HIGH GRADIENT CAVITIES FOR LONG BUNCH MUON BEAMS

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Abstract

Large energy spread of muon beams can be reduced by so-called phase rotation. It requires rather low frequency or long slope in the RF waveform and rather high gradient field against the short muon lifetime. Because such a low frequency high gradient RF cavity is difficult to construct, a high frequency bunching system is proposed. The bunching system located after 100m from a production target can bunch a muon beam of 100MeV ±50% with 200MHz RF, and a 100MHz bunched muon beam is obtained at 200m position.

1 INTRODUCTION

Muons are obtained as decay products from pions that are generated by high-energy protons hitting on a production target. Because the muons are tertiary products, muon beams have very bad quality. Among the six components in the phase space, only the initial time spread can be shortened (by shorten the bunch of primary protons) to reduce the longitudinal emittance of the pions. Thus, a large energy spread of muons can be reduced by so-called phase rotation, which decelerates early coming fast muons and accelerates late slow muons. A long drift space is needed for developing the good correlation between the TOF (time of flight) and its energy. It requires rather low frequency or long slope in the RF waveform for the de/ac-celeration. The short life of muons also requires rather high gradient field. Because of the high gradient, magnetic materials are difficult to apply for resonant cavities.

An air core cavity of less than 10MHz resonant frequency with more than 0.5MV/m will have a radius of more than a few meters. In order to eliminate such huge low frequency cavities and use high frequency cavities such as 100MHz, a high frequency bunching system was proposed[1,2]. The scheme is improved by an introduction of High-Frequency “Adiabatic” Buncher[3].

2 LOCAL PHASE SPACE DISTRIBUTION IN TIME SLICE

Figure 1 shows the muon energy as a function of TOF at the 100m downstream of production target with its original phase space at the production target. The full width of the original time spread is 3ns. Because the original phase space size of each 5ns slice at 100m increases as the energy increases, the slice emittance at 100m also increases. Each local phase space should be manipulated with appropriate RF voltage for each time slice.

3 MODULATING BUNCHER

Firstly, the bunching scheme is explained in Fig. 2. By applying a sawtooth waveform after the Drift-1(100m), the phase space can be flipped by the kick, where the amount of the kick depends on the energy of the slice. After the same drift length (Drift-2), the trapezoidal area rotates to upright position. Noting that the center energy for each slice does not change and propagates without any kick, the time distances between the slices develop twice for twice of the total traveling distance. The buncher period has to be large enough compared with the initial time spread to keep good bunching factor. Even if the buncher period is equal to the initial time spread, a bunch factor of 2 can be obtained(broken lines in Fig.2) by the sawtooth waveform.
3.1 Rough Simulation

Assuming that the initial time spread is 3 ns, a buncher frequency of 200 MHz should be a reasonable choice. The resulted bunch frequency at 200 m becomes 100 MHz. Use of single sine wave instead of the sawtooth waveform makes the buncher system less complex with some bunching loss. We can add higher order components later to recover it, if needed. The required waveform for the buncher system is as follows:

\[ V(t) = 2 \sin(\omega_0 t) \frac{dW_\mu(t)}{dt}, W_\mu(t) = m_\mu \left( \frac{ct}{\sqrt{c^2t^2 - L^2}} - 1 \right) \]

where \( \omega_0 \) is the angular frequency of the buncher (200 MHz) and \( m_\mu \) is the muon mass in electron volt (see Fig. 3). \( W_\mu(t) \) is the muon energy at the buncher position \( L \) and the time \( t \). The voltage of \( V(t) \) flips the time slice of the beam after 100 m drift. The slope of \( V(t) \) is similar to that of an exponential function but steeper, which corresponds to a very low Q value. Because rather high voltage is required, such a cavity is not practical. Figure 4 shows the envelop of \( V(t) = V_{\text{envelop}}(t) = V(t) \sin(\omega_0 t) \) and a fitted result with a sine wave and its harmonics, where the function \( V_{\text{envelop}}(t) \) is approximated by

\[ V_{\text{envelop}}(t) = 20(\sin(\omega_0 t + \varphi_1) + A \sin(2\omega_0 t + \varphi_2)) \text{ [MV]} \]

Thus \( V(t) \) can be written in a form:

\[ V(t) = 20 \sin(\omega_0 t)(\sin(\omega_0 t + \varphi_1) + A \sin(2\omega_0 t + \varphi_2)) \]

where \( \omega_0 \) correspond to about 2 MHz. With use of the trigonometric reduction, \( V(t) \) can be reduced to a sum of four sine functions:

\[ V(t) = 10\{ \cos((\omega_0 - \omega_0) t + \varphi_1) - \cos((\omega_0 + \omega_0) t + \varphi_1) \} + 10A \{ \cos(2(\omega_0 - \omega_0) t + \varphi_2) - \cos(2(\omega_0 + \omega_0) t + \varphi_2) \}. \]

These can be generated by four cavities with the frequency of 196, 204, 198 and 202 MHz, which generates the low frequency components as the beat. Figure 5 shows a simulation result of the modulating buncher. In the simulation, all the cavities are located at the same position and only drif ts and kicks are assumed (no transit time factor included). About 79% of the muons are collected within 5 ns and 69% within 3.5 ns at the 200 m point with 50% more voltage. Because the real cavity has finite length, the frequency \( f_\nu \) of the cavity has to be modified as the location changes:

\[ f' = \frac{L_0}{L_n} f_\nu \text{ (to the first order approximation),} \]

where \( L_0 \) is the buncher location (100 m) and \( L_n \) is the location of the \( n \)-th cavity. Assuming that all the cavities have length of 1 m, about 73% of the muons are still collected within 5 ns (60% within 3.5 ns). Because of the strong nonlinearity in the phase space motion, further numerical improvements have to be performed iteratively.

3.2 Dual Frequency Cavity (DFC)

It is possible to construct a cavity with two close resonant frequencies (see Fig. 6). The cavity has an extra resonator (\( \lambda/4 \) resonator for example) that has a resonance close to the central frequency. The central frequency is determined by the cavity size and the frequency difference can be adjusted by the coupling between the main cavity and the perturbator. The two modes appear as 0- and \( \pi \)-modes.
More than ±10% split was obtained in the MAFIA calculation for the example with >7MΩ/m at 100MHz. It should be noted that the frequency difference can be adjusted by the coupling between the two resonators and the adjustable range can be large.

The shorter cavity length reduces the discrepancy between the first and second simulation results, and 74% muons are collected within 5ns and 64% within 3.5ns with 40% more voltage. Sparking problem in such a cavity is not clear, because of the complex waveform of the electric field.

4 ADIABATIC MODULATING BUNCHER

‘Adiabatic’ buncher [3] can improve the bunching factor, which distributes the RF cavities along the beam line. As a preliminary study, two modulating bunching stations are situated at 80m and 120m position. Because of the locations, each cavity has different function $V(t)$. A rough simulation showed that 83% of the muon are collected within 5ns and 73% within 3.5ns(see Fig.7). The voltages and the frequencies are listed in the caption. Use of more stations will increase the bunching efficiency.

5 COMB PULSE ENERGY COMPRESSOR

The waveform $V_{EC}$ needed for the energy compression or “phase rotation” again can be fitted by two frequencies as follows:

$$V_{EC}(t) = 100 - 100 \left( \sin(\omega_l t + \psi_l) + B \sin(2\omega_l t + \psi_l) \right),$$

where $\omega_l$ corresponds to the low frequency. Because the modulating buncher does not break the global correlation between energy and TOF, we can still apply the “phase rotation” after it. Because the muons are now bunched, we do not need very low frequency RF such as a few MHz. Then the raw “phase rotation” waveform $V_{EC}$ can be modulated by an approximated square wave $V_{sq}$:

$$V_{sq}(t) = 1.25 \left( \cos(\omega_c t) - \cos(3\omega_c t)/4 \right)$$

and

$$V_{CPEC}(t) = V_{sq}(t)V_{EC}(t),$$

where $V_{CPEC}$ is the modulated waveform. The resulted waveform performs as a Comb Pulse Energy Compressor (CPEC). This also can be reduced to a sum of eight sine waves. With this function, about 40% muons can be compressed within 10% energy spread. When the CPEC is combined with the modulating buncher, the yield increases up to 68% (see Fig. 8).

6 DISCUSSIONS

The modulating buncher can be combined with the “Adiabatic” buncher and the distributed buncher system will bunch the beam efficiently. For a multi-bunch operation, the frequencies have to be multiples of the extraction cycle of the proton driver(100kHz?). Because of the voltage, CPEC will have a length of more than 10m. A procedure to optimize the parameters has to be developed.

REFERENCES