Solution to worldvolume action of D3 brane in pp-wave background

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Abstract

We find minimum energy nonsupersymmetric solution of D3 brane configuration in the background of pp wave obtained as a Penrose limit of $AdS_5 \times S^5$. The solution has a form as that of spike and is stable.
Recently, it has been shown that the study of maximally supersymmetric IIB supergravity background, pp waves[1], of various geometries gives rise to many interesting ideas in the context of AdS/CFT correspondence. Especially, the study of BMN[2] tells us how to see the duality in a particular sector of the four dimensional SU(N) gauge theory with $N = 4$ with type IIB superstring theory in pp wave background. The study of duality have been extended to various singular spaces with different supersymmetries in [3]-[11] and have been studied in [8], [9], [10] from holographic point of view. Moreover, string theory in this background is exactly solvable. The quantization of strings in the pp wave background with NS-NS and R-R fluxes is done in [12]-[14] and the quantization of Dp brane is studied in [15]. Solution to branes in the pp wave background have been studied in [16]-[17] and in [18],[19] from supergravity point of view. The authors of [17] have studied various configurations of Dp brane with different embeddings and found solutions of constant embeddings with zero worldvolume flux on the brane.

In this letter we shall derive a nonsupersymmetric solution of D3 brane whose worldvolume directions are extended along one of the light cone direction and the other three directions are along one SO(4) directions, in the pp wave background. The solution is a stable solution because for the kind of embeddings we are choosing produces a minimum energy configuration of this D3 brane which can couple to 5-from R-R field strength.

Before we start to present the solution of D3 brane in the pp wave background, let us discuss the geometry briefly. The geometry of $\text{AdS}_p \times S^q$ in global coordinate can be described as:

$$ds^2 = R_A^2(d\rho^2 + \sinh^2 \rho d\Omega^2_{p-2} - \cosh^2 dt^2) + R_S^2(d\theta^2 + \sin^2 \theta d\Omega^2_{q-2} + \cos^2 \theta d\psi^2), \quad (1)$$

where $R_A, R_S$ are the radius of curvature and radius of $\text{AdS}_p$ and $S^q$ respectively. To derive the pp wave, we have to consider a particle moving along the $\psi$ direction and sitting at $\rho = 0$ and $\theta = 0$ and the geometry seen by the particle while moving along this trajectory will give us the desired geometry. To do so, let us define coordinates as:

$$x^+ = \frac{1}{2}(t + \frac{R_S}{R_A} \psi), \quad x^- = \frac{R_A^2}{2}(t - \frac{R_S}{R_A} \psi), \quad x = R_A \rho, \quad y = R_S \theta. \quad (2)$$

Hence, the geometry in this coordinates becomes

$$ds^2 = \begin{align*}
R_A^2 & \left[ \frac{dx^+}{R_A^2} + \sinh^2 \left( \frac{x}{R_A} \right) d\Omega^2_{p-2} - \cosh^2 \left( \frac{x}{R_A} \right) \left\{ (dx^+)^2 + R_A^4(dx^-)^2 + 2R_A^2 dx^+ dx^- \right\} 
+ R_S^2 & \left[ \frac{dy^+}{R_S^2} + \sin^2 \left( \frac{y}{R_S} \right) d\Omega^2_{q-2} + \cos^2 \left( \frac{y}{R_S} \right) \left\{ \frac{R_A^2(dx^+)^2}{R_S^2} + \frac{(dx^-)^2}{R_S^2 R_A^2} - \frac{2}{R_S^2} dx^+ dx^- \right\} \right] \end{align*} \quad (3)$$

Let us define a limit

$$R_A \to \infty, \quad R_S \to \infty, \quad \text{and} \quad \text{keeping} \quad \frac{R_S}{R_A} = \text{fixed.} \quad (4)$$

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In this limit the metric becomes
\[ ds^2 = -4dx^+ dx^- \left( x^2 + \frac{R_p^2}{R_S^2} y^2 \right) (dx^+)^2 + dx^2 + x^2 d\Omega_{p-2}^2 + dy^2 + y^2 d\Omega_{q-2}^2, \] (5)
scaling \( x^+ \to \mu x^+ \) and \( x^- \to \frac{x^-}{\mu} \), we get the metric as
\[ ds^2 = -4dx^+ dx^- - \mu^2 \left( x^2 + \frac{R_A^2}{R_S^2} y^2 \right) (dx^+)^2 + dx^2 + x^2 d\Omega_{p-2}^2 + dy^2 + y^2 d\Omega_{q-2}^2. \] (6)

It’s easy to see from the Penrose limit of \( AdS_p \times S^q \) that the Penrose limit of spaces with different values of \( p \) and \( q \) but with same \( p+q \) have the same pp wave provided the radius of curvature of AdS is equal to the radius of sphere. For \( AdS_5 \times S^5 \) the corresponding R-R 5-form fieldstrength is
\[ F_{+1234} = F_{+5678} = \text{constant} \times \mu. \] (7)

The world volume theory of Dp brane supports solitonic configurations of lower dimensional branes. In flat space, in particular, in the context of D3 branes, BPS monopoles correspond to an orthogonal D1 brane. We shall describe D3 brane in the Penrose limit of \( AdS_5 \times S^5 \) background.

The low energy dynamics of a D3 brane in a pp wave background, i.e
\[ ds^2 = -4dx^+ dx^- - \mu^2 \left( x^2 + \sum_{i=1}^{8} x_i^2 \right) (dx^+)^2 + \sum_{i,j=1}^{8} dx^i dx^j \eta_{ij}, \] (8)
where we have taken the radius of curvature of AdS is same as the radius of the sphere, is described by
\[ S = -T_3 \int d^4\sigma \sqrt{-\det (P[G]_{ab} + \lambda F_{ab})}, \] (9)
where we have set \( B_{\mu\nu} = 0 \), dilaton=0. \( F_{ab} \) is the U(1) fieldstrength living on the brane, \( \lambda \equiv 2\pi\alpha' \), but we shall set \( \lambda = 1 \), henceforth, and \( P \) is the pullback which will pullback the bulk fields onto the worldvolume of the brane and \( P[G]_{ab} \) is given by
\[ P[G]_{ab} = -2\partial_a X^+ \partial_b X^- - 2\partial_a X^- \partial_b X^+ - \mu^2 (x_1^2 + \ldots + x_8^2) \partial_a X^+ \partial_b X^- + \sum_{i,j=1}^{8} \partial_a X^i \partial_b X^j \eta_{ij}. \] (10)

We shall choose the static gauge choice as\(^1\)
\[ X^+ = \tau, \quad X^- = \phi^9, \quad X^1 = \sigma^1, \quad X^2 = \sigma^2, \quad X^3 = \sigma^3, \quad X^m = \phi^m. \] (11)
\(^1\)In our notation a,b denotes the worldvolume coordinates of D3 brane and can take values 0,1,2,3 and m can take values from 4 to 8.
In this choice of static gauge pullback becomes

\[
P[G]_{ab} = -\mu^2 \left( (\sigma^1)^2 + (\sigma^2)^2 + (\sigma^3)^2 + \phi^m \phi^m \right) \partial_a \tau \partial_b \tau + \partial_a \sigma^1 \partial_b \sigma^1 + \partial_a \sigma^2 \partial_b \sigma^2 + \partial_a \sigma^3 \partial_b \sigma^3 \\
+ \partial_a \phi^9 \partial_b \phi^9 + \partial_a \phi^m \partial_b \phi^m,
\]  

(12)

where we have taken the spacetime coordinates as same as the embeddings, i.e. \( x^i = X^i, i = 1, \ldots, 8 \). Let us excite only one transverse scalar, say \( \phi = \phi^4 \) and a magnetic field \( B^\alpha = \frac{1}{2} \epsilon^{\alpha\beta\gamma} F_{\beta\gamma} \) \( (\alpha, \beta, \gamma = 1, 2, 3) \). For static configurations, the energy then becomes

\[
E = -L = T_3 \int d^3 \sigma \sqrt{\mu^2 \left( (\sigma^1)^2 + (\sigma^2)^2 + (\sigma^3)^2 + (\phi)^2 \right) \left( 1 \pm \tilde{B} \cdot \vec{\nabla} \phi \right)^2 + (\tilde{B} \mp \vec{\nabla} \phi)^2} \left( 1 \pm \tilde{B} \cdot \vec{\nabla} \phi \right) \]

\[
T_3 \geq \int d^3 \sigma \sqrt{\mu^2 \left( (\sigma^1)^2 + (\sigma^2)^2 + (\sigma^3)^2 + (\phi)^2 \right) \left( 1 \pm \tilde{B} \cdot \vec{\nabla} \phi \right)} \]

(13)

The lower bound is achieved when

\[
\tilde{B} = \pm \vec{\nabla} \phi,
\]

(14)

using this along with the Bianchi identity, \( \vec{\nabla} \cdot \tilde{B} = 0 \), we get the equation to scalar as

\[
\nabla^2 \phi = 0.
\]

(15)

The nontrivial solution to eq.(15) and magnetic field is

\[
\phi = \frac{N}{2r}, \quad \tilde{B} = \mp \frac{N}{2r^3} \vec{r},
\]

(16)

where \( r^2 = (\sigma^1)^2 + (\sigma^2)^2 + (\sigma^3)^2 \) and \( N \) is an integer due to charge quantization [20]. Hence, the energy to this configuration is the sum of energy of Born-Infeld and Chern-Simon part. Since, we are taking minimum energy for the nonlinear Born-Infeld action implies the total energy is minimum.

In order to see how much supersymmetry does this configuration preserve let us note that the supersymmetric analysis to D3 brane have been studied in [21],[22],[18] and in [15], these authors have noted that D3 brane preserves half of the supersymmetry in the background of pp waves provided the worldvolume directions of D3 are along the light cone directions along with two other directions are along either of SO(4) directions. If one of the direction is along one SO(4) and other is along the other SO(4) direction then the solution do not preserve any supersymmetry [17].

It has been argued that with every brane embedding there is a kappa symmetry projection operator which satisfies

\[
\Gamma \epsilon = \epsilon,
\]

(17)
if the given brane embedding preserves some fraction of the supersymmetry, where \( \epsilon \) is a Killing spinor of the supersymmetric background and \( \Gamma \) is kappa symmetric projector and is defined as in [23]-[25]

\[
d^{p+1}\sigma \Gamma = - \frac{\epsilon \cdot \phi}{L_{\text{DBI}}} e^{-\frac{\phi}{4}X} \wedge XY |_{\text{vol}} , \tag{18}
\]

with

\[
X = \bigoplus_n \gamma^{(2n+q)} P^{n+q} , \tag{19}
\]

where

\[
\text{IIA} : P = \gamma_1, \quad Y = 1, \quad q = 1 \\
\text{IIB} : P = K, \quad Y = I, \quad q = 0, \tag{20}
\]

where \(|_{\text{vol}}|\) means that term should be proportional to the volume form of the brane and \( F = F - B \). The operators \( I \) and \( K \) act on spinors as \( I \psi = -i \psi \) and \( K \psi = \psi^* \) respectively. These operators are anticommuting and satisfy the following properties: \( I^2 = -1, \quad J^2 = 1 = K^2 \) and \( IJ=K \). \( \frac{1}{L_{\text{DBI}}} \) is the value of the DBI Lagrangian evaluated in the background with the embeddings. \( \gamma^{(2n)} \) is a 2n-from gamma matrices defined on target space as

\[
\gamma^{(2n)} = \frac{1}{2n!} \gamma_{i_1...i_{2n}} d\sigma^{i_1} \wedge \ldots \wedge d\sigma^{i_{2n}} , \tag{21}
\]

and \( \gamma_{i_1...i_{2n}} \) is the pullback for the target space gamma matrices

\[
\gamma_{i_1...i_{2n}} = \partial_{i_1} X^{\mu_1} \ldots \partial_{i_{2n}} X^{\mu_{2n}} \gamma_{\mu_1...\mu_{2n}} . \tag{22}
\]

It has been shown in [23] that \( \Gamma \) satisfies the following properties. \( \Gamma^2 = 1 \) and \( \text{tr}(\Gamma) = 0 \), which enables to define projector of that kind which projects out half of the worldvolume fermions thus making the degrees of freedom of worldvolume fermions and bosons same.

Since, we are studying a D3 brane configuration where dilaton and 2-form anti-symmetric B field is zero in an IIB background then the projector written in eq. (18) becomes

\[
\Gamma = - \frac{\epsilon_{i_1...i_4}}{L_{\text{DBI}}} \left[ \frac{1}{24} \gamma_{i_1...i_4} I + \frac{1}{4} F_{i_1 i_2} \gamma_{i_3 i_4} J + \frac{1}{8} F_{i_1 i_2} F_{i_3 i_4} I \right] . \tag{23}
\]

Evaluating the projector in our static gauge choice, we get:

\[
\Gamma = - \frac{\epsilon_{0123}}{L_{\text{DBI}}} \left[ (\gamma_{+123} + \partial_1 \phi \gamma_{+238} + \partial_2 \phi \gamma_{+138} + \partial_3 \phi \gamma_{+128}) I \right. \\
\left. + \{B^1 (\gamma_{+1} + \partial_1 \phi \gamma_{+8}) + B^2 (\gamma_{+2} + \partial_2 \phi \gamma_{+8}) + B^3 (\gamma_{+3} + \partial_3 \phi \gamma_{+8}) \} \right] . \tag{24}
\]
The first and second line of eq. (24) follows from the first and second term of eq. (23) and the last term vanishes because of our choice of magnetic field. For negative chirality type IIB spinors
\[ \gamma_{-12345678} \epsilon = -\epsilon, \] (25)
the Killing spinor, \( \epsilon \), is derived by setting the supersymmetric variation of dilatino and gravitino to zero, of the pp-wave is given by [1], [17]
\[ \epsilon = \left\{ 1 - \frac{i}{2} \sum_{a=1}^{4} \gamma_{-}(y^{a} \gamma_{a} \gamma_{1234} + z^{a} \gamma_{(a+4)} \gamma_{5678}) \right\} \]
\[ \left( \cos \frac{x^{+}}{2} - i \sin \frac{x^{+}}{2} \gamma_{1234} \right) \left( \cos \frac{x^{+}}{2} - i \sin \frac{x^{+}}{2} \gamma_{5678} \right) (\lambda + i \eta), \] (26)
where \( \lambda \) and \( \eta \) are constant, real negative chiral spinors and \( y^{a}, z^{a} \) are the coordinates along the first and second SO(4) respectively. Substituting the value of \( \Gamma \) and \( \epsilon \) in eq. (17), and restricting to \( \phi = \phi(\sigma^{1}) \) and only one component of magnetic field i.e. to \( B^{1} \), for simplicity, we get
\[ \epsilon = i \frac{\epsilon^{0123}}{\mathcal{L}_{DBI}} \left[ (\gamma_{+123} + \partial_{1} \phi \gamma_{+238}) \epsilon - B^{1}(\gamma_{+1} + \partial_{1} \phi \gamma_{+8}) \epsilon^{*} \right]. \] (27)
Let us define
\[ P \equiv -\frac{1}{2} \sum_{a=1}^{4} \gamma_{-}(y^{a} \gamma_{a} \gamma_{1234} + z^{a} \gamma_{(a+4)} \gamma_{5678}) \]
\[ Q \equiv \gamma_{+123} + \partial_{1} \phi \gamma_{+238} \]
\[ R \equiv B^{1}(\gamma_{+1} + \partial_{1} \phi \gamma_{+8}). \] (28)
using these, we can rewrite eq. (27) as
\[ \epsilon = i \frac{\epsilon^{0123}}{\mathcal{L}_{DBI}} \left[ Q \epsilon - R \epsilon^{*} \right]. \] (29)
Since the Killing spinor eq. (26) holds for all values of \( x^{+} \), implies, on plugging into eq. (29) gives rise to
\[ \left( 1 - \frac{i \epsilon^{0123}}{\mathcal{L}_{DBI}} Q \right) (1 + iP) (\lambda + i \eta) = -\frac{i \epsilon^{0123}}{R} (1 - iP) (\lambda - i \eta) \]
\[ \left( 1 - \frac{i \epsilon^{0123}}{\mathcal{L}_{DBI}} Q \right) (1 + iP) (\gamma_{1234} + \gamma_{5678}) (\lambda + i \eta) = -\frac{i \epsilon^{0123}}{R} (1 - iP) (\gamma_{1234} + \gamma_{5678}) (\lambda - i \eta) \]
\[ \left( 1 - \frac{i \epsilon^{0123}}{\mathcal{L}_{DBI}} Q \right) (1 + iP) \gamma_{1...8} (\lambda + i \eta) = -\frac{i \epsilon^{0123}}{R} (1 - iP) \gamma_{1...8} (\lambda - i \eta). \] (30)
Equating real and imaginary parts of eq. (30) and using negative chirality of $\lambda$ and $\eta$, we get

$$
\left(1 + \frac{\epsilon^{0123}}{\mathcal{L}_{DBI}} Q P + \frac{\epsilon^{0123}}{\mathcal{L}_{DBI}} R P\right)\lambda + \left(\frac{\epsilon^{0123}}{\mathcal{L}_{DBI}} Q + \frac{\epsilon^{0123}}{\mathcal{L}_{DBI}} R - P\right)\eta = 0
$$

$$
\left(1 + \frac{\epsilon^{0123}}{\mathcal{L}_{DBI}} Q P + \frac{\epsilon^{0123}}{\mathcal{L}_{DBI}} R P\right)\lambda + \left(1 + \frac{\epsilon^{0123}}{\mathcal{L}_{DBI}} Q P - \frac{\epsilon^{0123}}{\mathcal{L}_{DBI}} R P\right)\eta = 0
$$

$$
\left(\frac{\epsilon^{0123}}{\mathcal{L}_{DBI}} Q - P - \frac{\epsilon^{0123}}{\mathcal{L}_{DBI}} R\right)\lambda + \left(1 + \frac{\epsilon^{0123}}{\mathcal{L}_{DBI}} Q P - \frac{\epsilon^{0123}}{\mathcal{L}_{DBI}} R P\right)\eta = 0
$$

$$
\left(1 + \frac{\epsilon^{0123}}{\mathcal{L}_{DBI}} Q P + \frac{\epsilon^{0123}}{\mathcal{L}_{DBI}} R P\right)\gamma_{1234}\gamma_{\gamma+} + \left(\frac{\epsilon^{0123}}{\mathcal{L}_{DBI}} Q + \frac{\epsilon^{0123}}{\mathcal{L}_{DBI}} R - P\right)\gamma_{1234}\gamma_{\gamma+}\eta = 0
$$

$$
\left(P - \frac{\epsilon^{0123}}{\mathcal{L}_{DBI}} Q + \frac{\epsilon^{0123}}{\mathcal{L}_{DBI}} R\right)\gamma_{1234}\gamma_{\gamma+} + \left(1 + \frac{\epsilon^{0123}}{\mathcal{L}_{DBI}} Q P - \frac{\epsilon^{0123}}{\mathcal{L}_{DBI}} R P\right)\gamma_{1234}\gamma_{\gamma+}\eta = 0,
$$

(31)

Using $P\gamma_- = 0$, we can rewrite fifth equation of eq. (31) as

$$
\gamma_+\lambda = -\frac{\epsilon^{0123}}{\mathcal{L}_{DBI}}\gamma_+\left(\gamma_{1234} - \partial_1 \phi_{\gamma_{1238}} + B^1(\gamma_{1238} - \partial_1 \phi_{\gamma_{1238}})\right)\eta = 0,
$$

(32)

Substituting $\gamma_+\lambda = 0$, which follows from eq. (32), in the last equation of eq. (31), we get

$$
\gamma_-\gamma_+\eta = 0.
$$

(33)

Multiplication of $\gamma_+$ from right side of first eq. (31) and using $\gamma_+Q = 0, \gamma_+R = 0$ gives

$$
\gamma_+\lambda - \gamma_+P\eta = 0,
$$

(34)

imposing the constraint that this equation should be in consistent with eq. (32) gives us

$$
\gamma_+\gamma_-\eta = 0.
$$

(35)

Compatibility of eq. (33) with eq. (34) implies

$$
\eta = 0,
$$

(36)

and multiplying $\gamma_+$ from right side of second eq. (31) and imposing its consistency with eq. (32) gives

$$
\lambda = 0.
$$

(37)

Same conclusion also follows from third and fourth equation of eq. (31). So, finally we see that the above type of brane embeddings break all supersymmetry of the background. Hence, the solution that we got is a nonsupersymmetric but the solution
is a stable solution because the solution has minimum energy and is charged under the R-R 5-form fieldstrength.

We have derived a nonsupersymmetric solution of D3 brane in the pp wave background by exciting only one transverse scalar and a nonzero magnetic field on the worldvolume of the brane. The solution that we found is a spike type solution like in the case of flat space [26] and this solution is a stable solution because of the given choice of embeddings gives us a minimum energy solution which couples to nonvanishing R-R potential. If we compare this solution with the flat space solution one notes that in flat space the spike type solution does preserve one half supersymmetry follows from the linearized supersymmetric variation of gaugino and hence a stable BPS solution. But in pp-wave case this breaks all supersymmetries of the supersymmetric background.

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References


