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**CHIRAL GAUGE THEORIES ON THE LATTICE WITHOUT GAUGE FIXING?**

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**ABSTRACT**

We discuss two proposals for a non-perturbative formulation of chiral gauge theories on the lattice. In both cases gauge symmetry is broken by the regularization. We aim at a dynamical restoration of symmetry. If the gauge symmetry breaking is not too severe this procedure could lead in the continuum limit to the desired chiral gauge theory.

1. **Restoration of gauge symmetry.** The non-perturbative formulation of chiral gauge theories on the lattice is still an unsolved problem in field theory. On a hypercubic lattice each Weyl fermion is accompanied by 15 species doublets, where half of these carry opposite chirality. A naive lattice transcription of a chiral gauge theory leads therefore to a vector-like theory, instead of the desired chiral gauge theory. Many of the methods, which have been proposed to solve this problem, conflict with the concept of gauge invariance. We shall discuss here two methods, Wilson’s approach and the staggered fermion approach, which have been used before in vector-like theories to reduce the number of fermion flavors. The first method tries to remove the species doublets from the spectrum by rendering them heavier than the cut-off, whereas the second one uses them as physical degrees of freedom. Both methods lead to a gauge non-invariant actions when applied to chiral gauge theories. One possibility to treat these non-gauge invariant models is to start from a gauge fixed continuum model and to adopt one of the two methods to transcribe it to the lattice. Since the resulting lattice action breaks BRST invariance, one has to add all counterterms with dimension \(\leq 4\) and tune their coefficients such that this symmetry gets restored in the scaling region. From a technical point of view however this method is very cumbersome and one also has to worry about non-perturbative gauge fixing. Alternatively one can also aim at a dynamical restoration of gauge invariance. Both models become automatically gauge invariant after integrating over the gauge fields in the path integral, however at the price of introducing an extra radially frozen scalar field \(V_x\). Let’s start first from a generic non-gauge invariant lattice action, \(S(U)\). It is easy to show that the partition function can be written in the following form \(Z = \int DU e^{S(U)} = \int DU D V e^{S(V_x U_{\mu x} V_{x+\mu})}\). The new form of the action \(S(V_x U_{\mu x} V_{x+\mu})\) is trivially invariant under the local gauge transformations \(U_{\mu x} \rightarrow \Omega_x U_{\mu x} \Omega_{x+\mu}^\dagger, V_x \rightarrow \Omega_x V_x\). The important question of symmetry restoration is whether the resulting model still can describe the physics of the underlying gauge invariant target model in the continuum.

2. **Wilson’s method.** As our target model we shall consider here a \(U(1)_{L}^{\text{local}} \otimes U(1)_{R}^{\text{global}}\) gauge-fermion model. Its lattice action is given by the following expression

\[
S = -\frac{1}{2} \sum_{x, \mu} \bar{\psi}_x \gamma_\mu \left[ (D^+_\mu + D^-_\mu) P_L + (\partial^+ - \partial^-) P_R \right] \psi'_x - \sum_{x} \bar{\psi}_x \gamma_5 \psi'_x + \frac{w}{2} \sum_{x} \bar{\psi}_x \nabla \psi'_x ,
\]  

(1)
where \( D^+ \psi' \equiv U_{\mu x} \psi'_{x+\mu} - \psi'_\mu \), \( D^- \psi' \equiv \psi'_x - U^{*}_{\mu x-\mu} \psi'_{x-\mu} \), \( \partial^+ = D^+ |_{\nu=1} \), \( \partial^- = D^- \), and a standard Wilson mass term has been added to remove the species doublers from the spectrum. Both, the Wilson mass and the bare mass term break chiral gauge invariance. The action which results after integrating over the gauge fields in the partition function is given by (1) with \( U_{\mu x} \rightarrow V^*_x U_{\mu x} V^*_{x+\mu} \). The \( \psi' \)-field in this action is screened from the gauge fields by the \( V \)-fields and is therefore neutral with respect to the \( U(1)_L \) gauge transformations. By the gauge transformation \( \psi'_x = (V^*_x P_L + P_R) \psi'_x \) we can remove the \( V \)-fields from the kinetic term. The action in terms of the \( \psi \)-fields is identical with (1) (with \( \psi' \rightarrow \psi \)), except that the bare and Wilson mass terms turn into Yukawa and Wilson-Yukawa terms, 

\[-y \sum_x (\overline{\psi}_{Lx} V_x \psi_{Rx} + \overline{\psi}_{Rx} V^*_x \psi_{Lx}) + \frac{\kappa}{2} \sum_x (\overline{\psi}_{Lx} V_x \psi_{Rx} + \overline{\psi}_{Rx} V^*_x \psi_{Lx}) \]

We have studied this model, which is known also as Smit-Swift model 3, in the global symmetry limit, \( U_{\mu x} = 1 \), and with the term \( 2\kappa \sum_{x,\mu} \text{Re} (V^*_x V_{x+\mu}) \) added to the action. The phase diagram which has been established by numerical and analytical calculations is shown in fig. 1. Besides the ferromagnetic (FM) phase there are two symmetric phases, PMS and PMW. Analytic and numerical calculations have shown that the physics in the PMW (PMS) phase is described by the action in terms of the \( \psi \) (\( \psi' \))-fields. The particles which are associated with the scalar fields decouple in both phases as long as one keeps far away from the phase boundaries. The \( \psi \)-fermion in the PMW phase is massless. However, its species doublers stay also massless and the resulting model is vector-like. In contrast in the PMS phase the species doublers of the \( \psi' \)-fermion can indeed be removed from the spectrum, but the physical \( \psi' \)-fermion is massive and decouples from the bosonic particles in the continuum limit 4. It has also been shown that a charged fermion, which would couple to gauge fields and may exist as a \( V \)-\( \psi' \) bound state, does not exist in the PMS phase 4. This result shows that a dynamical restoration of chiral gauge symmetry does nowhere occur in the phase diagram. The same model exhibits however also an example for dynamical gauge symmetry restoration: Let's consider for the moment the vector-like naive fermion model with \( w = y = 0 \) as our target model. The gauge invariance of this model gets broken for \( y > 0 \). The \( \kappa-y \) phase diagram is given by the \( w = 0 \)-plane in fig. 1. The \( \psi \)-fermion is massless in the PMW phase, even though the coefficient \( y \) of the symmetry breaking mass term is non-zero and gauge symmetry is restored in this phase provided that one keeps far away from the phase boundaries.

3. The staggered fermion approach. Staggered fermions describe four Dirac flavors in the scaling region. The Dirac and flavor components of these four staggered flavors do not appear in an explicit form since they are spread out over the lattice. It has been shown in ref. 2 how one can couple these spin and flavor components in an arbitrary manner to other fields. The staggered fermion action may symbolically be represented by the first term in (1), keeping in mind however that the spin-flavor components of the \( \psi' \) field are now spread out over the lattice. The model lacks gauge invariance since gauge transformations on the \( U \)-fields cannot be carried through to the \( \psi' \)-fields. A perturbative calculation in two dimensions has shown that the model performs well for smooth external gauge fields 5. The important issue however is whether this remains true also when taking into account
the full quantum fluctuations. The gauge invariant version of the staggered fermion model is again obtained by $U_{\mu x} \rightarrow V_{\rho} U_{\mu x} V_{\sigma+\rho}$. Since the staggered fields $\psi'$ are neutral with respect to the $\text{U}(1)$ transformations we have added also the mass term in (1) to the action. In contrast to the Wilson-Yukawa model we cannot relate the $\psi'$-fields by a gauge transformation to a “staggered” $\psi$-field. The crucial question however is whether the scalar fields are sufficiently smooth such that this transformation could effectively take place, leading to a PMW phase at small $y$ with a massless $\psi$-fermion. In this case gauge symmetry would be restored dynamically. An extensive investigation of the $\kappa$-$y$ phase diagram for the case of an axial-vector model (this model has a larger lattice symmetry group) has shown that the PMS phase extends down to $y = 0$ with no PMW phase opening up at small $y$ and thereto also in this case the desired restoration of gauge symmetry does not take place.

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5. References: