A proposal to resolve the black hole information paradox*

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The entropy and information puzzles arising from black holes cannot be resolved if quantum gravity effects remain confined to a microscopic scale. We use concrete computations in nonperturbative string theory to argue for three kinds of nonlocal effects that operate over macroscopic distances. These effects arise when we make a bound state of a large number of branes, and occur at the correct scale to resolve the paradoxes associated with black holes.

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Black holes have a ‘Bekenstein entropy’ \( S_{\text{Bek}} = A/4G \). Are there \( e^{S_{\text{Bek}}} \) microstates of the hole, and if so, then where do we see the differences between them? The Hawking radiation from the hole appears to be reliably given by a semiclassical calculation, but the radiation thus computed carries little information about the infalling matter. Is unitarity violated by black holes?

The essential strength of these puzzles lies in the fact that the more massive we make the hole, the smoother its geometry becomes at the horizon. Incoming matter disappears into the central singularity while Hawking radiation arises from vacuum fluctuations near the smooth horizon. But the horizon is a large distance – \( R_s \), the Schwarzschild radius – from the singularity, so the radiation is unable to encode the information of the infalling matter.

While everyone agrees that nonlocal physics can arise at a microscopic length scale like planck length or string length, what we need to effect the information transfer is nonlocal effects across spatial lengths of order \( R_s \), which is a length that increases with the mass of the hole. Thus in our theory of gravity if we make a bound state of a large number of quanta, then nonlocal effects should operate not at a fixed microscopic length but rather at a length that increases with the number of quanta in the bound state.

In this article we argue that we must modify our intuition about quantum gravity to include three phenomena where quantum gravity effects indeed reach out to macroscopic length scales:

(a) Bound states of branes have a nonzero size; it is important that this size grows with the number of branes.

(b) When we make a bound state from a large number \( N \) of branes then the excitations of this bound state are given in terms of fractionated branes which have a tension \( \sim 1/N \); thus the virtual fluctuations of these fractionated branes extend to macroscopic distances when \( N \) is macroscopic.

(c) The concept of limited stretchability, which says that if spatial hypersurfaces ‘stretch too much’ during evolution then we encounter a breakdown of the semiclassical approximation even though there are no large curvatures anywhere – rather, the degrees of freedom on the hypersurface have become too ‘dilute’.

We show that these principles are supported by concrete calculations in string theory, and that they fit together to provide a resolution of the paradoxes arising from black holes.\(^1\)

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\(^1\) For some other attempts at resolving the information puzzle (both related and unrelated) see [1].
We will extract evidence for these principles from computations in IIB string theory, which lives in 9+1 spacetime dimensions. Five space directions are compactified; we write these as $T^4 \times S^1$. One system of interest is the D1-D5 system – we wrap $n_5$ D5 branes on $T^4 \times S^1$, and $n_1$ D1 branes on $S^1$. The bound state of these branes has a highly degenerate ground state, with entropy $S_{\text{micro}} = 2\sqrt{2}\sqrt{n_1 n_5}$. The volume of $T^4$ is $(2\pi)^4 V$, the radius of $S^1$ is $R$, the string coupling is $g$, and the string tension is $\alpha'$. A set of dualities maps this ‘2-charge system’ to another 2-charge system – the FP bound state, where we have $n_5$ fundamental strings wrapped on $S^1$, carrying $n_1$ units of momentum along $S^1$.

Let us first look at the size of bound states. Consider a fundamental string (F) carrying momentum (P). The string possesses no longitudinal vibration modes, so to carry the momentum the string must bend away from its central axis. The FP state thus acquires a certain transverse size; the more the momentum, the larger this size.

Had the bound state been pointlike, the metric would have ended in a point singularity at $r = 0$. But the finite size of the bound state modifies this naive metric inside a region $r < r_0$. A recent analysis of such metrics yielded an interesting set of results. For the same total energy the string can carry different vibration profiles, and these give metrics that differ for $r < r_0$. [2][3]. Let us coarse-grain over these ‘microstates’ by truncating the geometry at the surface $r = r_0$. From the area $A$ of this ‘horizon’ we find [4]

$$S_{Bek} \equiv A/4G \sim \sqrt{n_1 n_5} \sim S_{\text{micro}}!$$

Further, a quantum that falls past this ‘horizon’ gets ‘trapped’ in the complicated geometry at $r < r_0$, for times $\sim 1/\hbar$. So from a classical perspective, this surface indeed behaves like a horizon.

Thus for the extremal 2-charge system we resolve the ‘entropy puzzle’: we directly relate the ‘area entropy’ to a coarse graining over microstates (‘hair’). Tracing back, the critical fact was that the bound states had a transverse size which grew with the charges at just the correct rate to always reach a ‘horizon’ which satisfies (1).

Can such a picture can be extended to the 3-charge system, which can be constructed by adding $n_p$ units of momentum along $S^1$ to the D1-D5 bound state? The 3-charge system has a classical size horizon radius, so we need to understand how the microstate develops an effective size that is classical. The key notion (for both two and three charge states) is fractionation. Consider a string wrapped on a circle of radius $R$. The minimum excitation energy (for no net momentum along the string) is $\frac{1}{R} + \frac{1}{R} = \frac{2}{R}$. But now let the string
wind $N$ times around the circle before closing on itself. Now the vibration modes can have a wavelength $2\pi RN$, and the energy gap drops to $\frac{2}{N \pi R}$ [5].

If we have a bound state of D1 and D5 branes then for vibrations along $S^1$ the energy spacing drops to $\frac{2}{n_1 n_5 R}$ [6]. Clearly, the larger we make the hole, the lighter will be its excitations. In [7] the effective size of the D1-D5-momentum bound state resulting from ‘fractionated’ degrees of freedom was estimated at weak coupling, and was found to be

$$r_0 \sim \left[ \frac{(n_1 n_5 n_n)^{1/2} g^2 \alpha'^4}{V R} \right]^{1/3}$$

But this is the same algebraic function of these six parameters that gives (upto a factor of order unity) the Schwarzschild radius of the 3-charge hole! These results suggest that fractionation gives bound states of branes a size that always equals the horizon radius, We picture a ‘fuzz’ of virtual brane-antibrane pairs extending out to the horizon. A massive infalling particle falls through this fuzz to the center, but over a longer time scale (order $1/h$) these virtual fluctuations can transport the information in the particle to horizon and into the outgoing Hawking radiation.

Let us now address the information paradox. Broadly speaking, we can consider two kinds of foliations in analyzing the radiation. In the first kind, slices are $t =$ constant for $r >> 2m$, but inside the horizon they extend into the singularity. Can the singularity at $r = 0$ modify radiation at $r = 2M$? Yes, since fractionation generates effects that extend to the horizon!

But an apparently stronger argument for information loss can be made by using a foliation where the slices never approach the singularity, and there are no large curvatures anywhere. It was shown in [8] that in such foliations the spacelike slices stretch in the course of evolution by a large factor $\sim 1/h$. It was suggested that spacelike hypersurfaces should be regarded as ‘rubber sheets’ that have not only a shape but a density of degrees of freedom. Stretching dilutes these degrees, and if more matter data is placed on the slice than the number of available ‘bits’ then nonlocal effects occur. Quantitatively, if we assume that a volume $V$ in flat space can stretch to a ‘throat’ with maximum depth $\sim V/G$ then the information paradox is resolved – every foliation has slices that have a singularity or ‘stretch too much’, and the semiclassical derivation of Hawking radiation is invalidated in each case.

Remarkably, a similar picture emerged in the exact analysis of throat geometries of the D1-D5 system, which was mapped by AdS/CFT duality to a 1+1 sigma model [3].
The throat had different depths for different microstates. The dual sigma model described the geometry in terms of a collection of ‘component strings’. Longer throats (‘stretched geometries’) were described in the CFT by fewer component strings. When enough quanta were placed in the throat so that each component string in the dual description was excited (all ‘bits’ were used up), the physics underwent a qualitative change. Lastly, the maximum depth of the throat was exactly $V/2G$!

To summarize, fractionation generates very light degrees of freedom; the virtual fluctuations of these degrees can transport the information in an infalling particle all the way to the horizon (over times of order $1/\hbar$). If we consider a foliation by ‘regular slices’ that avoid the singular source of these virtual quanta then the slices ‘stretch’ by a factor $O(1/\hbar)$. AdS/CFT duality supports the idea that degrees of freedom along the slice are limited and get ‘overdiluted’ under such extreme stretching, so that semiclassical behavior breaks down. Physics in the absence of black holes, studied on ‘regular’ foliations, remains unaffected by all these phenomena, and we resolve all puzzles associated with black holes.
References
