RESISTIVE-WALL COUPLED-BUNCH INSTABILITY DRIVEN BY IN-VACUUM INSERTION DEVICES IN THE SPRING-8 STORAGE RING

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Abstract
More than ten of in-vacuum type insertion devices are installed in the SPring-8 storage ring. When their gap was closed, we observed the low frequency vertical coupled bunch instability. We consider that this instability is driven by resistive-wall impedance caused by the magnet blocks made of NdFeB, of which conductivity is two order lower than Aluminum. We calculated the resistive-wall impedance and measured its characteristics.

1 INTRODUCTION
More than ten of in-vacuum type insertion devices[1] including a 25m long one, are installed in the SPring-8 storage ring. The material of their magnet is NdFeB of which conductivity is two order lower than aluminum that the normal beam chamber is made of. A thin Cu-Ni sheet is placed on the surface of the magnet blocks to shield the high frequency component of a beam field to avoid heating of the magnet blocks[1] and to suppress the high frequency impedance driven by gaps and steps between magnet blocks. However, the vertical transverse resistive wall impedance is high at low frequency and the skin depth of the sheet at the frequency of the vertical coupled-bunch instability resonance is larger than its thickness. We calculated the impedance and measured the characteristics of the instability.

2 RESISTIVE-WALL IMPEDANCE
We estimated the impedance of the in-vacuum type insertion devices(IVID) analytically with the aid of simulation. The schematic structure of the surface of the magnets of IVID is shown in Fig. 1. In the following discussion, we assume skin depth is much smaller than gap or radius of the structure.

![Diagram](image1.png)

Figure 1: Schematic view of the structure of the surface of in-vacuum type insertion device.

The minimum gap of IVIDs is 8mm, which is smaller enough compared to the width of the magnet surface, 71mm, hence we can treat the system as two parallel infinite plates and a beam between them. We compared the impedance of this system with that of the circular pipe of which case the impedance is analytically known[2]. The beam field on the surface of the IVID diffuses into the inner volume of the wall and the distribution of the field in depth is the same in any points on the surface but the strength may differ from point to point and are proportional to the beam field there and the beam field is proportional to the wall current density induced by the beam. The power loss by the wall current at a frequency is proportional to the real part of longitudinal impedance.

Then we uses following discussion;
1) the frequency dependence of the impedance is the same between two shapes.
2) the ratio of the longitudinal impedance of the two shapes; parallel plate and circular pipe, is equal to the ratio of the power loss of them. As shown in later, the power loss is the same in these two.
3) From dimensional analysis, the transverse impedance $Z_T$ is proportional to $Z_L/d^2$ where $Z_L$ is the longitudinal impedance and $d$ is a constant of the dimension of length[2].
4) Therefore, we set the constant $d$ to be the gap or the radius.
5) With the assumption(1), The shape of longitudinal wake function is the same then we can says that the transverse wake is proportional to the transverse gradient of the longitudinal wake, using Panovski-Wentzel theorem. The gradient of wake normalized by wake should be the same as the resistive-wall wake or wake by simple shape like a small groove of which dimension is smaller enough than gap, radius and bunch length. The wake of such small structure can be obtained using three-dimensional simulation code, such as MAFIA. We have the vertical impedance of the parallel plates is 0.8 times that of the round pipe of the radius gap/2.

For the argument 2) above, we will show the distribution of the current density and its power loss. The system is two-dimensional, $(x, y)$ as shown in Fig 2, and using the image current method, we had the surface current...
distribution on the parallel plate surface produced by a beam current $\lambda e^{ik(z-ct)}[3]$, 
$$K(x) = \frac{e^{ik(z-ct)}}{2\pi b} \frac{\pi}{2} \text{sech}\left(\frac{\pi}{2b} x\right). \quad (1)$$

The power loss distribution is
$$\frac{dP}{dz}(x) = \frac{|K(x)|^2}{2\sigma} = \frac{1}{2\sigma \delta} \left(\frac{e^{ik(z-ct)}}{2\pi b}\right)^2 \text{sech}^2\left(\frac{\pi}{2b} x\right) \quad (2)$$
and total power is
$$\frac{dP}{dz} = \frac{1}{2\sigma \delta} \left(\frac{e^{ik(z-ct)}}{2\pi b}\right)^2 \int \text{sech}^2\left(\frac{\pi}{2b} x\right) dx = \frac{1}{2\sigma \delta} \frac{\lambda^2}{2b} \quad (3)$$
which is the same value as a round beam pipe of radius $b$.

This shows that the longitudinal impedance of the parallel plate structure is the same as that of the round beam pipe.

Next, we consider the effect of the sheet in a round beam pipe as shown in Fig. 2.

![Figure 2: Vacuum-conductor-conductor system for the calculation of the resistive-wall impedance. Region I: vacuum, region II: conductivity $\sigma_2$ and region III: conductivity $\sigma_3$.](image)

Following the method in mono layer case[2], we obtain the equations for the field in region I, II, III

(I) $\tilde{E}_r^I = A^I$, $\tilde{B}_\theta^I = -\frac{1}{2} \frac{ik}{r} \frac{\mu_0 c^2 q}{2\pi} r$

(II) $\tilde{E}_r^I = A_r e^{i\lambda_2(r-b)} + A_s e^{-i\lambda_2(r-b)}$

$\tilde{B}_\theta^I = \left(\frac{k}{\lambda_2} + \frac{\lambda_2}{k}\right) A_r e^{i\lambda_2(r-b)} - A_s e^{-i\lambda_2(r-b)}$

(III) $\tilde{E}_r^I = A_3 e^{i\lambda_3(r-c)}$, $\tilde{B}_\theta^I = \left(\frac{k}{\lambda_3} + \frac{\lambda_3}{k}\right) A_3 e^{i\lambda_3(r-c)} \quad (4)$

and $k = \frac{\omega}{c}$, $\lambda_i = \frac{1+i}{2\mu_0\sigma_i \omega}$ with boundary conditions; $\tilde{E}_r^I = \tilde{E}_r^I$, $\tilde{B}_\theta^I = \tilde{B}_\theta^I$ at $r=b$, $\tilde{E}_r^I = \tilde{E}_r^I$, $\tilde{B}_\theta^I = \tilde{B}_\theta^I$ at $r=c$. In above equations, $\omega$ is the angular frequency and $\delta_i$ is the skin depth of the $i$-th layer and we assume $\delta_i << b$. Solving above equation, we have

$$Z_{||} = -\frac{1}{\epsilon q} \tilde{E}_r = \frac{2}{\lambda_2} \left(\frac{\beta + \frac{1}{\beta}}{C_2} \frac{\beta - \frac{1}{\beta}}{C_3} C_2 \right) \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{\lambda_2}{k} \frac{\lambda_3}{k} \frac{

\begin{align}
C_1 &= \frac{1}{2} \frac{ik}{b} \\
C_2 &= \frac{2\pi}{b} \frac{\mu_0 c^2 q}{k} \\
C_3 &= \frac{1}{k} + \frac{\lambda_3}{k} + \frac{\lambda_2}{k} + \frac{\lambda_1}{k} \\
\end{align}

and $\beta = e^{i\lambda_3 b}$. We applied this to the case that region I) is a vacuum beam pipe, region II) is a Ni sheet and region III) is a NdFeB magnet. The relative permeability of the Ni sheet is set to 1 because it saturates by the magnetic field of IVID. The result for different thickness of region II (shield) is shown in Fig. 3 for standard 4.5m IVID. The 50µm thick Ni sheet reduces the impedance by 75%. For the 25m long IVID, a 50µm thick Cu + 10µm thick Ni sheet is used to reduce the impedance by nearly 25%. The conductivity of NdFeB, Ni, Cu and Al in unit 1/Ωm are $6.9\times10^5$, $1.5\times10^7$, $5.9\times10^7$ and $3.5\times10^7$, respectively.

![Figure 3: Transverse impedance of a round pipe in Fig. 2 vs. shield thickness (region II) where $b=4.8$mm, length 4.5m, frequency $(1-\Delta\nu)\nu_0$ is 142kHz.](image)

3 EXPERIMENT

The following measurements were performed with the stored beam current, 100mA with zero vertical chromaticity, in which the instability has maximum growth rate, and with full-bucket filling for coupled-bunch mode identification.

From theories, the resonance of the transverse coupled-bunch instability is at $(m-\Delta\nu)\nu_0$ where $m=1,2,3,...$ and the transverse resistive-wall impedance is higher at low frequency[1]. Therefore, the strongest resonance of the vertical coupled-bunch mode is $(1-\Delta\nu)\nu_0$ which is ~140kHz in the SPRing-8 storage ring.

We used the signal from button type pick-up antenna of which signal shows amplitude modulation when the beam shows betatron motion. Because the button type pick-up antenna does not have sensitivity at low frequency, we measured the frequency $(1-\Delta\nu)\nu_0$ where the base band spectrum is repeated by bunch structure of the beam.

Using a spectrum analyzer, we measured the spectrum of the signal and observed the instability peak as the gap of IVIDWs was closed.

First, we confirmed that the difference of the impedance of the IVIDWs of the same shape is small. Second, using the IVIDWs checked at the first step, we measured the dependence of the impedance of IVIDWs on gap and the dependence of the strength of the instability on beta function.
We observed that the growth of the instability saturated at some amplitude and we considered this suppression was came from the decoherence of the electrons in a bunch by tune spread produced by amplitude dependent tune shift. Therefore, as the measure for the relative strength of the instability, we used the peak height of the lowest resonance of the instability and if the peak height is the same, we consider the strength of the instability was also the same.

The result is shown in Table 1 which shows the gap of the IVIDs when the betatron peak at $(1-\Delta\nu_y)f_0$ is $-70\text{dBm}$ while the carrier of the 508.58 MHz is $+10\text{dBm}$. The ratio between two signals is $-80\text{dBc}$ and the amplitude of the center of mass of the bunch is the order of a few micro meters.

### 3.1 Individuality of IVIDs

First, we checked the difference of IVIDs of the same shape. From Table 1, the difference is less than 3%.

### 3.2 Gap Dependence

Using the IVIDs checked at the first step, we measured the dependence of the impedance on the gap of IVIDs. Using the result for several combination of the number of IVID and their gap shown in Table 1, which they should have the same total impedance, we had the result shown in Fig. 4 and found the vertical impedance is proportional to $1/gap^3$.

### 3.3 Beta Function Dependence

From theories, the strength of the instability is proportional to the beta function at the impedance. At the hybrid-lattice which is the one of operation mode of the ring, we had the IVIDs at the straight sections of different vertical beta function, $\beta_y=8\text{m}$ in odd cells and $\beta_y=12\text{m}$ in even cells. Using the gap dependence of the impedance, $1/gap^3$, and the result of run 13,15,16 and run 18, we obtain 1.5 for the ratio of the beta function at ID9,11,39 and ID40 and this value is close to the design value above. The surface of ID40 has shallow longitudinal groove of a few mm depth in the center of magnet surface and the groove is filled with Cu. This explains that the gap of ID40 is smaller than ID44 when the same impedance is produced or ID40 has smaller impedance than ID44 at the same gap.

### 3.4 Absolute Value

From the theory, the margin of the sum of transverse impedance and beta function for the coupled-bunch instability is $94\text{M\Omega}$ at stored current 100mA. The rough estimation of the contribution of the resistive-wall impedance of Al beam pipe in the normal section($b=20\text{mm}$, elliptical shape, $\beta_y \approx 25\text{m}$) is $54\text{M\Omega}$. Then the margin left for IVIDs is $40\text{M\Omega}$. IVIDs are placed in $\beta_y = 8\text{m}$ hence the margin for impedance of IVIDs is $5\text{M\Omega}/\text{m}$. Using the gap dependence of the impedance, $1/gap^3$, we have $4 \text{ M\Omega/m}$ for IVIDs from Table 1 when the amplitude of the instability is a few micro meters. However, because we still do not know the value and the effect of tune spread caused by amplitude dependent tune shift, absolute value is rather hard to say.

### 6 REFERENCES