ENERGY LOSS MEASUREMENTS AT LEP2

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Abstract

The accurate measurement of the W Boson mass at LEP requires to determine the beam energy to the highest possible precision. Present schemes rely on accurate energy determinations in the range from 40 to 60 GeV using resonant depolarization and on precise extrapolations to high energy. Several methods based on measurements of the energy loss due to synchrotron radiation have been studied. Different approaches such as the study of the damping time of transverse oscillations, the radio frequency sawtooth and the dependence of the synchrotron tune on the total accelerating voltage are described and results are discussed.

1 MOTIVATION

LEP2 provides a rather unique opportunity to study energy loss and synchrotron frequency in an environment with major energy losses and high $Q'$. The main motivation for the studies presented below however is the development of a reliable energy determination at the highest energies with an accuracy of 20 MeV or better. The presently used extrapolation methods using magnetic measurements cross-calibrated with resonant depolarization in the range from 40 to 60 GeV show systematic effects of the order of 20 MeV at highest energies [1]. The methods mentioned below are alternatives, based mainly on determinations of the energy loss and using existing LEP equipment.

2 MEASUREMENTS

2.1 Damping of Coherent Oscillations

A coherent horizontal oscillation is excited by a single kick and the center-of-charge position of the bunch is observed over 1024 consecutive turns. A fit to the data by a damped oscillation with amplitude dependent frequency yields the coherent damping time $\tau_{coh}$ as described in [2]. The coherent damping at LEP is composed of radiation and head-tail damping:

$$1/\tau_{coh} = 1/\tau_0 + 1/\tau_{head-tail} \quad \text{with} \quad 1/\tau_{head-tail} \sim \frac{Q'}{E_0} l_s$$

where $Q'$ is the chromaticity, $l_s$ the bunch current and $E_0$ the beam energy. An extrapolation to $l_s = 0$ and $E_0$ the energy loss or energy can be extracted. Table 1 gives the results for measurements at 60 and 45.625 GeV. Although the measurements are in good agreement with the MAD [3] predictions, the resulting relative energy uncertainty is of the order of $O(1\%)$.

2.2 The Energy Sawtooth

The horizontal beam position is a function of the local momentum. The continuous energy loss in the arc sections leads to sawtooth-like horizontal orbits in the LEP ring. The difference between the positron and electron orbits can be used to determine the energy loss with the help of the horizontal dispersion. Results of fits to the sawtooth are shown in figure 1 where the energy loss is plotted as a function of the day in the year. Details on the fit method can be found in [4]. The fit results seem to scatter around a central value but there are clear “jumps” some of which correspond to BPM calibrations (dashed lines). The other jumps could not yet be accounted for. The RMS of the energy loss distribution before day 275 is relatively small and corresponds to a relative uncertainty of the energy of around $O(5 \cdot 10^{-4})$. This method however is strongly limited by systematic effects. The fit results differ between the octants, depend on the selection of rejected (faulty) BPMs and exhibit “jumps”

<table>
<thead>
<tr>
<th>energy [GeV]</th>
<th>energy loss [MeV]</th>
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<tbody>
<tr>
<td>45.625</td>
<td>127</td>
</tr>
<tr>
<td>60.000</td>
<td>380</td>
</tr>
</tbody>
</table>

Table 1: Results and MAD predictions of the energy loss due to synchrotron radiation at 45.6 and 60 GeV beam energy.

Figure 1: Energy loss from the sawtooth fits in MeV as function of day in year. The dashed lines denote BPM calibrations.

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sometimes correlated to BPM calibrations. The good intrinsic accuracy and the parasitic measurement favor this method but the systematics are not yet under control.

2.3 \( Q_s \) and total RF Voltage

As the synchrotron tune depends on the beam energy as well as on energy loss \( U_0 \) and total RF voltage \( V_{RF} \), measurements of these dependencies can be used to determine the beam energy. The upper plot of fig. 2 shows a measurement of the synchrotron tune as function of total RF voltage at 50.005 GeV. The curve is a best fit according to

\[
Q_s^2 = \left( \frac{\alpha_c \hbar}{2\pi E} \right) \sqrt{\frac{V_{RF}^2 - U_0^2}{\Delta K}} \tag{1}
\]

with \( U_0 = C_e / \rho E^4 \), \( \alpha_c \) being the momentum compaction factor, \( h \) the harmonic number and \( \rho \) the average magnetic radius. This analytical model is only valid when the RF voltage is homogenously distributed along the ring and for slow synchrotron oscillations. It has to be refined to take into account the large energy loss of \( U_0 / E \sim 2\% \) at highest energies. The bottom plot of fig. 2 shows the difference between data and fit. Residuals and \( \chi^2 \) show the sensitivity of the data to additional corrections which can be determined in the fit or included as constraints from separate measurements and calculations. A first step is the correction of the energy for differences between the central frequency and the actual RF frequency

\[
E_c = E \left( 1 - \frac{1}{\alpha_c} \left( \frac{f_{RF} - f_{RF}^c}{f_{RF}^c} \right) \right) \tag{2}
\]

and the introduction of a “voltage correction factor” \( V_{RF} \rightarrow g V_{RF} \) to take care of RF voltage calibration and phasing errors. In addition to the synchrotron radiation loss in dipoles other energy losses have to be taken into account: energy loss from quadrupoles due to sawtooth and closed orbit distortions, energy loss from correctors, parasitic mode losses, corrections due to finite beam size and to the momentum offset due to central frequency and tides. The finite beam size adds a contribution equivalent to a shift of the beam by one RMS beam size. The sum of these losses is \( K = 2.57 \text{ MeV at 50.005 GeV and } K = 2.65 \text{ MeV at 60.589 GeV with an overall uncertainty of } \Delta K = \pm 0.50 \text{ MeV.} \) The total energy loss \( U_0 \) used in eq.(1) is finally

\[
\tilde{U}_0 = \frac{C_e}{\rho} E^4 + K \tag{3}
\]

To study the fit quality and to find a correction for the unequally distributed RF voltage, the model was tested with the MAD simulation program. Figure 3 shows \( Q_s \) generated for a beam energy of 50.005 GeV with different RF configurations: a realistic case with the normal LEP RF distribution, a case where the same total voltage is concentrated in one point and the limit of a homogenous distribution where the voltage is distributed over the whole ring. To account for the RF distribution, a term proportional to \( V_{RF}^4 \) has to be added to eq.(1). The weight factor \( M \) is taken from the fit to the MAD dataset. The final model reads as follows:

\[
Q_s^4 = \left( \frac{\alpha_c \hbar}{2\pi} \right)^2 \left\{ \frac{g^2 V_{RF}^c}{E_c^4} + M g^4 V_{RF}^4 - \frac{1}{E_c^4} \tilde{U}_0^2 \right\} \tag{4}
\]

with the relations from equations (2,3). The energy extracted from a fit to simulation data is in good agreement with the input energy. A systematic uncertainty of \( \pm 10 \text{ MeV} \) is assigned to the fit results. Figure 4 shows the measurements at 50.005 and 60.589 GeV. The curves are best fits using the “final” model of eq.(4). The momentum compaction factor \( \alpha_c \) and the voltage nonlinearity factor \( M \) are taken from MAD. All other parameters were allowed to vary in the fit. Figure 5 shows the residuals of the fit to the 50.005 GeV data. It is clearly visible that the model is able to reproduce the measurements quite accurately. External knowledge was incorporated in the fit by introducing constraints of the type \( (a - a_{\text{corr}})/\sigma_a \) where \( a \) stands for a...
fit parameter and \( \sigma_q \) for its estimated uncertainty to the \( \chi^2 \) function. \( q_{\text{nom}} \) is the value the parameter is constrained to. The beam energy was constrained to the nominal energy, \( \sigma_E \) was set to \( \pm 50 \text{ MeV} \). The central value of \( K \) was set to the given values with an error of \( \pm 0.5 \text{ MeV} \). The voltage correction factor \( g \) was constrained to the average value obtained from the measurements. \( \sigma_g \) was estimated from the spread of the results. The value of \( g = 0.95415 \pm 0.00005 \) implies that the effective voltage is about \( 5\% \) less than the nominal voltage. In table 2 fit results are compared to the

![Figure 4](image)

Figure 4: Measurements of \( Q_x \) as function of total RF voltage. The curves are the results of the fits according to eq. (4).

<table>
<thead>
<tr>
<th>( E_{\text{nom}} )</th>
<th>( E_{\text{fit}} )</th>
<th>( \Delta E/E )</th>
<th>( E_{\text{pol}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.005 GeV</td>
<td>50.013 ( \pm 0.026 )</td>
<td>( 5.2 \cdot 10^{-4} )</td>
<td>50.020</td>
</tr>
<tr>
<td>60.589 GeV</td>
<td>60.576 ( \pm 0.021 )</td>
<td>( 3.5 \cdot 10^{-4} )</td>
<td>60.597</td>
</tr>
</tbody>
</table>

Table 2: Results of the fits using the model of eq. (4). All energies are given in GeV. The systematic uncertainty assigned to the results from studies with MAD is \( \pm 10 \text{ MeV} \).

nominal machine energy (obtained from the magnet calibration curves) and to the energies measured with resonant depolarization in the following fill. The impact of the energy constraint on the final errors is small. For both measurements the fitted energies are lower than the polarization energies but still agree within their errors. The error is of the required magnitude. Preliminary measurements at high energy indicate that the absolute error is essentially energy independent, \( \Delta E \approx 25 \text{ MeV} \). This value can be improved if tighter bounds can be set on \( K \) and \( g \). If the energy is known from other measurements (resonant depolarization) the same fits can be used to extract the momentum compaction factor. The relative uncertainty resulting from this method is \( \Delta q / q \approx 1 \cdot 10^{-3} \) whereas conventional measurements have relative uncertainties of 1 - 2\%. All measurements of the momentum compaction factor are in good agreement with the MAD value.

3 SUMMARY

Several methods to measure the energy from the energy loss using existing LEP equipment have been studied. To be useful, the relative calibration uncertainty should not exceed a few times \( 10^{-4} \). The determination of the energy loss from the damping of coherent oscillations gives a relative error of \( O(1\%) \). The measurement of the energy loss using the energy sawtooth has a good intrinsic accuracy (\( O(5 \cdot 10^{-4}) \)) and does not require dedicated beam time. However the systematics are not yet under control. The most promising method is the measurement of the synchrotron tune as function of the total RF voltage. External information is introduced into the fit in a controlled way which also allows to assess sensitivities to input parameters and simulation biases. A relative energy error of \( \Delta E/E = 2.8 \cdot 10^{-4} \) has been reached and further improvements are possible.

4 ACKNOWLEDGEMENTS

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5 REFERENCES