Abstract

As one of the figures of merit, muon polarization and its correlation to the particle arrival time was studied for the high intensity muon beam source of a Neutrino Factory. Muon polarization, 100% polarized in the parent pion rest system, was tracked down the pion capture, phase rotation, and ionization cooling channels, using the BMT equation. A study was done of the dependence of the muon polarization and its correlation on the configuration of induction linac channels in the phase rotation channel. Depolarization effects of the muon polarization through absorbers in the ionization cooling channel was simulated.

1 INTRODUCTION

Muon polarization provides one handle on the flux of muon neutrinos and electron neutrinos in a long baseline detector, for it affects the decay kinematics of the muons into electrons, muon neutrinos, and electron neutrinos. The muon polarization may allow a better understanding of the systematics in a neutrino oscillation experiment. We determine the muon average polarization and the muon polarization correlation to the muon arrival time in the front end channel of the neutrino factory design.[1, 2, 3, 4]

Charged pions are generated in the interactions of 24 GeV or 16 GeV primary protons in heavy metal targets in a 20 T solenoidal field. Pions are captured in a solenoid where the longitudinal magnetic field decreases gradually from 20 T to 1.25 T. The phase rotation channel consists of long solenoids with a magnetic field strength of 1.25 T and an induction linac system which surrounds the solenoid coils.

In the charged pion rest system, because the neutrino mass is negligibly small and its helicity is +1 or -1, muons also have the helicity of +1 or -1, reflecting the fact that the pion spin is zero. Hence the muons in the pion decays are polarized 100% in the pion rest system. Because of the Lorentz boost of the pion rest system to the laboratory system and the limited acceptance of the pions/muons in the pion capture, phase rotation, bunching, and cooling channels, the muon average polarization is different from the asymptotic muon polarization(with infinite pion momentum), and the muon polarization correlation as a function of the muon arrival time depends on the front end channel design.

2 MUON POLARIZATION IN THE LABORATORY FRAME

2.1 Muon polarization in pion decay

We define the muon polarization as the projection of the 3 dimensional muon spin in the muon rest system along the muon momentum in the laboratory system.

$$h = \frac{\vec{s}_{\mu} \cdot \vec{p}_{\mu}}{||\vec{p}_{\mu}||}$$

The muon polarization is expressed by using the Wigner angle $\omega$ as:[5]

$$h = \cos\omega = \frac{\{E_{\pi} p_{\mu} + E_{\pi} m_{\pi} \cos\theta_{\pi} - E_{\mu} p_{\mu} - E_{\mu} m_{\mu}\}}{p_{\mu} m_{\pi}}$$

where $p_{\pi}$, $E_{\pi}$ are the pion momentum and energy in the laboratory frame, and $p_{\mu}$, $E_{\mu}$ are muon momentum and energy in the pion rest frame, $p_{\mu}$, $m_{\pi}$ are muon momentum in the laboratory system and the pion rest mass.[5] An equivalent expression of the muon polarization is given in a reference.[6]

Figure 1 shows a schematic diagram of the components of the front end channel of the Neutrino Factory in the r-z plane.

Figure 2 shows the kinetic energy of $\pi^+$s at $z = 0$ m with/without the muon tags at the end of the 2nd induction linac and at the end of the cooling channel (top figure), and the kinetic energy of $\mu^+$s at $z = 36$ m with/without the same muon tags in the Study II model. Because of the kinematics in the $\pi^+$ decay into $\mu^+$ and $\nu_\mu$, the higher tail of the kinetic energy of tagged $\pi^+$s at $z = 0$ m is larger than that of the tagged $\mu^+$s at $z = 36$ m, where backward decayed muons are captured in the front end channel. The MARS simulation code was used to generate the secondary pion data at the target.[8] Tracking of the pions and decayed muons are simulated with the ICOOL simulation code.[9]

Figure 3 shows the $p_t$ spectra of $\pi^+$s, and the $p_t$ vs. $p_z$ 2 dimensional lego plots at the target with/without tags by the $\mu^+$s at the end of 2nd induction linac and at the end of the cooling channel in the Study II model. $p_t$ spectra are similar if the pions at the target are tagged by muons.
getting to the end of the second induction linac channel, and if the pions at the target are tagged by muons at the end of the cooling channel.

### Figure 2: Kinetic energy of $\pi^+$ s with/without tags at $z = 0$ m, 36 m

#### Table 1

<table>
<thead>
<tr>
<th>Kin. Energy (GeV)</th>
<th>Plan A</th>
<th>No Tag</th>
<th>Tagged at End of Cooling</th>
<th>Tagged at End of 2nd Ind. Linac</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 K primary proton</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
</tr>
</tbody>
</table>

#### Figure 3: Kinetic energy of $\pi^+$ s with/without tags at $z = 0$ m, 36 m

### 2.2 Muon spin tracking

The muon spin in the muon rest system undergoes Thomas precession when a muon travels through magnetic and electric fields. The spin direction change is expressed by the Thomas-BMT equation:

$$
\frac{d\vec{s}}{dt} = \frac{e}{m} \vec{s} \times \left[ (a + \frac{1}{\gamma}) \vec{B} - a \frac{\gamma}{\gamma + 1} (\vec{\beta} \cdot \vec{B}) \vec{\beta} - (a + \frac{1}{\gamma + 1}) \vec{\beta} \times \vec{E} \right]
$$

(1)

where $a \equiv \frac{g}{2} - 1$ is the muon anomalous magnetic moment and is $1.165 \times 10^{-3}$.

The change of the muon helicity is given by:

$$
\frac{dh}{dt} = -\frac{e}{m} \vec{s}_\perp \cdot \left[ a \vec{\beta} \times \vec{B} + ((a + 1)\beta - \frac{1}{\beta}) \vec{E} \right]
$$

(2)

### Figure 4: Muon helicity vs. $c \Delta t$ at the end of the cooling channel

Polarized muons has non-zero probability of spin flipping in traversing through absorbers, liquid $H_2$ for example, in the processes of energy loss and the multiple scattering. But for the muons with a typical momentum of 200 MeV/c, the spin flip probability of muons is less than $\sim 5 \times 10^{-3}$, which is simulated in ICOOL. [11]
3 CONCLUSION

The muon polarization in the storage ring of the neutrino factory affects the production yield of muon neutrinos and electron neutrinos at the far detector site. We simulated the muon polarization in the front end channel of the neutrino factory by generating polarized muons in pion decay and tracking the muon spin through electromagnetic fields, and absorbers.

In the front end model of the neutrino factory, $\mu^+$s at the end of the ionization cooling channel have the average effective helicity of $0.175 \pm 0.004$ and the muon helicity correlation to the muon arrival time is almost linear with a slope of the muon helicity of $0.50 \pm 0.04$ per 100 m in $c\Delta t$.

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