CP violation in charged Higgs decays in the MSSM with complex parameters

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Abstract

Supersymmetric loop contributions can lead to different decay rates of $H^+ \to t\bar{b}$ and $H^- \to b\bar{t}$. We calculate the decay rate asymmetry $\delta^{CP} = [\Gamma(H^+ \to t\bar{b}) - \Gamma(H^- \to b\bar{t})] / [\Gamma(H^+ \to t\bar{b}) + \Gamma(H^- \to b\bar{t})]$ at next-to-leading order in the MSSM with complex parameters. We analyse the parameter dependence of $\delta^{CP}$ with emphasis on the phases of $A_t$ and $A_b$. It turns out that the most important contribution comes from the loop with stop, sbottom, and gluino. If this contribution is present, $\delta^{CP}$ can go up to $\sim 10\text{--}15\%$ for $\tan\beta \sim 10$, and to $\sim 5\%$ for very large values of $\tan\beta$. 

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1 Introduction

Already for some time, it has been customary to look for CP-violating effects beyond the Standard Model (SM). All extensions of the SM contain possible new sources of CP violation through additional CP-violating phases. In particular, in the Minimal Supersymmetric Standard Model (MSSM), the Higgs mixing parameter $\mu$ in the superpotential, two of the soft SUSY-breaking Majorana gaugino masses $M_i$ ($i = 1, 2, 3$), and the trilinear couplings $A_f$ (corresponding to a fermion $f$) can have physical phases, which cannot be rotated away without introducing phases in other couplings [1]. From the point of view of baryogenesis, one might hope that these phases are large [2]. On the other hand, the experimental limits on electron and neutron electric dipole moments (EDMs) [3], $|d_e| \leq 2.15 \times 10^{-13}$ e/GeV, $|d_n| \leq 5.5 \times 10^{-12}$ e/GeV, place severe constraints on the phase of $\mu$, $\phi_\mu < \mathcal{O}(10^{-2})$ [4], for a typical SUSY mass scale of the order of a few hundred GeV. A larger $\phi_\mu$ imposes fine-tuned relationships between this phase and other SUSY parameters [5]. CP-violating effects that might arise from $A_{u,d}$ (where $u,d$ are light quarks) are very much suppressed as they are proportional to $m_{u,d}$. On the other hand, the trilinear couplings of the third generation $A_{t,b,\tau}$ can lead to significant CP-violation effects, especially in top quark physics [6]. Phases of $\mu$ and $A_{t,b,\tau}$ also affect the Higgs sector in a relevant way. Although the Higgs potential of the MSSM is invariant under CP at tree level, at loop level CP is sizeably violated by complex couplings [7, 8]. As a consequence, the three neutral Higgs mass eigenstates are superpositions of the CP eigenstates $h^0$, $H^0$, and $A^0$.

In this paper, we study CP violation in the decays $H^+ \rightarrow t\bar{b}$ and $H^- \rightarrow b\bar{t}$ in the MSSM with complex parameters. In particular, we calculate the CP-violating asymmetry

$$\delta^{CP} = \frac{\Gamma(H^+ \rightarrow t\bar{b}) - \Gamma(H^- \rightarrow b\bar{t})}{\Gamma(H^+ \rightarrow t\bar{b}) + \Gamma(H^- \rightarrow b\bar{t})},$$

(1)
due to one-loop exchanges of $\tilde{t}$, $\tilde{b}$, $\tilde{g}$, $\tilde{\chi}^\pm$, $\tilde{\chi}^0$, and $H^0$, see Fig. 1, taking into account CP violation in the neutral Higgs system according to [8]. Of course, the diagrams of Fig. 1 only contribute to $\delta^{CP}$ if they have an absorptive part. Since $\phi_\mu$ is highly constrained, the most important phases in our analysis are $\phi_t$ and $\phi_b$, the phases of $A_t$ and $A_b$. Therefore, we expect the graph with $\tilde{t}$, $\tilde{b}$, and $\tilde{g}$ in the loop to be the most important one, and $\delta^{CP}$ to be large in the case
The paper is organized as follows: In Section 2 we give the basic formulae for the $H^\pm \rightarrow tb$ decays and define the decay rate asymmetry $\delta^{CP}$ at the 1-loop level in terms of CP-violating form factors $\delta Y_{i}^{CP}$ ($i = t, b$). The explicit expressions for $\delta Y_{t,b}^{CP}$ due to the diagrams of Fig. 1 are given in Section 3. In Section 4, we perform a detailed numerical analysis. In Section 5, we summarize our results and comment on the feasibility of measuring the CP-violating asymmetry $\delta^{CP}$. Appendices A, B, and C contain the necessary mass and mixing matrices, the couplings, and the definition of the two- and three-point functions used in this paper.

2 The $H^\pm$ decay

The matrix elements of the $H^+ \rightarrow t\bar{b}$ and $H^- \rightarrow b\bar{t}$ decays can be written as

\[
M_{H^+} = \bar{u}(p_t) \left[ Y_b^+ P_R + Y_t^+ P_L \right] v(-p_b),
\]

\[
M_{H^-} = \bar{u}(p_b) \left[ Y_t^- P_R + Y_b^- P_L \right] v(-p_t),
\]

with $P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$ and the loop-corrected couplings

\[
Y_i^\pm = y_i + \delta Y_i^\pm \quad (i = t, b);
\]

\[
y_t = h_t \cos \beta, \quad y_b = h_b \sin \beta,
\]

with $h_t$ and $h_b$ the top and bottom Yukawa couplings. The decay widths at tree level are given by

\[
\Gamma^0 (H^\pm \rightarrow tb) = \frac{3\kappa}{16\pi m_{H^+}^3} \left[ (m_{H^+}^2 - m_t^2 - m_b^2)(y_t^2 + y_b^2) - 4m_t m_b y_t y_b \right],
\]

where \( \kappa = \kappa(m_{H^+}^2, m_t^2, m_b^2), \kappa(x, y, z) = [(x - y - z)^2 - 4yz]^{1/2} \). Since there is no CP violation at tree level, $\Gamma^0(H^+ \rightarrow t\bar{b}) = \Gamma^0(H^- \rightarrow b\bar{t})$. At next-to-leading order (NLO) we have

\[
\Gamma (H^\pm \rightarrow tb) = \frac{3\kappa}{16\pi m_{H^+}^3} \left[ (m_{H^+}^2 - m_t^2 - m_b^2)(y_t^2 + y_b^2 + 2y_t \text{ Re} \delta Y_t^\pm + 2y_b \text{ Re} \delta Y_b^\pm) \right.
\]

\[
- 4m_t m_b (y_t y_b + y_t \text{ Re} \delta Y_b^\pm + y_b \text{ Re} \delta Y_t^\pm) \bigg].
\]
The form factors $\delta Y^\pm_i (i = t, b)$ have, in general, both CP-invariant and CP-violating contributions:

$$\delta Y^\pm_i = \delta Y^{\text{inv}}_i \pm \frac{1}{2} \delta Y^{\text{CP}}_i.$$  \hspace{1cm} (8)

Both the CP-invariant and the CP-violating contributions have real and imaginary (absorptive) parts. CP invariance implies that the form factors of $H^+$ and $H^-$ are equal. Using Eqs. (7) and (8), we can write the CP-violating asymmetry $\delta^{CP}$ of Eq. (1) as

$$\delta^{CP} = \frac{2\Delta (y_t \Re \delta Y^{CP}_t + y_b \Re \delta Y^{CP}_b) - 4m_t m_b (y_t \Re \delta Y^{CP}_b + y_b \Re \delta Y^{CP}_t)}{\Delta (y_t^2 + y_b^2) - 4m_t m_b y_t y_b},$$  \hspace{1cm} (9)

where $\Delta = m_{H^+}^2 - m_t^2 - m_b^2$.

At one loop, there are six generic diagrams that may contribute to $\delta^{CP}$. These are (A) triangle diagrams with (i) two fermions and a scalar, (ii) two scalars and a fermion, and (iii) a scalar, a vector boson and a fermion in the loop, and (B) $H^+ - W^+$ self-energy diagrams with (i) two fermions, (ii) two scalars, and (iii) a scalar and a vector boson in the loop. In the following, we work out the formulae for $\delta Y^{CP}_i$ for the specific case of the MSSM with complex phases. The relevant Feynman diagrams are shown in Fig. 1.

## 3 CP-violating contributions

### 3.1 Generic diagrams

According to Eqs. (2) and (4) we write the matrix elements of the 1-loop diagrams of Fig. 1 as

$$\mathcal{M}_{H^+} = \bar{u}(p_t) \left[ \delta Y^+_b P_R + \delta Y^+_t P_L \right] v(-p_b),$$  \hspace{1cm} (10)

and analogously for the $H^-$ decay. In fact, we are only interested in the CP-violating parts $\Re \delta Y^{CP}_i$. Since $\Re \delta Y^{CP}_i = \Re (\delta Y^+_i - \delta Y^-_i)$, we just need the structure $\Im (g_0 g_1 g_2) \times \Im \mathcal{P} \mathcal{V} \mathcal{E}$ of the form factors $\delta Y^+_b$ and $\delta Y^+_t$. Here $g_0 g_1 g_2$ stands for the product of the couplings and $\mathcal{P} \mathcal{V} \mathcal{E}$ for the Passarino–Veltman two- and three-point functions [9] $B_0$ and $C_{0,1,2}$. In the following, we give the formulae for the various contributions to $\Re \delta Y^{CP}_{t,b}$. The necessary MSSM mass and
mixing matrices, the couplings, as well as the definition of the two- and three-point functions are given in Appendices A, B, and C.

3.2 Vertex graphs

3.2.1 Neutralino–chargino–stop (sbottom) loop

The graph of Fig. 1a, with a neutralino, a chargino, and a stop in the loop, leads to

\[
\text{Re} \delta Y_b^{CP}(\tilde{\chi}_k^0\tilde{\chi}_j^{\pm}\tilde{t}_i) = \frac{1}{8\pi^2} \left\{ \begin{array}{l}
m_t m_b \text{Im}(F_j^{\tilde{R}} a_{ik}^t l_i^t) + m_{\tilde{\chi}_j^0} m_{\tilde{\chi}_k^0} \text{Im}(F_j^{\tilde{R}} a_{ik}^t l_i^t) + m_t m_{\tilde{\chi}_j^0} \text{Im}(F_j^{\tilde{R}} a_{ik}^t l_i^t) \\
+ m_b m_{\tilde{\chi}_k^0} \text{Im}(F_j^{\tilde{R}} a_{ik}^t l_i^t) + m_{\tilde{\chi}_k^0} \text{Im}(F_j^{\tilde{R}} a_{ik}^t l_i^t) \end{array} \right\} \text{Im}(C_0) \\
+ m_t \left[ m_t \text{Im}(F_j^{\tilde{L}} a_{ik}^t l_i^t) + m_{\tilde{\chi}_j^0} \text{Im}(F_j^{\tilde{L}} a_{ik}^t l_i^t) + m_b \text{Im}(F_j^{\tilde{R}} a_{ik}^t l_i^t) + m_{\tilde{\chi}_j^0} \text{Im}(F_j^{\tilde{R}} a_{ik}^t l_i^t) \right] \text{Im}(C_1) \\
+ m_b \left[ m_t \text{Im}(F_j^{\tilde{L}} a_{ik}^t l_i^t) + m_{\tilde{\chi}_j^0} \text{Im}(F_j^{\tilde{L}} a_{ik}^t l_i^t) + m_b \text{Im}(F_j^{\tilde{R}} a_{ik}^t l_i^t) + m_{\tilde{\chi}_j^0} \text{Im}(F_j^{\tilde{R}} a_{ik}^t l_i^t) \right] \text{Im}(C_2) \\
+ \text{Im}(F_j^{\tilde{L}} a_{ik}^t l_i^t) \text{Im}(B_0(m_{H^0}^2, m_{\tilde{\chi}_j^0}^2, m_{\tilde{\chi}_j^0}^2)) \right\}, \tag{11}
\]

with \(C_X = C_X(m_t^2, m_{H^0}^2, m_b^2, m_{\tilde{\chi}_j^0}^2, m_{\tilde{\chi}_k^0}^2, m_{\tilde{\chi}_j^0}^2), X = 0, 1, 2,\) the three-point functions [9] in the notation of [10]; \(\text{Re} \delta Y_t^{CP}(\tilde{\chi}_k^0\tilde{\chi}_j^{\pm}\tilde{t}_i)\) is obtained from Eq. (11) by interchanging \(F_{jk}^{\tilde{R} \leftarrow \tilde{L}}, a_{ik}^t \leftrightarrow b_{ik}^t,\)
Therefore, the contribution from the neutralino–chargino–sbottom loop has exactly the same structure. The contribution from the diagram with a stop, a sbottom, and a gluino in Fig. 1b is

\[ \text{Re } \delta Y^C_P (\tilde{\chi}^0_k \tilde{\chi}^0_j \tilde{b}_i) \] obtained from Eq. (11) by the following substitutions: for the masses of the loop particles \( m_\tilde{\chi}_k \rightarrow m_\tilde{\chi}^+_j, m_\tilde{\chi}_j \rightarrow m_\tilde{\chi}^+_i, m_{\tilde{t}_i} \rightarrow m_{\tilde{b}_i}; \) for the couplings \( \tilde{a}^{\tilde{t}}_{ik} \rightarrow \tilde{t}_{ij}, \tilde{b}^{\tilde{t}}_{ik} \rightarrow \tilde{k}_{ij}, \tilde{t}^{\tilde{t}}_{ij} \rightarrow \tilde{t}^{\tilde{b}}_{ik}, \) and \( \tilde{t}^{\tilde{t}}_{ij} \rightarrow \tilde{a}^{\tilde{b}}_{ik}; \) and analogously for \( \text{Re } \delta Y^C_P (\tilde{\chi}^0_k \tilde{\chi}^0_j \tilde{b}_i). \)

### 3.2.2 Stop–sbottom–neutralino loop

The stop–sbottom–neutralino loop of Fig. 1b gives

\[
\text{Re } \delta Y^C_P (\tilde{t}_i \tilde{b}_j \tilde{\chi}^0_k) = \frac{1}{8\pi^2} \left[ m_{\tilde{\chi}_k} \text{Im}(G_{4ij} \tilde{a}^{\tilde{t}}_{ik} \tilde{b}^{\tilde{s}}_{jk}) \text{Im}(C_0) - m_t \text{Im}(G_{4ij} \tilde{b}^{\tilde{t}}_{ik} \tilde{b}^{\tilde{s}}_{jk}) \text{Im}(C_1) - m_b \text{Im}(G_{4ij} \tilde{a}^{\tilde{t}}_{ik} \tilde{b}^{\tilde{s}}_{jk}) \text{Im}(C_2) \right], \tag{12}
\]

\[
\text{Re } \delta Y^C_P (\tilde{t}_i \tilde{b}_j \tilde{\chi}^0_k) = \frac{1}{8\pi^2} \left[ m_{\tilde{\chi}_k} \text{Im}(G_{4ij} \tilde{b}^{\tilde{t}}_{ik} \tilde{a}^{\tilde{s}}_{jk}) \text{Im}(C_0) - m_t \text{Im}(G_{4ij} \tilde{a}^{\tilde{t}}_{ik} \tilde{a}^{\tilde{s}}_{jk}) \text{Im}(C_1) - m_b \text{Im}(G_{4ij} \tilde{b}^{\tilde{t}}_{ik} \tilde{a}^{\tilde{s}}_{jk}) \text{Im}(C_2) \right], \tag{13}
\]

with \( C_X = C_X(m_\tilde{t}_i, m^2_{H^+}, m_b, m^2_{\tilde{\chi}_k}, m^2_{\tilde{t}_i}, m^2_{\tilde{b}_j}). \)

### 3.2.3 Stop–sbottom–gluino loop

The contribution from the diagram with a stop, a sbottom, and a gluino in Fig. 1b is

\[
\text{Re } \delta Y^C_P (\tilde{t}_i \tilde{b}_j \tilde{g}) = -\frac{4}{3} \frac{\alpha_s}{\pi} \left[ m_{\tilde{g}} \text{Im}(G_{4ij} \tilde{R}^{\tilde{t}}_{i1} \tilde{R}^{\tilde{b}}_{j2} e^{i\phi_3}) \text{Im}(C_0) + m_t \text{Im}(G_{4ij} \tilde{R}^{\tilde{t}}_{i1} \tilde{R}^{\tilde{b}}_{j2}) \text{Im}(C_1) + m_b \text{Im}(G_{4ij} \tilde{R}^{\tilde{t}}_{i1} \tilde{R}^{\tilde{b}}_{j2}) \text{Im}(C_2) \right], \tag{14}
\]

\[
\text{Re } \delta Y^C_P (\tilde{t}_i \tilde{b}_j \tilde{g}) = -\frac{4}{3} \frac{\alpha_s}{\pi} \left[ m_{\tilde{g}} \text{Im}(G_{4ij} \tilde{R}^{\tilde{t}}_{i2} \tilde{R}^{\tilde{b}}_{j1} e^{-i\phi_3}) \text{Im}(C_0) + m_t \text{Im}(G_{4ij} \tilde{R}^{\tilde{t}}_{i2} \tilde{R}^{\tilde{b}}_{j1}) \text{Im}(C_1) + m_b \text{Im}(G_{4ij} \tilde{R}^{\tilde{t}}_{i2} \tilde{R}^{\tilde{b}}_{j1}) \text{Im}(C_2) \right], \tag{15}
\]

with \( C_X = C_X(m_\tilde{t}_i, m^2_{H^+}, m_b, m^2_{\tilde{g}}, m^2_{\tilde{t}_i}, m^2_{\tilde{b}_j}) \) and \( \alpha_s = g^2_s/(4\pi). \)
### 3.2.4 W boson–neutral Higgs–bottom (top) loop

There are two contributions, one with a bottom and one with a top quark in the loop (with $H_1^0$ and $W$ interchanged), see Fig. 1c. We use the $\xi = 1$ gauge. The $WH_b$ loop gives:

$$\text{Re} \delta Y_b^{CP}(WH_b) = -\frac{\sqrt{2} g^2}{32\pi^2} \left\{ \text{Im}(X_b^R) \left[ (3m_b^2 - 2m_{H_1}^2) \text{Im}(C_0) + m_t^2 \text{Im}(C_1) + 2m_b^2 \text{Im}(C_2) \right. \\
+ \text{Im} \left( B_0(m_{H_1}^2, m_W^2, m_{H_1}^2) \right) - 2 \text{Im} \left( B_0(m_t^2, m_b^2, m_W^2) \right) \left. \right] \\
+ m_b^2 \text{Im}(X_b^L) \text{Im}(2C_0 + C_2) \right\},$$

(16)

$$\text{Re} \delta Y_t^{CP}(WH_b) = -\frac{\sqrt{2} g^2}{32\pi^2} m_t m_b \left[ \text{Im}(X_t^R) \text{Im}(2C_1 + C_2) + \text{Im}(X_t^L) \text{Im}(C_1 - C_0) \right],$$

(17)

where $X_b^R = g_{H_bW} s_t^b$, $X_b^L = g_{H_bW} s_t^b$, and $C_X = C_X(m_t^2, m_{H_1}^2, m_b^2, m_W^2, m_{H_1}^2)$. Analogously, the $H_bW_t$ loop gives

$$\text{Re} \delta Y_b^{CP}(H_bW_t) = \frac{\sqrt{2} g^2}{32\pi^2} m_t m_b \left[ \text{Im}(X_t^L) \text{Im}(2C_1 + C_2) + \text{Im}(X_t^R) \text{Im}(C_1 - C_0) \right],$$

(18)

$$\text{Re} \delta Y_t^{CP}(H_bW_t) = \frac{\sqrt{2} g^2}{32\pi^2} \left\{ \text{Im}(X_t^L) \left[ (3m_t^2 - 2m_{H_1}^2) \text{Im}(C_0) + m_b^2 \text{Im}(C_1) + 2m_t^2 \text{Im}(C_2) \right. \\
+ \text{Im} \left( B_0(m_{H_1}^2, m_W^2, m_{H_1}^2) \right) - 2 \text{Im} \left( B_0(m_b^2, m_t^2, m_W^2) \right) \left. \right] \\
+ m_t^2 \text{Im}(X_t^R) \text{Im}(2C_0 + C_2) \right\},$$

(19)

with $X_t^R = g_{H_bW} s_t^b$, $X_t^L = g_{H_bW} s_t^b$, and $C_X = C_X(m_t^2, m_{H_1}^2, m_b^2, m_{H_1}^2, m_W^2)$.

### 3.2.5 Ghost–neutral Higgs–bottom (top) loop

Since the above graphs with a $W$ boson are calculated in the $\xi = 1$ gauge, we also have to include the corresponding graphs with $W^\pm \to G^\pm$. These lead to

$$\text{Re} \delta Y_b^{CP}(GH_t) = -\frac{1}{8\pi^2} \left[ m_b h_b \cos \beta \text{Im}(\hat{X}_b^R) \text{Im}(C_0) + m_t h_t \sin \beta \text{Im}(\hat{X}_b^R) \text{Im}(C_1) \\
- m_b h_b \cos \beta \text{Im}(\hat{X}_b^L) \text{Im}(C_2) \right],$$

(20)

$$\text{Re} \delta Y_t^{CP}(GH_t) = \frac{1}{8\pi^2} \left[ m_b h_t \sin \beta \text{Im}(\hat{X}_b^L) \text{Im}(C_0) + m_t h_b \cos \beta \text{Im}(\hat{X}_b^L) \text{Im}(C_1) \\
- m_b h_t \sin \beta \text{Im}(\hat{X}_b^R) \text{Im}(C_2) \right],$$

(21)
\[ \text{Re } \delta Y_b^CP(H_t G t) = - \frac{1}{8\pi^2} \left[ m_t h_b \cos \beta \text{Im} \left( \hat{X}_t^R \right) \text{Im} \left( C_0 \right) - m_t h_b \cos \beta \text{Im} \left( \hat{X}_t^L \right) \text{Im} \left( C_1 \right) \right. \]
\[ \left. + m_b h_t \sin \beta \text{Im} \left( \hat{X}_t^R \right) \text{Im} \left( C_2 \right) \right], \quad (22) \]
\[ \text{Re } \delta Y_t^CP(H_t G t) = \frac{1}{8\pi^2} \left[ m_t h_t \sin \beta \text{Im} \left( \hat{X}_t^L \right) \text{Im} \left( C_0 \right) - m_t h_t \sin \beta \text{Im} \left( \hat{X}_t^R \right) \text{Im} \left( C_1 \right) \right. \]
\[ \left. + m_b h_b \cos \beta \text{Im} \left( \hat{X}_t^L \right) \text{Im} \left( C_2 \right) \right]. \quad (23) \]

Here, \( \hat{X}_q^{R/L} = g_{H_i H^+ G} s_q^{R/L} \) for \( q = b, t \). The \( C \) functions are \( C_X = C_X(m_t^2, m_{H^+}^2, m_b^2, m_b^2, m_W^2, m_{H_1}^2) \) for \( q = b \) and \( C_X = C_X(m_t^2, m_{H^+}^2, m_b^2, m_t^2, m_{H_1}^2, m_W^2) \) for \( q = t \).

### 3.3 Self-energy graphs

#### 3.3.1 Neutralino–chargino loop

The self-energy graph with a neutralino and a chargino of Fig. 1d gives

\[ \text{Re } \delta Y_b^{CP} \left( \tilde{\chi}_k \tilde{\chi}_j^\pm - W \right) = \pm \frac{1}{8\pi^2} \frac{g^2 m_b(t)}{\sqrt{2} m_{H^+}^2 m_W^2} \text{Im} \left( B_0(m_{H^+}^2, m_{\tilde{\chi}_k}^2, m_{\tilde{\chi}_j}^2) \right) \times \]
\[ \left[ \text{Im} \left( c_{II} \right) m_{\tilde{\chi}_j}^2 (m_{H^+}^2 + m_{\tilde{\chi}_k}^2 - m_{\tilde{\chi}_j}^2) - \text{Im} \left( c_{IJ} \right) m_{\tilde{\chi}_k}^2 (m_{H^+}^2 - m_{\tilde{\chi}_k}^2 + m_{\tilde{\chi}_j}^2) \right] \quad (24) \]

with \( c_{II} = F_{jk}^R O_{kj}^R + F_{jk}^L O_{kj}^L \) and \( c_{IJ} = F_{jk}^R O_{kj}^L + F_{jk}^L O_{kj}^R \). The overall plus sign is for \( \delta Y_b^{CP} \), and the overall minus sign for \( \delta Y_t^{CP} \).

#### 3.3.2 Stop–sbottom loop

The graph of Fig. 1e leads to

\[ \text{Re } \delta Y_b^{CP} \left( \tilde{t}_i \tilde{b}_j - W \right) = \pm \frac{g^2 m_b(t)}{16\pi^2} \frac{m_b(t) m_{H^+}^2 m_W^2 (m_{\tilde{t}_i}^2 - m_{\tilde{b}_j}^2)}{m_{\tilde{t}_i}^2 m_{\tilde{b}_j}^2} \times \]
\[ \text{Im} \left( G_{\tilde{t}_i \tilde{b}_j \tilde{R} \tilde{L}} \right) \text{Im} \left( B_0(m_{H^+}^2, m_{\tilde{b}_j}^2, m_{\tilde{b}_j}^2) \right) \right). \quad (25) \]

#### 3.3.3 \( W^\pm - H_1^0 \) and \( G^\pm - H_1^0 \) loops

The self-energy graph with \( W^+ \) and \( H_1^0 \) is shown in Fig. 1f. Since we use \( \xi = 1 \) gauge for the \( W \) in the loop, we have to add the corresponding graph with a ghost, i.e. \( W^\pm \to G^\pm \) in the
loop. (The second $W$ propagator can be calculated in the unitary gauge. Hence, no ghost is necessary in this case.) The two contributions together give:

$$\Re \delta Y_{b(t)}^{CP} (WH^0_l - W) = \mp \frac{1}{32\pi^2} \frac{g^3 m_{b(t)}}{\sqrt{2} m_{H^+}^2 m_W} (2m_W^2 - 2m_{H^t}^2 - 3m_{H^+}^2) \times$$

$$O_{3l} (\cos \beta O_{1t} + \sin \beta O_{2t}) \Im (B_0(m_{H^+}^2, m_{H^t}^2, m_W^2)).$$

(26)

4 Numerical results

Let us now turn to the numerical analysis. In order not to vary too many parameters, we fix part of the parameter space at the electroweak scale by the choice

$$M_2 = 200 \text{ GeV}, \ \mu = 350 \text{ GeV}, \ M_G : M_Q : M_{\tilde{D}} = 0.85 : 1 : 1.05,$$

$$A_t = -500 \text{ GeV} = -A_b. \quad (27)$$

Moreover, we assume GUT relations for the gaugino mass parameters $M_1, M_2, M_3$. In this case, the phases of the gaugino sector can be rotated away. Since $\phi_\mu$, the phase of $\mu$, is highly constrained by the EDMs of electron and neutron, we take $\phi_\mu = 0$ unless mentioned otherwise. The phases relevant to our study are thus $\phi_t$ and $\phi_b$, the phases of $A_t$ and $A_b$.

The choice in Eq. (27) together with $M_\tilde{Q} = 490 \text{ GeV}$ and $\tan \beta = 10$ gives a mass spectrum very similar to the Snowmass point SPS1a [11]. Figure 2 shows $\delta^{CP}$ for this case as a function of $m_{H^+}$ in the range $m_{H^+} = 200-1400 \text{ GeV}$ and various values of $\phi_t$ ($\phi_b = \phi_\mu = 0$). The sparticle masses are given explicitly in Table 1.

For $m_{H^+} < m_{\tilde{t}_1} + m_{\tilde{b}_1}$, $\delta^{CP}$ is very small, $O(10^{-3})$ or smaller. The contributions to $\delta^{CP}$ come from the diagrams of Figs. 1a, 1c, and 1f; the diagram of Fig. 1d only contributes if there is a non-zero phase in the chargino/neutralino sector. In the region $m_{H^+} = 200-800 \text{ GeV}$, one can distinguish the thresholds of $\tilde{\chi}_1^0 \tilde{\chi}_1^\pm$ at $m_{H^+} \simeq 280 \text{ GeV}$, $\tilde{\chi}_1^0 \tilde{\chi}_2^\pm$ at $m_{H^+} \simeq 470 \text{ GeV}$, $\tilde{\chi}_3^0 \tilde{\chi}_1^\pm$ at $m_{H^+} \simeq 550 \text{ GeV}$, $\tilde{\chi}_2^0 \tilde{\chi}_2^\pm$ and $\tilde{\chi}_4^0 \tilde{\chi}_1^\pm$ at $m_{H^+} \simeq 560 \text{ GeV}$, and $\tilde{\chi}_{3,4}^0 \tilde{\chi}_2^\pm$ at $m_{H^+} \simeq 730-750 \text{ GeV}$. Below the $\tilde{\chi}_1^0 \tilde{\chi}_1^\pm$ threshold, $\delta^{CP}$ originates only from the graphs with $W$ and a neutral Higgs boson of Figs. 1c and 1f. Here note that the graph with $WH^0_l b$ of Fig. 1c always contributes, since $m_t > m_W + m_b$. 

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Table 1: Sparticle masses (in GeV) for parameter sets used in the numerical analysis for \( \phi_t = 0 \left( \frac{\pi}{2} \right) \).

However, once the \( H^+ \to \tilde{t} \tilde{b} \) channel is open, \( \delta^{CP} \) can go up to several per cent. The thresholds of \( H^+ \to \tilde{t}_1 \tilde{b}_1 \) at \( m_{H^+} \simeq 860 \) GeV, and of \( H^+ \to \tilde{t}_2 \tilde{b}_2 \) at \( m_{H^+} \simeq 1100 \) GeV are clearly visible in Fig. 2. For \( m_{H^+} = 1 \) TeV, we obtain \( \delta^{CP} = -3.4\% \), \(-6.4\% \), and \(-9.6\% \) for \( \phi_t = \frac{\pi}{8}, \frac{\pi}{4}, \) and \( \frac{\pi}{2} \), respectively. The dominant contribution comes from the stop–sbottom–gluino loop of Fig. 1b. Also the stop–sbottom–neutralino loop of Fig. 1b and the stop–sbottom self-energy of Fig. 1e can give a relevant contribution and should thus be taken into account. The contribution of the graphs with \( \tilde{\chi}^\pm \tilde{\chi}^0 \) or \( H^0W \) (Fig. 1a,c,f) exchange can, however, be neglected in this case.

The relative importance of the various contributions is illustrated in Fig. 3, where we plot the form factors \( \text{Re} \delta Y_b^{CP} \) and \( \text{Re} \delta Y_t^{CP} \) as a function of \( m_{H^+} \) for the parameters of Fig. 2. To calculate the contributions with neutral Higgs bosons, we have used [8, 12]. This is sufficient for our purpose, since we are mainly interested in large CP-violating effects that occur for \( m_{H^+} > m_{\tilde{t}_1} + m_{\tilde{b}_1} \) because of \( \phi_{t,b} \). However, once precision measurements of \( H^\pm \) decays become feasible, a more complete calculation of the \( H^0_i \) masses and couplings [13] might be used.

We next lower the stop/sbottom mass scale to \( M_{\tilde{Q}} = 350 \) GeV. The resulting masses are given in Table 1. Figures 4a and 4b show \( \delta^{CP} \) for this case as functions of \( m_{H^+} \) and tan \( \beta \),
Figure 2: Absolute value of $\delta^{CP}$ as a function of $m_{H^+}$, for $M_{\tilde{Q}} = 490$ GeV and $\tan \beta = 10$. The solid, dashed, and dotted lines are for $\phi_t = \frac{\pi}{2}$, $\frac{\pi}{4}$, and $\frac{\pi}{8}$, respectively. The other parameters are fixed by Eq. (27).

Figure 3: Absolute values of $\text{Re}\, \delta Y_{\tilde{b}}^{CP}$ and $\text{Re}\, \delta Y_{\tilde{t}}^{CP}$ as functions of $m_{H^+}$, for $M_{\tilde{Q}} = 490$ GeV, $\tan \beta = 10$ and $\phi_t = \frac{\pi}{2}$. The other parameters are fixed by Eq. (27). The blue lines show the contribution from the $\tilde{\chi}^\pm \tilde{\chi}^0$ exchanges of Fig. 1a; the red lines are those from the $\tilde{t}\tilde{b}$ exchanges of Figs. 1b and 1e; the green lines are those from the diagrams with $H^0$ and $W$ of Figs. 1c and 1f. The red dashed lines show $\text{Re}\, \delta Y_{\tilde{t}}^{CP}$ due to the $\tilde{t}\tilde{b}\tilde{g}$ loop only.
respectively. The threshold behaviour of Fig. 4a is very similar to that of Fig. 2a. The threshold of $H^+ \rightarrow \tilde{t}_1 \tilde{b}_1$ is shifted to $m_{H^+} \simeq 540$ GeV, and $\delta^{CP}$ reaches larger values for lighter squarks. Even for a small phase $\phi_t$, $\delta^{CP}$ can be of $\mathcal{O}(5\%)$. Figure 4b shows the tan $\beta$ dependence of $\delta^{CP}$ for $m_{H^+} = 700$ GeV and the cases $\phi_t = \pi/2$, $\phi_b = 0$ (full line) and $\phi_t = \phi_b = \pi/2$ (dashed line). It turns out that the asymmetry has a maximum around $\tan \beta \simeq 10$ and approaches a constant value for large $\tan \beta$. For $\phi_t = \pi/2$ and $\phi_b = 0$, we have $\delta^{CP} \sim -12\%$ at $\tan \beta = 10$ and $\delta^{CP} \sim -5\%$ at $\tan \beta = 50$. An additional phase of $A_b$ can enhance (or reduce) the asymmetry. The relative importance of $\phi_b$ increases with $\tan \beta$. This is because the $\tilde{t}_L \tilde{b}_R H^+$ coupling is proportional to $A_b \tan \beta$ while the $\tilde{b}_L \tilde{b}_R H^+$ coupling goes with $A_t \cot \beta$, see Eq. (49).

For $\phi_t = \phi_b = \pi/2$, we thus have $\delta^{CP} \sim -14\%$ at $\tan \beta = 10$ and $\delta^{CP} \sim -8\%$ at $\tan \beta = 50$. For completeness, we also show as a dotted line the case of a large phase of $\mu$: $\phi_\mu$ can have a large effect for small to medium values of $\tan \beta$, mainly through the $\tilde{t}_L \tilde{b}_R H^+$ coupling, see again Eq. (49). For $\phi_t = \phi_b = \mu = \pi/2$ and $\tan \beta = 10$, $\delta^{CP} \sim -22\%$.

Here we also note that the branching ratio of $H^+ \rightarrow t\bar{b}$ increases with $\tan \beta$. In the case of vanishing phases we have $\text{BR}(H^+ \rightarrow t\bar{b}) \simeq 13\%$ (12\%) at $\tan \beta = 10$ and 38\% (44\%) at $\tan \beta = 40$ for $M_{\tilde{Q}} = 350$ (490) GeV and $m_{H^+} = 700$ (1000) GeV.

The dependence on $\phi_t$ is shown explicitly in Fig. 5, where we plot $\delta^{CP}$ as a function of $\phi_t$ for $M_{\tilde{Q}} = 350$ GeV, $m_{H^+} = 700$ GeV, $\tan \beta = 10$ and 40 and various choices of $\phi_b$. As expected, $\delta^{CP}$ shows a $\sim \sin \phi_t$ dependence. Note again the influence of $\phi_b$: in case of a maximal stop phase, $\phi_b$ can change $\delta^{CP}$ by up to 20\% for $\tan \beta = 10$, and by up to 40\% for $\tan \beta = 40$.

Last but not least we relax the GUT relations between the gaugino masses and take $M_3$ as a free parameter (keeping, however, the relation between $M_1$ and $M_2$ and taking $M_3$ real). Figure 6 shows the dependence of $\delta^{CP}$ on the gluino mass for $M_{\tilde{Q}} = 350$ GeV, $m_{H^+} = 700$ GeV, $\phi_t = \pi/2$, and $\tan \beta = 10$ and 40. It is interesting that there is still an effect for a large gluino mass: for the large $\tan \beta$ case, $\delta^{CP}$ is reduced from about $-5\%$ to about $-3\%$ for $m_{\tilde{g}} = 600 \rightarrow 1200$ GeV. Also for $\tan \beta = 10$, $\delta^{CP}$ is decreased by about one half when the gluino mass is doubled: from $-11\%$ to $-6\%$ for $m_{\tilde{g}} = 600 \rightarrow 1200$ GeV.
Figure 4: $\delta^{CP}$ for $M_{Q} = 350$ GeV; in (a) as a function of $m_{H^+}$, for $\tan \beta = 10$, and in (b) as a function of $\tan \beta$, for $m_{H^+} = 700$ GeV. The other parameters are fixed by Eq. (27).

Figure 5: $\delta^{CP}$ as a function of $\phi_t$, for $M_{Q} = 350$ GeV, $m_{H^+} = 700$ GeV, $\tan \beta = 10$ in (a) and $\tan \beta = 40$ in (b); full lines: $\phi_b = 0$, dashed lines: $\phi_b = \phi_t$, dotted lines: $\phi_b = \pi/2$. The other parameters are fixed by Eq. (27).
Figure 6: $\delta^{CP}$ as a function of $m_{\tilde{g}}$, for $M_{\tilde{Q}} = 350$ GeV, $m_{H^+} = 700$ GeV, $\phi_t = \pi/2$, and $\tan \beta = 10$ and 40.

5 Conclusions

We have calculated the difference between the partial rates $\Gamma (H^+ \rightarrow t\bar{b})$ and $\Gamma (H^- \rightarrow \bar{t}b)$ due to CP-violating phases in the MSSM. The resulting decay rate asymmetry $\delta^{CP}$, Eq. (1), could be measured in a counting experiment. If $m_{H^+} < m_{\tilde{t}_1} + m_{\tilde{b}_1}$, $\delta^{CP}$ is typically of the order of $10^{-3}$. However, for $m_{H^+} > m_{\tilde{t}_1} + m_{\tilde{b}_1}$, $\delta^{CP}$ can go up to $\sim 10-15\%$, depending on the phases of $A_t$ and $A_b$, and on $\tan \beta$.

At the Tevatron, no sensitivity for detecting $H^\pm$ is expected for a mass $m_{H^+} \gtrsim 200$ GeV. The LHC, on the other hand, has a discovery reach up to $m_{H^+} \sim 1$ TeV, especially if QCD and SUSY effects conspire to enhance the cross section [14]. With a luminosity of $L = 100$ fb$^{-1}$, about 325 signal events can be expected for $pp \rightarrow H^+\bar{t}b$ with $S/\sqrt{B} = 9.5$ ($B$ being the background), for $m_{H^+} \simeq 700$ GeV and $\tan \beta = 50$. In $e^+e^-$ collisions, the dominant production mode is $e^+e^- \rightarrow H^+H^-$. Therefore, one would need a centre-of-mass energy $\sqrt{s} > 2m_{H^+}$. This would certainly be realized at a multi-TeV linear collider such as CLIC [15]. Hence, a CP-violating asymmetry $\delta^{CP}$ of a few per cent should be measurable at the LHC or CLIC.
Acknowledgements

We thank Johann Kühn and Werner Porod for helpful discussions. We also thank Marco Fabbrichesi for his participation in the initial stage of this work. The work of E. C. was supported by the Bulgarian National Science Foundation, Grant Ph–1010.

A Masses and mixing matrices

The neutralino mass matrix in the basis of

\[ \Psi_j^0 = (-i\lambda', -i\lambda^3, \psi_{H_1}^0, \psi_{H_2}^0) \]  

is:

\[ M_N = \begin{pmatrix}
  M_1 & 0 & -m_Z \sin \theta_W \cos \beta & m_Z \sin \theta_W \sin \beta \\
  0 & M_2 & m_Z \cos \theta_W \cos \beta & -m_Z \cos \theta_W \sin \beta \\
  -m_Z \sin \theta_W \cos \beta & m_Z \cos \theta_W \cos \beta & 0 & -\mu \\
  m_Z \sin \theta_W \sin \beta & -m_Z \cos \theta_W \sin \beta & -\mu & 0 
\end{pmatrix} \]  

with \( \tan \beta = v_2/v_1 \). This matrix is diagonalized by the unitary mixing matrix \( N \):

\[ N^* M_N N^\dagger = \text{diag}(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}) \]  

where \( m_{\tilde{\chi}_k^0}, k = 1, ..., 4 \), are the (non-negative) masses of the physical neutralino states.

The chargino mass matrix is:

\[ M_C = \begin{pmatrix}
  M_2 & \sqrt{2} m_W \sin \beta \\
  \sqrt{2} m_W \cos \beta & \mu 
\end{pmatrix} \]  

It is diagonalized by the two unitary matrices \( U \) and \( V \):

\[ U^* M_C V^\dagger = \text{diag}(m_{\tilde{\chi}_1^+}, m_{\tilde{\chi}_2^+}) \]  

where \( m_{\tilde{\chi}_{1,2}^+} \) are the masses of the physical chargino states.
The mass matrix of the \textbf{stops} in the basis \((\tilde{t}_L, \tilde{t}_R)\) is
\[
\mathcal{M}_t^2 = \begin{pmatrix}
M_Q^2 + m_Z^2 \cos 2\beta (\frac{1}{2} - \frac{2}{3} \sin^2 \theta_W) + m_t^2 & (A_t^* - \mu \cot \beta) m_t \\
(A_t - \mu^* \cot \beta) m_t & M_b^2 + \frac{2}{3} m_Z^2 \cos 2\beta \sin^2 \theta_W + m_t^2
\end{pmatrix}.
\]
(33)

\(\mathcal{M}_t^2\) is diagonalized by the rotation matrix \(R^t\) such that \(R_t^{\dagger} \mathcal{M}_t^2 R_t^t = \text{diag}(m_{t_1}^2, m_{t_2}^2)\) and \((\tilde{t}_L, \tilde{t}_R) = R^t (\tilde{t}_1, \tilde{t}_2)\). We have:
\[
R_t^t = \begin{pmatrix}
R_{t1}^t & R_{t2}^t \\
R_{R1}^t & R_{R2}^t
\end{pmatrix} = \begin{pmatrix}
\sin \theta_t & -\cos \theta_t \\
\cos \theta_t & \sin \theta_t
\end{pmatrix}.
\]
(34)

Analogously, the mass matrix of the \textbf{sbottoms} in the basis \((\tilde{b}_L, \tilde{b}_R)\),
\[
\mathcal{M}_b^2 = \begin{pmatrix}
M_Q^2 + m_Z^2 \cos 2\beta (\frac{1}{2} - \frac{2}{3} \sin^2 \theta_W) + m_b^2 & (A_b^* - \mu \tan \beta) m_b \\
(A_b - \mu^* \tan \beta) m_b & M_D^2 + \frac{1}{3} m_Z^2 \cos 2\beta \sin^2 \theta_W + m_b^2
\end{pmatrix},
\]
(35)

is diagonalized by the rotation matrix \(R_b^t\) such that \(R_b^{\dagger} \mathcal{M}_b^2 R_b^t = \text{diag}(m_{b_1}^2, m_{b_2}^2)\).

In the \textbf{neutral Higgs sector}, we have two CP-even states \(\phi_i = \sqrt{2} \text{Re} (\mathcal{H}_i^0) - v_i, i = 1, 2,\) and and one CP-odd state \(a = \sqrt{2} (\sin \beta \text{Im} (\mathcal{H}_1^0) + \cos \beta \text{Im} (\mathcal{H}_2^0))\), where \(\mathcal{H}_1\) and \(\mathcal{H}_2\) are the two Higgs doublets in the interaction basis. In the basis \((\phi_1, \phi_2, a)\), the neutral Higgs mass matrix \(\mathcal{M}_H^2\) can be written as the well-known tree-level part, which has a block form in this basis, plus a general \(3 \times 3\) matrix containing the loop corrections:
\[
\mathcal{M}_H^2 = \begin{pmatrix}
s^2 \beta m_A^2 + c^2 \beta m_Z^2 & -s\beta c\beta (m_A^2 + m_Z^2) & 0 \\
-s\beta c\beta (m_A^2 + m_Z^2) & c^2 \beta m_A^2 + s^2 \beta m_Z^2 & 0 \\
0 & 0 & m_A^2
\end{pmatrix} + (\mathcal{M}_H^{\text{loop}})^2,
\]
(36)

where \(s\beta \equiv \sin \beta, c\beta \equiv \cos \beta,\) etc. In the case of complex parameters, the loop contributions of \((\mathcal{M}_H^{\text{loop}})^2\) lead to a mixing of the CP-even and CP-odd states. The mass eigenstates then are
\[
\begin{pmatrix}
H_1^0 \\
H_2^0 \\
H_3^0
\end{pmatrix} = \mathcal{O}^T \begin{pmatrix}
\phi_1 \\
\phi_2 \\
a
\end{pmatrix},
\]
(37)
The real $3 \times 3$ rotation matrix $O$ diagonalizes the mass matrix $\mathcal{M}^2_H$,

$$O^T \mathcal{M}^2_H O = \text{diag} \left( m_{H_1}^2, m_{H_2}^2, m_{H_3}^2 \right),$$

with $m_{H_1} < m_{H_2} < m_{H_3}$. The transformations of the Higgs fields from the interaction basis to the mass eigenstate basis are given by

$$H_1^1 = v_1 + \frac{1}{\sqrt{2}} \left[ (O_{1j} + i \sin \beta O_{3j}) H_0^0 - i \cos \beta G_0^0 \right],$$

$$H_1^2 = -\cos \beta G^- + \sin \beta H^-,$$

$$H_2^1 = \sin \beta G^+ + \cos \beta H^+,$$

$$H_2^2 = v_2 + \frac{1}{\sqrt{2}} \left[ (O_{2j} + i \cos \beta O_{3j}) H_0^0 + i \sin \beta G_0^0 \right],$$

with the implicit sum over $j = 1, 2, 3$,

$$v_1 = v \cos \beta = \frac{\sqrt{2}}{g} m_W \cos \beta, \quad v_2 = v \sin \beta = \frac{\sqrt{2}}{g} m_W \sin \beta.$$  

For the numerical evaluation of the physical Higgs masses and the rotation matrix $O$ in the 1-loop effective potential approach [8], we use the program chp.f [12].

**B Interaction Lagrangian**

In this section we give the parts of the interaction Lagrangian that we need for our calculation.

We start with the interaction of Higgs bosons with quarks and squarks:

$$\mathcal{L}_{Hqq} = H^+ \bar{t} (y_t P_R + y_t P_L) b + H^- \bar{b} (y_b P_R + y_b P_L) t + H_0^0 \bar{q} (s^q R P_R + s^q L P_L) q,$$

$$\mathcal{L}_{H\bar{q}q} = (G_4)_{ij} H^+ \bar{t}_i \tilde{b}_j + (G_4^*)_{ij} H^- \tilde{b}_j \bar{t}_i,$$

with $i, j = 1, 2, l = 1, 2, 3$ and

$$P_L = \frac{1}{2}(1 - \gamma_5), \quad P_R = \frac{1}{2}(1 + \gamma_5).$$

For the $H^\pm$ couplings to quarks we have

$$y_t = h_t \cos \beta, \quad y_b = h_b \sin \beta,$$
with
\[
\begin{align*}
    h_t &= \frac{g \bar{m}_t}{\sqrt{2} m_W \sin \beta}, \quad h_b = \frac{g \bar{m}_b}{\sqrt{2} m_W \cos \beta}, \\
\end{align*}
\]
where \( \bar{m}_q \) is the DR running quark mass. For the \( H^0 \) couplings to quarks we have
\[
\begin{align*}
    s^q_{iR} &= -\frac{g \bar{m}_q}{2 m_W} \left( g^S_{H^0 q q} + i g^P_{H^0 q q} \right), \\
    s^q_{iL} &= -\frac{g \bar{m}_q}{2 m_W} \left( g^S_{H^0 q q} - i g^P_{H^0 q q} \right),
\end{align*}
\]
with \( g^S_{H^0 q q} \) and \( g^P_{H^0 q q} \) given by Eqs. (4.11)–(4.14) in [8]. The \( H^\pm \) couplings to squarks are given by the matrix
\[
G_4 = R^{\tilde{1}} \hat{G}_4 R^{\tilde{b}},
\]
with
\[
\hat{G}_4 = \begin{pmatrix}
    h_b m_b \sin \beta + h_t m_t \cos \beta - \sqrt{2} g m_W \sin \beta \cos \beta & h_b (A^*_b \sin \beta + \mu \cos \beta) \\
    h_t (A_t \cos \beta + \mu^* \sin \beta) & h_t m_b \cos \beta + h_b m_t \sin \beta
\end{pmatrix}.
\]
The interactions of charginos and neutralinos are described by
\[
\begin{align*}
    \mathcal{L}_{H^+ \tilde{\chi}^+} &= H^+ \tilde{\chi}^+ (F^R_{ik} P_R + F^L_{ik} P_L) \tilde{\chi}^0_k + H^- \tilde{\chi}^- (F^R_{ik} P_L + F^L_{ik} P_R) \tilde{\chi}^+_i, \\
    \mathcal{L}_{q \tilde{q} \tilde{\chi}^+} &= \bar{t} (i \tilde{b}^T P_R + k \tilde{b}^T P_L) \tilde{\chi}^+_j \tilde{\chi}^-_i + \bar{b} (i \tilde{t}^T P_R + k \tilde{t}^T P_L) \tilde{\chi}^+_j \tilde{\chi}^-_i \\
    &+ \tilde{\chi}^+_j (i \tilde{b}^T P_L + k \tilde{b}^T P_R) \bar{t} \tilde{b}^*_i + \tilde{\chi}^-_j (i \tilde{t}^T P_L + k \tilde{t}^T P_R) \bar{b} \tilde{t}^* _i, \\
    \mathcal{L}_{q \tilde{q} \tilde{\chi}^0} &= \bar{q} (a \tilde{a}^T P_R + b \tilde{b}^T P_L) \tilde{\chi}^0_k \tilde{\chi}^-_i + \bar{\tilde{\chi}^0}_k (a \tilde{a}^T P_L + b \tilde{b}^T P_R) \tilde{q} \tilde{q}^*_i,
\end{align*}
\]
with \( i, j = 1, 2 \) and \( k = 1, \ldots, 4 \). The couplings of \( H^\pm \) to charginos and neutralinos are
\[
\begin{align*}
    F^R_{ik} &= -g \left[ V_{i1} N_{k4} + \frac{1}{\sqrt{2}} (N_{k2} + N_{k1} \tan \theta_W) V_{i2} \right] \cos \beta, \\
    F^L_{ik} &= -g \left[ U_{i1}^* N_{k3}^* - \frac{1}{\sqrt{2}} (N_{k2}^* + N_{k1}^* \tan \theta_W) U_{i2}^* \right] \sin \beta.
\end{align*}
\]
The chargino–squark–quark couplings are
\[
\begin{align*}
    \tilde{t}^i_{ij} &= -g V_{j1} R_{1i}^i + h_t V_{j2} R_{2i}^i, \\
    \tilde{b}^i_{ij} &= -g U_{j1} R_{1i}^b + h_b U_{j2} R_{2i}^b, \\
    \tilde{t}^i_{ij} &= h_b U_{j2}^* R_{1i}^b, \\
    \tilde{b}^i_{ij} &= h_t V_{j2}^* R_{1i}^b.
\end{align*}
\]
The neutralino–squark–quark couplings are

\[
\begin{align*}
\tilde{q}^i_{lk} &= gf_{Lk}^i R_{1i}^\tilde{q} + h_{Lk}^i R_{2i}^\tilde{q}, \\
\tilde{b}^i_{lk} &= h_{Lk}^i R_{1i}^\tilde{q} + gf_{Rk}^i R_{2i}^\tilde{q},
\end{align*}
\]

(57)-(58)

with

\[
\begin{align*}
f_{Lk}^i &= -\frac{1}{\sqrt{2}} (N_{k2} + \frac{1}{3} \tan \theta_W N_{k1}), \\
f_{Rk}^i &= \frac{1}{\sqrt{2}} (N_{k2} - \frac{1}{3} \tan \theta_W N_{k1}), \\
h_{Lk}^i &= \frac{2}{3} \tan \theta_W N_{k1}, \\
h_{Rk}^i &= -\frac{2}{3} \tan \theta_W N_{k1},
\end{align*}
\]

(59)-(61)

Finally, the squark–quark–gluino interaction is given by

\[
\begin{align*}
\mathcal{L}_{q\tilde{q}g} &= -\sqrt{2} g_s T^{\alpha}_{L} \left[ \bar{q}_s \left( R_{1i}^\tilde{q} e^{-i\phi_3} P_L - R_{2i}^\tilde{q} e^{i\phi_3} P_R \right) q_s \tilde{q}_{i,t} \\
&+ \tilde{q}_s \left( R_{1i}^{\tilde{q}*} e^{i\phi_3} P_R - R_{2i}^{\tilde{q}*} e^{-i\phi_3} P_L \right) \tilde{g}^a \tilde{q}_{i,t} \right],
\end{align*}
\]

(62)

and the quark interaction with $W$ bosons is

\[
\mathcal{L}_{qW} = -\frac{g}{\sqrt{2}} \left( W^+ \bar{t} \gamma^\mu P_L b + W^- \bar{b} \gamma^\mu P_L t \right).
\]

(63)

We next turn to the interaction of Higgs bosons with $W$ bosons and ghosts. The Lagrangian of two Higgs particles and one $W$ boson is given by

\[
\mathcal{L}_{HHW} = -i \frac{g}{\sqrt{2}} \left[ W^+_\mu \left( \mathcal{H}_1^* \partial^\mu \mathcal{H}_1 + \mathcal{H}_2^* \partial^\mu \mathcal{H}_2 \right) + W^-_\mu \left( \mathcal{H}_1 \partial^\mu \mathcal{H}_1^* + \mathcal{H}_2 \partial^\mu \mathcal{H}_2^* \right) \right],
\]

(64)

where

\[
A \partial^\mu B = A (\partial^\mu B) - (\partial^\mu A) B.
\]

(65)

Using the transformations Eq. (39) we get

\[
\mathcal{L}_{HHW} = i \frac{g}{2} \left[ W^+_\mu \left( g_{H, H W^+} H_j H_j^0 \partial^\mu H^- + g_{H, G^- W^+} H_j^0 \partial^\mu G^- + i G^0 \partial^\mu G^- \right) + h. c. \right]
\]

(66)

where

\[
g_{H, H W^+} = -\sin \beta O_{1j} + \cos \beta O_{2j} + i O_{3j} \quad \text{and} \quad g_{H, G^- W^+} = \cos \beta O_{1j} + \sin \beta O_{2j}.
\]

(67)-(68)
Moreover, \( g_{H, H^+ W^-} = g_{H, H^- W^+}^* \) and \( g_{H, G^+ W^-} = g_{H, G^- W^+} \). Note that there is no \( G^0 W^+ H^- \) coupling. We further need the couplings of two charged and one neutral Higgs bosons. They are derived from the \( D \)-term interaction Lagrangian of the Higgs sector,
\[
\mathcal{L} = -\frac{1}{2} (D'D' + D^1 D^1 + D^2 D^2 + D^3 D^3),
\]
where \( D' \) and \( D^i \) are the \( U(1)_Y \) and \( SU(2)_L \) \( D \)-terms, respectively. In terms of Higgs fields in the interaction basis we have
\[
\mathcal{L}_{HHH} = -\frac{1}{8} \left( g^2 + g'^2 \right) \left( \mathcal{H}^{1*} \mathcal{H}_1^{1*} + \mathcal{H}^{2*} \mathcal{H}_2^{2*} - \mathcal{H}_1^{2*} \mathcal{H}^{1*} - \mathcal{H}_2^{2*} \mathcal{H}^{1*} \right)
- \frac{g^2}{2} \left( \mathcal{H}^{1*} \mathcal{H}_2^{2*} + \mathcal{H}^{2*} \mathcal{H}_1^{2*} \right) \left( \mathcal{H}^{1*} \mathcal{H}_1^{2*} + \mathcal{H}^{2*} \mathcal{H}_2^{2*} \right). \tag{69}
\]
The Lagrangian in the mass eigenstate basis is again obtained by applying the transformations of Eqs. (39) and (40). We are only interested in the combinations of \( (H^0_l, G^0) \times (H^\pm, G^\pm) \times (H^\mp, G^\mp) \). The couplings to \( G^0 \), e.g. \( G^0 H^+ H^-, G^0 H^+ G^-, G^0 G^+ H^-, \) and \( G^0 G^+ G^- \), are zero. The couplings to \( H^0_l \) are:
\[
\mathcal{L}_{HHH} = g_{H, H^+ H^-} H^0_l H^+ H^- + g_{H, H^+ G^-} H^0_l H^+ G^- \\
+ g_{H, G^+ G^-}^* H^0_l G^+ H^- + g_{H, G^+ G^-} H^0_l G^+ G^-, \tag{70}
\]
with
\[
g_{H, H^+ H^-} = \frac{g m_W}{2} \left\{ [(1 + t_W^2) c 2 \beta - 2] c \beta O_{1j} - [(1 + t_W^2) c 2 \beta + 2] s \beta O_{2j} \right\}, \tag{71}
g_{H, H^+ G^-} = \frac{g m_W}{2} \left[ c 2 \beta (s \beta O_{1j} + c \beta O_{2j}) - i O_{3j} + t_W^2 s 2 \beta (c \beta O_{1j} - s \beta O_{2j}) \right], \tag{72}
g_{H, G^+ G^-} = -\frac{g m_W}{2} (1 + t_W^2) c 2 \beta (c \beta O_{1j} - s \beta O_{2j}). \tag{73}
\]
Note that only \( g_{H, H^+ G^-} \) is complex. In Eqs. (71) – (73), we have used the abbreviations \( s \beta \equiv \sin \beta, s 2 \beta \equiv \sin 2 \beta, c \beta \equiv \cos \beta, c 2 \beta \equiv \cos 2 \beta, \) and \( t_W^2 \equiv \tan^2 \theta_W \).

### C Passarino–Veltman integrals

Here we give the definition of the Passarino–Veltman one-, two-, and three-point functions \([9]\) in the convention of \([10]\). For the general denominators we use the notation
\[
\mathcal{D}^0 = q^2 - m_0^2 \quad \text{and} \quad \mathcal{D}^j = (q + p_j)^2 - m_j^2. \tag{74}
\]
Then the loop integrals in $D = 4 - \epsilon$ dimensions are as follows:

$$A_0(m_0^2) = \frac{1}{i\pi^2} \int \frac{d^Dq}{D^0} ,$$  \hspace{1cm} (75)

$$B_0(p_1^2, m_0^2, m_1^2) = \frac{1}{i\pi^2} \int \frac{d^Dq}{D^0D^1} ,$$  \hspace{1cm} (76)

$$B_\mu(p_1^2, m_0^2, m_1^2) = \frac{1}{i\pi^2} \int \frac{d^Dq}{D^0D^1} \frac{q_\mu}{D^0D^1} = p_{1\mu} B_1(p_1^2, m_0^2, m_1^2) ,$$  \hspace{1cm} (77)

and

$$C_0 = \frac{1}{i\pi^2} \int \frac{d^Dq}{D^0D^1D^2} ,$$  \hspace{1cm} (78)

$$C_\mu = \frac{1}{i\pi^2} \int \frac{d^Dq}{D^0D^1D^2} \frac{q_\mu}{D^0D^1D^2} = p_{1\mu} C_1 + p_{2\mu} C_2 ,$$  \hspace{1cm} (79)

$$C_{\mu\nu} = \frac{1}{i\pi^2} \int \frac{d^Dq}{D^0D^1D^2} \frac{q_\mu q_\nu}{D^0D^1D^2}$$
$$= g_{\mu\nu} C_{00} + p_{1\mu} p_{1\nu} C_{11} + (p_{1\mu} p_{2\nu} + p_{2\mu} p_{1\nu}) C_{12} + p_{2\mu} p_{2\nu} C_{22} .$$  \hspace{1cm} (80)

where the $C$’s have $(p_1^2, (p_1 - p_2)^2, p_2^2, m_0^2, m_1^2, m_2^2)$ as their arguments. The function $B_1$ can be expressed as a combination of the functions $A_0$ and $B_0$:

$$2p_1^2 B_1(p_1^2, m_0^2, m_1^2) = A_0(m_0^2) - A_0(m_1^2) - (p_1^2 - m_1^2 + m_0^2) B_0(p_1^2, m_0^2, m_1^2) .$$  \hspace{1cm} (81)

**References**


[12] The Fortran program cph.f can be obtained from

http://pilaftsi.home.cern.ch/pilaftsi/


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