . The tunes correspond to $\epsilon = -0.0044$ [Eq. (10)]. The first coupling peak occurs at about the 275th turn and $|y_{\max}|/|x_0| = 0.70$, where $|x_0|$ is the initial horizontal amplitude. From the spectra, we can easily distinguish the peaks corresponding to betatron tunes ν_x and ν_y . The tunes are rather strongly amplitude dependent due to the second order sextupole effect and the effects of the fringe fields at quadrupole ends. Figure 5 shows two plots of measured $|y_{\max}|$ versus $|x_0|$ at two different sextupole settings. The threshold in $|x_0|$ is clearly shown, and the curves suggest that there is also an upper limit as predicted in the previous section.

4 SIMULATION

Computations of the motion of a single particle through the Aladdin lattice confirm the theory presented in Section 2 in all respects. In Fig. 6 we show the computed x and y motions versus turn number for a typical case above threshold. They correspond to what we deduced from Fig. 1(b), for motion with a small initial y amplitude. We also show in Fig. 6(c) the *Poincaré* map in the (γ_1, J_1) phase plane for the computed orbits, for comparison with Fig. 1(b).

The single-particle results suggest but do not match exactly the experimental results in Fig. 4. The experimental results represent the centroid of the beam bunch containing about 10^{10} electrons. After the kicker is fired the beam centroid does indeed reach a minimum in x and a maximum in y within a few hundred turns. However, due to the amplitude dependence of the tunes, particles tend to spread out over the phase space in such a way that the beam centroid exhibits a damped oscillatory motion.

In order to simulate the decoherent motion, a computer code was developed to track the motion of the centroid of a bunched beam. This code uses the following algebraic transformation:

$$\begin{aligned} X_{j+1} &= X_j \cos \phi_{xj} + P_{xj} \sin \phi_{xj}, \\ P_{xj+1} &= -X_j \sin \phi_{xj} + P_{xj} \cos \phi_{xj} + \Delta P_{xj}, \end{aligned} \quad (27)$$

where X and P_x are defined as $X = x/\sqrt{\beta_x}$, $P_x = (\beta_x p_x + \alpha_x x)/\sqrt{\beta_x}$. Similar transformation is used for the y motion. Nonlinear field effects are included in the code as nonlinear kicks ΔP_x and ΔP_y given by

$$\Delta P_{x,y} = -\sqrt{\beta_{x,y}} \frac{\Delta B_{x,y} \ell}{B \rho}, \quad (28)$$

FIGURE 4: Experimental measurements at $\nu_x = 7.266$, $\nu_y = 7.135$ with sextupole currents SD7: -7.5 A, SD10: 22.5 A.

where the multipole fields ΔB_x and ΔB_y are given by

$$\Delta B_x + i \Delta B_y = \sum_n \frac{B^{(n)}}{n!} (x + iy)^n \quad . \quad (29)$$

Only normal multipoles are included.

Using this transformation, one can track the particle motion from one nonlinear element to the next. The linear lattice parameters are derived from the program COMFORT. The amplitude dependencies of the tunes due to the fringe fields of the quadrupole ends are evaluated from the tracking program MARYLIE under the assumption of the hard edge fringe fields and are simulated by appropriate octupoles. Three octupoles are included in each superperiod. The beam size is represented by a Gaussian distribution in the initial X and P_x phase space, which is generated by a random number generator. The coordinates of the centroid are then calculated after each turn. In accordance with the experimental conditions, we have set the initial y amplitude very small for all particles.

FIGURE 5: Peak y amplitude vs initial x amplitude

FIGURE 6: Computed motion of a single particle

Figure 7 shows the simulation results corresponding to the measurements shown in Fig. 4. The first coupling peak occurs at about the 300th turn and it gives $|y_{\max}|/|x_{\max}| = 0.71$. The octupole components damp the oscillations of the beam centroid and contribute to the nonlinear tune shifts. The simulated results agree rather well with the experimental measurements, showing all the observed features with roughly the correct magnitudes. It has not been possible to extract the resonance phase plane coordinates from the experimental data, because they depend sensitively on the relative magnitudes and phases of the x and y oscillations.

5 CONCLUSION

We have measured the motion of the electron beam in the storage ring Aladdin under the influence of the nonlinear forces due to sextupole magnets. The general features of the single particle dynamics near the third-order difference resonance $\nu_x - 2\nu_y = m$ are

FIGURE 7: Simulation results for $\nu_x = 7.266/\nu_y = 7.135$, 100 particles

qualitatively confirmed by the measurement. These features include: (1) the existence of both lower (threshold) and upper limits in x amplitude for the resonant coupling between the x motion and the y motion; and (2) the relatively fast rise and fall of the y amplitude when resonant coupling occurs. Detailed quantitative comparison between analysis and measurement is difficult because of the unknown errors in the ring and the large emittance of the beam covering a fairly large area in the phase space. Computer simulation using multiparticle tracking shows a good agreement with measurements. We have thus established the validity of the analytical and numerical treatments of the nonlinear motion near the difference resonance. Earlier we derived similar conclusions for the third-integral resonance.

On the experimental side a large part of the effort was devoted to compensating the observable error-effects in the ring and to calibrating the BPM readings to obtain the true position of the beam with adequate precision. The precise setting of the betatron tunes and chromaticities was also time consuming. The beam kicking and the recording of the digitized readout of the four BPMs for 4096 turns was, however, rather straightforward.

We have modified the kicker to kick the beam in both the horizontal and vertical planes. This will provide the capabilities for future extended studies which will include also the sum resonance $2\nu_y + \nu_x = 22$.

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