SIMULATION INVESTIGATIONS OF THE LONGITUDINAL SAWTOOTH INSTABILITY AT SURF*

K. Harkay †, K.-J. Kim, and N. Sereno, Advanced Photon Source, Argonne National Laboratory, Argonne, IL

U. Arp and T. Lucatorto, National Institute of Standards and Technology, Gaithersburg, MD

Abstract

Strong evidence of self-excited emission of coherent synchrotron radiation in the microwave spectral region was observed at the Synchrotron Ultraviolet Radiation Facility (SURF III), the electron storage ring at the National Institute of Standards and Technology (NIST). The microwave emission — between 25 mm and 35 mm wavelength (about 1/10 the rms bunch length) — was dominated by intense bursts of radiation. The bursts occur only when the beam is unstable; namely, during bunch length relaxation oscillations. Furthermore, the shape, width, and period of these bursts depend strongly on the operational parameters of the storage ring. In this study, simulations were performed to address the main features of the longitudinal instability and to explain the detailed dynamics in the phase space during the bunch blowup phase. We show that, driven by the rf cavity higher-order modes (HOMs), the beam shows development of high-frequency features in the nonlinear time evolution of the bunch mode oscillations. These features suggest the possibility of spontaneous, self-induced microbunching of the electrons in the bunch, leading to the observed coherent enhancement of the emitted radiation.

1 INTRODUCTION

SURF III is a weak-focusing ring with a radius of 0.8382 m in which electrons with energies from 10 to 380 MeV can be stored. The rf voltage (V_{rf}) ranges from 10–20 kV, and the rf frequency (f_{rf}) is 113.846 MHz with a harmonic number of two. Typical injection currents are 350–650 mA. Excitation of vertical betatron oscillations increases the beam size and, thus, improves the beam lifetime, which is dominated by Touschek scattering.

A simulation investigation was undertaken in an effort to understand the mechanism driving the long-observed sawtooth instability. The period of the oscillations is observed to coincide with the period of microwave bursts, strongly suggesting a correlation [1]. To determine whether nonlinear development of high-frequency structure could account for the coherent radiation emission, the simulation results were post-processed to analyze the evolution and spectral content of the bunch distribution.

2 EXPERIMENTAL RESULTS

The measured temporal and spectral features of the relaxation oscillations of the sawtooth instability are shown in Figs. 1–3. A capacitive beam monitor electrode (BME) signal (Fig. 1) was spectrally analyzed to give the rms bunch length (σ_t) and growth of synchrotron sidebands (SB) around the rf harmonics (Fig. 2). The microwave emission was measured using a broad-band detector in the 7–12 GHz band [1]. Figure 3 shows that microwave bursts occur at the onset of coherent bunch mode oscillations, just after maximum bunch compression, and as the oscillations saturate when the bunch length blows up.

Figure 1: Sawtooth instability signature: 256 MeV, 113 mA, V_{rf} 10 kV, φ_{rf} 54.2° (BME signal envelope ≈1/σ_t).

Figure 2: Time evolution of σ_t and quadrupole synchrotron sideband (2f_s) for conditions in Fig. 1.

Figure 3: Correlation of BME, microwave, and 2f_s signals.

† harkay@aps.anl.gov
At SURF, high shunt impedance ($R_L$) and quality factors ($Q_L$) measured for the HOMs make the rf cavity the most likely source of coupling impedance to induce the instability. In particular, the $R_L$ of $f_1$ (HOM near $3f_{\text{rf}}$) is sufficient to give the instability growth rates observed [2]. In addition, the sawtooth instability and coherent microwave emission occur only at certain phase angles ($\phi_d$) between the rf generator and the rf cavity. Adjusting the phase alters the matching condition between the rf source and the cavity; this results in detuning the cavity for unmatched phase angles. Frequency shifts of up to 20 and 80 kHz were measured for the fundamental and $f_1$, respectively. The SURF cavity is a quarter-wave structure with HOMs only near and slightly above the odd harmonics of the fundamental (see Table 1). This detuning satisfies the condition for Robinson instability at these higher harmonics; namely, that the impedance is greater at the upper synchrotron sidebands than at the lower sidebands of the rotation harmonics. Because the HOM impedances peak only near the $f_1$ harmonics, only the in-phase modes can be excited, i.e., the two bunches oscillate in phase.

Table 1: Bench measurements of the SURF rf cavity fundamental and HOM parameters [2]

<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>$Q_L$</th>
<th>$R_L$ (k$\Omega$)</th>
<th>Detuning (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>113.8</td>
<td>680</td>
<td>12</td>
<td>–</td>
</tr>
<tr>
<td>342.0</td>
<td>540</td>
<td>3.1</td>
<td>0.97</td>
</tr>
<tr>
<td>571.8</td>
<td>190</td>
<td>0.7</td>
<td>1.0</td>
</tr>
<tr>
<td>797.5</td>
<td>230</td>
<td>0.6</td>
<td>0.32</td>
</tr>
<tr>
<td>1027</td>
<td>143</td>
<td>0.3</td>
<td>0.59</td>
</tr>
<tr>
<td>1257</td>
<td>193</td>
<td>0.4</td>
<td>0.97</td>
</tr>
</tbody>
</table>

3 SIMULATION

In an attempt to understand the experimental results at SURF, simulations were performed using the *elegant* program [3] to address the main features of the longitudinal relaxation oscillations and explain the detailed dynamics of the phase space during the bunch blowup phase. We show that HOMs indeed give rise to both high-frequency structure in the bunch and relaxation oscillations of the bunch length through the nonlinear evolution of the distribution. The vacuum chamber for SURF is large compared to the beam dimensions; thus, the broadband impedance is not considered to be important.

In the simulation, the input resonator fundamental and HOM impedance parameters were obtained from Table 1. The beam current used in the simulation was 97 mA distributed in two bunches, each containing 0.85 nC. Each bunch was represented by $10^4$ macroparticles, and tracking proceeded for up to $3 \times 10^6$ turns for various beam energies. This was enough time to simulate a few damping periods. A particle bin size ($N$) of 50 ps was chosen to resolve features up to 10 GHz.

The results for 256-MeV beam energy and 10.8-kV rf voltage illustrate how the simulation reproduced many features of the observations. The phase space evolution of particle momentum vs. time showed not only the overall bunch length relaxation oscillations, but also the multiple, smaller blowups of synchrotron oscillations. In the initial blowup, a large part of the core of the phase space starts to execute synchrotron oscillations, which grow to large amplitude. After the initial blowup has filamented in the non-linear rf potential, the core left behind blows up in a similar fashion. The filaments of higher-density, small-scale structure persist for many synchrotron periods. Finally, the modulation in the distribution completely decoheres in phase space after both blowup stages. Radiation damping reduces the bunch length and the cycle repeats on time scales close to those observed.

The evolution of the bunches is shown in Fig. 4. The dynamics of the two bunches are almost identical, as expected for the in-phase mode; for clarity, only one bunch is shown. The relaxation period of the rms bunch length (top) is very similar to that shown in Fig. 2, although the blowup is only about 30% compared to measured values of about a factor of two. Dipole synchrotron oscillations occurring during the blowup are evident in the average centroid offset, $<t>$, also shown in the figure (middle). The first and second synchrotron moments of the distribution were analyzed for the first blowup (bottom of Fig. 4); the dipole mode is dominant.

Inclusion of all the HOMs appears to be necessary to account for the details of the multiple bursts. For example,
simulations with only $f_3$ resulted in a bunch length blowup of only half that observed with all HOMs. Simulations in which the HOM frequency was shifted to fall exactly on the upper quadrupole sideband ($f_3+2f_s$) were the most unstable. One bunch with half the current (55 mA) gave similar results to those obtained with two bunches, but the results did not match the observations as well (e.g., relaxation period and relaxed bunch length). The better agreement with two bunches suggests that weak bunch-coupling may play a role in the observed bunch dynamics.

**Figure 5: Bunch distributions in *elegant* simulations.**

**Figure 6: Spectra of 1st and 3rd bunch distributions in Fig. 5.**

### 4 DISCUSSION

Rakowsky [5] first suggested that the sawtooth instability at SURF could be attributed to a Robinson instability. The sawtooth instability has been observed in a number of other electron rings, most notably in the Stanford Linear Collider damping rings and the Stanford electron accelerating ring, SPEAR. At SPEAR, a narrowband rf cavity tuned near the fundamental was shown to give rise to relaxation oscillations caused by the Robinson instability [6]. Robinson-like instabilities can occur at the HOMs as well; in particular, when these parasitic modes are near the harmonics of the fundamental, as at SURF.

An interesting analogy can be drawn between the sawtooth instability at SURF and instabilities at electron storage rings with higher-harmonic rf cavities. Both the Advanced Light Source (ALS) and Aladdin rings have 3rd harmonic cavities installed for bunch lengthening. In passive operation, what are believed to be nonlinear Robinson instabilities are observed at high current [7][8]. At SURF, the beam-induced voltage in the rf cavity HOMs $f_3$ appears to produce beam dynamics similar to the passive bunch lengthening cavities. Depending on machine conditions and the tuning angle of the 3rd harmonic at Aladdin, sometimes the dipole and sometimes the quadrupole synchrotron mode is dominant [8]. In the SURF simulations, the rf phase $(\phi_{rf})$ was not modeled, which may account for that fact that the dipole mode dominated rather than the quadrupole mode seen in the data for the same conditions. The simulated beam-induced voltages at $f_3$ and $f_{11}$ (multiplied by 10 for clarity) are shown in Fig. 7.

**Figure 7: Induced HOM voltages at 342 and 1257 MHz.**

### 5 CONCLUSIONS

Simulations of the SURF sawtooth instability driven by rf cavity HOMs show the development of high-frequency features in the nonlinear time evolution of bunch mode oscillations. This finding suggests the possibility of wakefield-induced microbunching during the turbulent bunch length oscillations during which enhanced microwave emission is observed. Similarities are noted between the instability at SURF and nonlinear Robinson instabilities observed at ALS and Aladdin with passive harmonic cavities. Additional modeling is planned to help address remaining discrepancies in comparisons with the SURF data (e.g., including the rf phase, modeling impedances beyond cutoff, and adding the CSR wakefield computation to *elegant*). It should be noted that, under normal operating conditions (no detuning of rf phase), SURF III is a stable synchrotron radiation source.

### 6 ACKNOWLEDGMENTS

The authors are very grateful for discussions with E. Lessner, A. Nassiri, J. Byrd, and R. Bosch.

### 7 REFERENCES