Spin-Induced Disk Precession in the Supermassive Black Hole at the Galactic Center

Siming Liu\(^1\) and Fulvio Melia\(^{1,2,3}\)

\(^1\)Physics Department, The University of Arizona, Tucson, AZ 85721
\(^2\)Steward Observatory, The University of Arizona, Tucson, AZ 85721

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ABSTRACT

Sgr A* is a compact radio source at the Galactic Center, thought to be the radiative manifestation of a $2.6 \times 10^6 M_\odot$ supermassive black hole. At least a portion of its spectrum—notably the mm/sub-mm “bump”—appears to be produced within the inner portion ($r < 10 r_S$) of a hot, magnetized Keplerian flow, whose characteristics are also consistent with the $\sim 10\%$ linear polarization detected from this source at mm wavelengths. (The Schwarzschild radius, $r_S$, for an object of this mass $M$ is $2GM/c^2 \approx 7.7 \times 10^{11}$ cm, or roughly 1/20 A.U.) The recent detection of a 106-day cycle in Sgr A*’s radio variability adds significant intrigue to this picture, since it may signal a precession of the disk induced by the spin $a$ of the black hole. The dynamical time scale near the marginally stable orbit around an object with this mass is $\approx 20$ mins. Thus, since the physical conditions associated with the disk around Sgr A* imply rigid-body rotation, a precession period of 106 days may be indicative of a small black hole spin if the circularized flow is confined to a region $\sim 30 r_S$, for which $a \approx (M/10) (r_o/30 r_S)^{5/2}$. The precession of a larger structure would require a bigger black hole spin. We note that a small value of $a/M (< 0.1)$ would be favored if the non-thermal ($\sim 1 – 20$ cm) portion of Sgr A*’s spectrum is powered with energy extracted via a Blandford-Znajek type of process, for which the observed luminosity would correspond to an outer disk radius $r_o \sim 30 r_S$. Such a small disk size is also suggested by earlier hydrodynamical simulations, and is implied by Sgr A*’s spectral and polarimetric characteristics.

Subject headings: accretion—black hole physics—Galaxy: center—gravitation—radiation mechanisms: non-thermal—relativity
1. Introduction

Sgr A*’s radio emission is known to be variable, with a fluctuation amplitude that increases toward high frequencies (Zhao et al. 1993). At mm wavelengths, large-amplitude (∼ 100%) variations have also been observed (Wright & Backer 1993; Miyazaki et al. 1999). Its time-averaged spectrum is roughly a power law below 100 GHz, with a flux density $S_\nu \propto \nu^\alpha$, and $\alpha \sim 0.19–0.34$.

In the mm/sub-mm region, however, Sgr A*’s radiative output is dominated by a “bump” extending above this power law (Zylka et al. 1992; Falcke et al. 1998).

Unlike the high level of polarization seen in this object at mm/sub-mm wavelengths (Aitken et al. 2000), Sgr A* reveals a lack (< 1%) of linear polarization below 112 GHz, though some circular polarization (∼ 1%) has been detected (Bower et al. 1999; Bower et al. 2001). These prominent spectral and polarimetric differences (Melia, Bromley, & Liu 2001) between the cm and the mm/sub-mm bands suggest two different emission components in Sgr A*. Because higher frequencies correspond to smaller spatial scales (Melia 1992; Melia et al. 1992; Narayan et al. 1995), the mm/sub-mm radiation is likely produced in the vicinity of the black hole. Earlier work (e.g., Melia 1992, 1994; Coker & Melia 1997) has indicated that Sgr A* is accreting from the stellar winds surrounding the black hole and that the infalling gas circularizes at a radius of $\sim 20–800 \, r_S$. We recently (Melia, Liu, & Coker 2001; Bromley et al. 2001) demonstrated that the inner $10 \, r_S$ of the resultant Keplerian structure can not only account for the mm/sub-mm properties via thermal synchrotron emission, but it may also produce Sgr A*’s X-ray spectrum in the quiescent state (Baganoff et al. 2001) via Comptonization of the mm/sub-mm photons. On the other hand, the cm radio emission appears to be produced by non-thermal synchrotron emission (Liu & Melia 2001). Meanwhile, the recently detected X-ray flare (Baganoff et al. 2001) from Sgr A* appears to be produced by thermal bremsstrahlung radiation resulting from a transient injection of additional mass flowing through the disk (Liu & Melia 2002).

The 106-day cycle seen at 1.3 and 2.0 cm (Zhao et al. 2001) is intriguing because high resolution VLA observations have already ruled out the possibility that such a period might be produced by an orbiting emitting object, which would have been distinguished spatially from
the main source (Bower et al. 1998). This inference is supported by the observed lack of proper motion (Backer et al. 1999; Reid et al. 1999) of Sgr A*, which precludes its possible association with rapidly moving components. In addition, any stellar origin for such a source falls short of the power required to account for the measured radio emission. It is reasonable to conclude, therefore, that the periodic variations in Sgr A* are intrinsic to the source. In this Letter, we examine the possibility that the observed 106-day radio modulation is associated with a precession of the disk in a Kerr metric and explore the implications this may have on the nature of Sgr A*.

2. Nature of the Long-Term Modulation

The characteristics of this 106-day cycle constrain the nature of its origin rather tightly (Zhao et al. 2001). First, the observed period is independent of the wavelength, and recent Submillimeter Array (SMA) and VLA observations of Sgr A* show that the mm and cm variations are correlated (Zhao et al. 2001a). Thus, the fact that the emission at different frequencies is produced on different spatial scales (Melia et al. 1992), suggests that this period should be induced by a single process, which can cause correlated fluctuations across a broad range of wavelengths. Second, the period is four orders of magnitude longer than the dynamical time scale at the marginally stable orbit \( r_{ms} \). So it can either be produced on a much larger spatial scale, or it may be associated with an intrinsic property of the supermassive black hole itself, such as a nonzero spin. However, recent VLA observations have indicated that the 2 cm emission is produced within a region no larger than 140 \( r_S \) (Bower et al. 2002), for which the corresponding dynamical time scale is about one and a half days, which is much smaller than the observed period. Higher frequency emission is expected to be produced within still smaller regions, associated with even smaller time scales.

We may ask then, whether this modulation could be produced by a corrugation wave in an accretion disk, which is used to account for the quasi-periodic oscillations (QPOs) seen in low-mass X-ray binaries. These waves have periods that are much longer than the corresponding dynamical time scale (on a comparable spatial scale; Kota 1990), but the periods depend on radius and thus may not be able to account for the first feature described above. Moreover, the observed light
curves show quite stable periodic fluctuations, rather than the uncorrelated segments constituting QPOs. We thus conclude that the period must be associated with a single process evolving on a small spatial scale (i.e., certainly less than $\sim 100 r_S$ in size).

A schematic diagram of the assumed geometry for this system is shown in Figure 1. The oscillation frequency of a perturbation in the perpendicular direction to a circular orbit in the equatorial plane of a Kerr black hole has been derived by Kato (1990), and may be written in the form $\nu^2_{\theta} = \nu^2_{\phi} (1 \mp 4 a M^{1/2} r^{-3/2} + 3 a^2 r^{-2})$, where $G = c = 1$, and $\nu_{\phi} = \pm M^{1/2} r^{-3/2} \left[2\pi (1 \pm a M^{1/2} r^{-3/2}) \right]^{-1}$ is the frequency of a circular orbit at radius $r$ measured by a static observer at infinity (Bardeen et al. 1972). For small values of $a/M$, the nodal precession frequency is then $\nu_{\rho \theta} \equiv \nu_{\phi} - \nu_{\vartheta} \simeq a M r^{-3}/\pi$. Because this frequency depends on $r$, the final configuration of a disk that is misaligned with the black hole spin at large radii depends on whether the viscous time scale $t_{vis}$ is shorter or longer than the precession period $\nu_{\rho \theta}^{-1}$. For a thin cold disk, the differential Lense-Thirring precession will dominate the internal coupling of the plasma at small radii and will therefore lead to the so-called Bardeen-Petterson effect (Bardeen & Petterson, 1975), in which the inner region flattens toward the equatorial plane, producing a warped accretion pattern. As shown by Nelson & Papaloizou (2000), however, thicker disks with a mid-plane Mach number of 5 or less can suppress warping effects due to the coupling provided by pressure gradients in the gas. The mid-plane Mach number in Sgr A* is $\sim 3$ (Melia, Liu, & Coker 2001), so it appears that the disk in this system precesses more or less as a rigid body.

Since the surface density of the disk is roughly a constant (Melia, Liu, & Coker 2001), the total precessional torque acting on it is

$$T = \sin \xi \int_{r_i}^{r_o} 2\pi \nu_{\rho \theta} v_k r 2\pi r \Sigma dr = 8\pi \sin \xi a M (G M)^{1/2} \Sigma \left[1 - (r_i/r_o)^{1/2} \right] / r_i^{1/2} \ , \quad (1)$$

where $\xi$ is the angle between the axis of the disk and the black hole spin, $\Sigma$ is the surface density, $v_k$ is the Keplerian velocity and $r_i$ and $r_o$ are, respectively, the inner and outer radii. The disk’s total angular momentum is given by $L = 0.8\pi (G M)^{1/2} \Sigma r_o^{5/2} \left[1 - (r_i/r_o)^{5/2} \right]$. Thus, the
precession period of the disk is

\[ P = 2\pi \sin \xi (L/T) = \frac{\pi r_o^{5/2} r_i^{1/2} [1 - (r_i/r_o)^{5/2}]}{5 a M [1 - (r_i/r_o)^{1/2}]} \].

(2)

In these calculations we have neglected the higher order general relativistic effects, which will introduce corrections of order \( r_S/r \). The gas will fall into the black hole very quickly once it crosses \( r_{ms} \). The inner boundary \( r_i \) of the accretion disk may therefore be taken to be \( r_{ms} \), which is \( \sim 3 r_S \) for a black hole with small spin. Corrections introduced by the higher order general relativistic effects should then be no bigger than \( \sim 30\% \).

The disk’s outer radius, on the other hand, is not well constrained, even though several observational lines of evidence point to a compact, hot, magnetized, Keplerian region as the dominant radiator (Melia, Liu, & Coker 2000; 2001; Liu & Melia 2001). Earlier hydrodynamical simulations, however, do suggest that the accretion flow in Sgr A* circularizes at a radius of \( 20 - 800 r_S \) (Coker & Melia 1997). In the following discussion, we will take \( r_o = 30 r_S \) as a fiducial outer boundary. It is straightforward to see that an identification of this precession period with the detected radio modulation then implies a black hole spin

\[ a/M = 0.088 (r_i/3 r_S)^{1/2} (r_o/30 r_S)^{5/2} \frac{1 - (10 r_i/r_o)^{5/2}}{1 - (10 r_i/r_o)^{1/2}} \].

(3)

(Note that in these units, \( r_S = 2 M \).)

There are several questions that arise from this discussion. The first concerns whether or not the accreted angular momentum vector can disrupt the small inner disk as it precesses around the black hole’s spin axis, since \( t_{vis} \ll P \) (Liu, & Melia 2002). It should be emphasized, however, that the viscous time scale is the time required for the gas to flow through the disk, while the time required to realign the disk via accretion is determined by the disk’s total angular momentum vector and the angular momentum flux through it. This depends primarily on whether or not angular momentum is transported efficiently through the disk. The infalling gas carries angular momentum inward. However, a magnetic stress applied to the flow by the accretor can introduce an outward angular momentum flux that can effectively cancel much of the inward flux. Recent work on processes below \( r_{ms} \) has shown that the torque at the inner edge of the disk is not
necessarily zero, and that angular momentum can therefore be extracted from the magnetized gas below $r_{ms}$ (see, e.g., Krolik 1999; Gammie, 1999). Consequently, the angular momentum flux through the disk may be sufficiently small (perhaps even zero) that its effect on the inner structure may be negligible. However, the disk can be disrupted by significant fluctuations in the angular momentum induced by variations in the large scale accretion flow. As we alluded to earlier, this type of event is expected to occur roughly every few hundred years (Coker & Melia 1997).

Second, how does a precessing disk affect the cm emission, given that earlier work has already established the implausibility of an accretion flow producing this radiation via a thermal synchrotron process (Liu & Melia 2001)? One possibility is that the $\sim 1 - 20$ cm spectrum of Sgr A* is produced by shock-accelerated non-thermal particles in the region outside $\sim 30 r_S$, where the infalling gas is circularizing. In that case, the radio modulation could be the result of optical depth effects as the flattened distribution of particles precesses about the black hole’s spin axis. One should note, however, that due to the weak internal coupling in the quasi-spherical region (Liu, & Melia 2001), the precessing disk should not affect this large scale accretion. A more interesting possibility is suggested by the fact that the integrated $\sim 1 - 20$ cm luminosity of Sgr A* would be comparable to the power extracted from its spin energy via a Blandford-Znajek type of electromagnetic process if $a/M \approx 0.088$. The 106-day modulation would then presumably arise when the precessing disk periodically shadows the non-thermal particles flooding the region surrounding the black hole as they escape from their creation site near the event horizon.

For this mechanism to work, however, the vacuum must break down as $r$ approaches $r_S$, but one can see right away that if $a/M$ is indeed as low as the value inferred from the radio luminosity of Sgr A*, this break down cannot happen with $\gamma$-rays produced via curvature radiation (Blandford & Znajek 1977). Instead, the prevalence of mm/sub-mm photons offers the possibility that inverse Compton scatterings may produce a sufficient number of energetic $\gamma$-rays that can materialize to form the required sea of electron-positron pairs. To quantify this thought, we note that Sgr A*’s sub-mm spectrum turns over at $\nu_{mm} \approx 236$ GHz with a flux density of 2.8 Jy (Falcke et al. 1998). The corresponding radiation energy density near
the event horizon is therefore 1.02 erg cm\(^{-3}\) (assuming a distance to the Galactic Center of 8.5 kpc) which is larger than the critical radiation energy density of 0.57 ergs cm\(^{-3}\) required to initiate the cascade of electron-positron pair creation (see Eq. 2.9 of Blandford & Znajek 1977). To break down the vacuum, this photon number density must be matched to an adequately intense magnetic field: \(B \geq 29\left(0.088 \frac{M}{a}\right)\left(2.6 \times 10^6 \frac{M_\odot}{M}\right)\left(236\text{GHz/}\nu_{mm}\right)\) gauss.

By comparison, the magnetic field in the accretion disk is inferred to be about 20 gauss (Melia, Liu, & Coker 2001). However, Krolik (1999) has argued that below \(r_{ms}\), the magnetic field energy density will increase inward and should saturate to the local rest mass energy density of the infalling gas. Gammie (1999) has built a specific model where the magnetic field energy density increases significantly inside \(r_{ms}\). Based on these estimates, we would therefore expect the magnetic field to be even larger than 29 gauss near the event horizon. Under these conditions, the power extracted from the spinning black hole would scale in the following fashion:

\[
L \sim 1.1 \times 10^{34} \left(\frac{B}{29 \text{ gauss}}\right)^2 \left(\frac{a}{0.088 \text{M}_\odot}\right)^2 \left(\frac{M}{2.6 \times 10^6 \text{M}_\odot}\right) \text{ergs s}^{-1},
\]

a value that is based on the integrated \(\sim 1 - 20\) cm radio emissivity of Sgr A*.

Unfortunately, the actual process by which the energy is transferred from the black hole to the particles is not known with sufficient certainty for us to predict with confidence the characteristics of their radiative output, other than their maximum available power. Nonetheless, by assuming an equipartition between their energy density and that of the magnetic field \(B\), one can show that the angular size \(\theta\) and the value of \(B\) for this non-thermal source may be inferred from its observed turnover frequency \(\nu_{mm}\) and the corresponding flux density \(F_{mm}\) (Marscher 1983):

\[
\theta = \left[8\pi \times 10^{10} \frac{n(\alpha)}{b(\alpha)^2} (2\alpha - 1) (\gamma_1 m_e c^2)^{2\alpha-1} D_{\text{Gpc}}\right]^{1/(4\alpha+15)} \nu_{mm}^{-1} F_{mm}^{-1/(4\alpha+15)} \text{ mas},
\]

\[
B = 10^{-5} b(\alpha) \left[8\pi \times 10^{10} \frac{n(\alpha)}{b(\alpha)^2} (2\alpha - 1) (\gamma_1 m_e c^2)^{2\alpha-1} D_{\text{Gpc}}\right]^{-4/(4\alpha+15)} \nu_{mm} F_{mm}^{2/(4\alpha+15)} \text{ gauss},
\]

where \(\nu_{mm}\) and \(F_{mm}\) are in units of GHz and Jy respectively, \(\alpha\) is the spectral index of the power-law synchrotron emission in the optically thin region, \(n(\alpha)\) and \(b(\alpha)\) are parameters given in Marscher (1983), \(\gamma_1 m_e c^2\) is the low-energy cutoff of the electron distribution, and \(D_{\text{Gpc}}\) is the distance to the source in Gpc. The results are not very sensitive to the spectral index \(\alpha\). For Sgr A*, adopting an \(\alpha\) of 1, we find that the 22 cm radio emission \((\approx 0.53 \text{ Jy})\) is produced within a region whose projected size is 4.7 mas, corresponding to \(\approx 760 r_S\). Its magnetic field intensity
is 0.32 gauss. At 1.3 cm (with a flux of \( \approx 1.1 \) Jy), where the 106-day period was detected, the corresponding source size is \( 65 r_S \). We therefore conclude that a precessing disk with a diameter of \( \sim 60 r_S \) can shadow a significant fraction of the emitting region at 1.3 cm, and thereby produce a noticeable modulation at this wavelength, but not at 22 cm. This expected behavior is consistent with what was observed, in that the amplitude of the modulation was greatest at 1.3 cm, dropping monotonically toward longer wavelengths, and finally falling below statistically significant levels beyond 6 cm.

3. Discussion and Conclusions

If this picture is correct, there are several immediate consequences for the nature of Sgr A*’s spectrum at mm/sub-mm and X-ray wavelengths. If we assume that the non-thermal emission from this process extends into the sub-mm region with a spectral index of 0.2 and it turns over at 236 GHz, then the flux density at the turnover is \( \approx 1.5 \) Jy. The calculated source size and magnetic field intensity at that frequency are then \( 7.4 r_S \) and 49 gauss, respectively, which are also consistent with the magnetic field inferred from our accretion model. But the fact that the observed 236 GHz flux density (\( \approx 2.8 \) Jy) is larger than that produced by the non-thermal particles then points to the need for a hot accretion disk and its contribution to the mm/sub-mm bump.

Thinking about the spectrum produced by the non-thermal particles in the region beyond the observed break at \( \nu_{mm} \approx 236 \) GHz, we note that extending the best fit X-ray spectral index (Baganoff et al. 2001) of 1.2 backward toward the mm/sub-mm region predicts a flux density of \( \approx 1.4 \) Jy at \( \nu_{mm} \), which is very close to the value one obtains by extrapolating the non-thermal radio emission toward this frequency (Liu & Melia 2001). Although this may itself be a coincidence, the convergence of Sgr A*’s low- and high-energy spectral components at \( \approx 236 \) GHz suggests that this may therefore be the turnover frequency that separates the optically thick and thin non-thermal emission components in Sgr A*. The Chandra-detected X-rays in the quiescent state would then be due to optically-thin non-thermal synchrotron processes. In this instance, however, \( a/M \) would need to be somewhat larger than \( \approx 0.088 \) in order to accommodate
the implied non-thermal particle luminosity of $2.9 \times 10^{34}$ ergs s$^{-1}$.

There is clearly much to be done with this picture. Not coincidentally, the physics of accretion below the marginally stable orbit has attracted some attention in recent years (Krolik 1999; Gammie 1999; Agol & Krolik 2000). It appears that a nonzero stress at this radius can change the properties of the accretion disk significantly, and strong dissipation may be possible inside this orbit. Since the sharp increase in radial velocity toward the event horizon leads to a significantly lower number density, a continued equipartition between the particles and the magnetic field may produce very high-energy particles via magnetic dissipation. Under some circumstances, this combination of low particle density and magnetic reconnection may permit some particles to escape and form a directed, relativistic outflow, a scenario that provides an alternative to the trapped non-thermal particle model we have been describing here. Further study of these various possibilities is warranted.

We note, finally, that the shadowing effect from the precessing disk should be even more pronounced at 43 GHz than at 22 GHz, based on our estimate for the relative sizes of the emitting regions, suggesting that an observation of a 106-day period at this higher frequency should reveal an even stronger modulation than has been seen thus far at the longer wavelengths.

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REFERENCES


Baganoff, F. et al. 2001, Nature, 413, 45


Zhao, J. H., & Goss, W. M. 1993, Sub-Arcsecond radio astronomy (Cambridge: Cambridge Univ. Press), 38

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Fig. 1.— Schematic diagram showing the pertinent characteristics of a system in which the spin-induced precession of a compact disk accounts for the observed 106-day cycle in the $\lambda > 1$ cm emission from Sgr A*. The mm/sub-mm bump in Sgr A*'s spectrum is thought to originate from the hot, magnetized Keplerian structure, whereas the $\sim 1 - 20$ cm emission is probably due to the synchrotron emission of non-thermal particles, possibly energized by a Blandford-Znajek process near the event horizon. Whether or not these particles escape from the system and form a directed outflow may depend on the strength of the magnetic field and its configuration. In either case, the 106-day modulation may be due to the periodic shadowing of the non-thermal emitting region by the precessing disk. Here, $\vec{L}$ is the angular momentum vector of the disk, $a$ is the spin parameter for the black hole, and $\nu_{p\theta}$ is the precession frequency of the disk.