Abstract
For CESR operation around ψ, Charm and Tau thresholds, the superconducting wigglers with 2.1T peak field of ~20 m total length will be installed around the ring, allowing necessary damping. We describe here optimization process for the wiggler magnet parameters to meet the field value and quality required.

1 INTRODUCTION
Development of this subject was initiated by proposal considered to modernize the CESR for the low energy operation [1]. Mostly results and procedures developed can be applied to any wiggler however. It is known [2], that a passage with an angle through the edge field of a multipole, acting to the particle as the next order multipole. This can be expressed as the following

\[ \int_{\text{outside}} G_m(s)ds = \frac{\ln a}{m} G_{m-1}(in). \]  

(1)

Here \( G_{m-1}(in) \) is the value of original multipole at the center, and \( G_m(s) \) is treated as effective next order multipole as a function of longitudinal distance \( s \). \( \alpha \) is a crossing angle. Dipole corresponds to \( m=1, G_0 = B \), quadrupole to \( m=2 \ldots \) Mostly used edge focusing in a dipole field is a lowest among this kind of relation. In a wiggler the effective angle is significantly changes during the passage trough the edge, Fig.1. Each wiggler with sinusoidal dependence of the field on longitudinal coordinate \( s \), \( B(s) = B_0 \sin(s/\lambda) \), where \( \lambda = 2\pi \) \( \lambda \) is a wiggler period, \( B_0 \) stands for the field amplitude, can be characterized by the dimensionless wiggler parameter \( K \), what is \( K = eB_0\lambda/mc^2 \approx 93.4xB_0[T\times \lambda[m]. \)

![Figure 1: The top view onto the pole of the wiggler. The y-axis is looking up from the plane of the drawing. Dashed line describes the trajectory in a low bend magnet, say quadrupole. The particle’s trajectory in infinite sin-like field with amplitude \( B_0 \) could be described as \( x = X + x_0 \cos(s/\lambda), \alpha = \alpha_0 + \alpha_0 \sin(s/\lambda), \) where \( x_0 = \lambda K/\gamma, \alpha_0 = K/\gamma, c \) is a speed of light, \( \beta_{L0} \equiv K/\gamma, X \) is a constant which depends on details of the entrance in the wiggler field such as tapering. For example, if the wiggler tapered with \(+\frac{1}{2}, -\frac{1}{2}, -1, +1,..., \) the constant \( X \equiv 0 \) [4]. The physical nature of appearance of octupole (and higher) type dependence of kick versus displacement is in wigging with angle in sextupole (and higher) field. This sextupole generated intrinsically by the field dependence on longitudinal coordinate (along the wiggler). When the pole is wide, the sextupole has strict value defined by second derivative of the wiggler field along the longitudinal coordinate \( s \).

2 FIELD IN A WIGGLER
Static magnetic field existing in a wiggler can be represented as the following [3] \( S \) and \( D \) stand for sextupole and decapole respectively

\[ B_x(x,y,s) = -\frac{xy}{4} B^{(s)}(s)+\frac{x^3+y^3}{48} B^{(iv)}(s) - \frac{1}{12} S^{(s)}(s) + \frac{1}{32-10} y^x + \frac{3}{4} z^y + \frac{7}{6} y^6 D^{(s)}(s) + \cdots \]

\[ B_y = \frac{1}{8} \left[ B^x - \frac{y^2}{4} \right] + \frac{x^3+y^3}{48} B^{(iv)}(s) - \frac{1}{12} S^{(s)}(s) + \frac{1}{32-6} y^x + \frac{17}{3} x^2 y^4 - 9 y^6 S^{(iv)}(s) - \cdots \]

\[ D_x(x,y,s) = -\frac{1}{4} \left( \frac{10x^2+y^2}{32} y^3 + 9 y^6 D^{(s)}(s) + \cdots \right) \]

\[ D_y = \frac{1}{8} \frac{1}{16} \left[ \frac{5}{32} x^3 + \frac{1}{4} y^3 + \frac{3}{2} y^6 D^{(s)}(s) + \cdots \right] \]

For the wiggler with infinitely wide poles in \( x \) (transverse) direction, there must be no dependence on \( x \), however. This yields the following relations between multipoles

\[ S(s) = \frac{1}{16} \left[ \frac{5}{32} x^3 + \frac{1}{4} y^3 + \frac{3}{2} y^6 D^{(s)}(s) + \cdots \right] \]

Under these conditions all components of magnetic field from different multipoles along transverse axis \( x \) equates to zero, \( B_x(s)=0 \). If we suggest in addition, that the dipole field have sinusoidal dependence on longitudinal coordinate \( B(s) = B_0 \sin(s/\lambda) \), the multipoles become

\[ S(s) = -\frac{B_0}{8}\lambda^2 \sin(s/\lambda), \quad D(s) = \frac{B_0}{384} \lambda^4 \sin(s/\lambda), \cdots \]

Substitute these values into expression for the fields, one can find, that the field dependence becomes
\[ B_{y}(y,s) = 0, \quad B_{y}(y = 0, s) = \sum B_{n}^{y} \cdot \sin(s/\lambda), \]
\[ B_{y}(y, s) \equiv (y/\lambda) \cdot B_{0} \cdot \cos(s/\lambda) - \frac{s}{\lambda^2} \cdot (y/\lambda) \cdot \sin(s/\lambda) - \frac{s^2}{\lambda^3} \cdot (y/\lambda)^3 \cdot B_{0} \cdot \cos(s/\lambda) + \cdots. \]

First term here responsible for linear, the second one for octupole and the last one for twelve pole type focusing along vertical direction. So one can see, that cancellation of the field dependence along one coordinate, \( x \), makes dependence along other one, \( y \), more stiff. Vertical force acting to the particle with instant position \( (x, y, s) \) is proportional to the
\[ F_{y} \equiv e \bar{v} \times \left( - B_{y}(x, y, s) \cdot \sin(\alpha(x, y, s)) + B_{y}(x, y, s) \right), \]
where \( \bar{v} \) is the particle’s speed. Vertical kick experienced by a particle, passed though the multipole magnet off its center with angle \( \alpha \), can be expressed as the following
\[ y' \equiv \frac{1}{pc} \int_{\text{inside}}^{\text{outside}} \frac{F_{y}(y, s)dS}{\cos \alpha} \equiv - \frac{1}{BR} \int_{\text{outside}}^{\text{outside}} B_{y}(y, s) \cdot \tan \alpha \cdot ds, \]
where \( e/pc = BR \) is a magnet rigidity. So sin-like motion brings additional factor \( \frac{1}{2} \) in this formula (and in formula (1)).

### 3 THE OCTUPOLE VALUE

The tune shift generated by octupole \( O(s) \) is [5]
\[ \Delta Q \equiv \frac{1}{2} \beta_{o} \cdot a^{2} \cdot N \cdot \int O(s) \cdot ds / (HR), \quad (2) \]
where \( \beta_{o} \) is an envelope function at the wiggler’s location, \( N \) is the number of wiggler in all machine (\( N = 14 \) in our case). We suggested for simplicity, that all wiggler (octupoles) be displaced around the ring having the same envelope function and phase. So this is majorette estimation. Meanwhile we could see from previous considerations, that
\[ \int_{s_{in}}^{s_{out}} O(s)ds \equiv \frac{5}{2} \cdot \frac{a_{0}B_{o}}{\lambda^2} \cdot (L/\pi\lambda - 1) \cdot N. \quad (3) \]

With additional real octupole installed in the ring one may try to reduce the influence of nonlinear term. Problem here is that wiggler focusing only in vertical direction. The simplest way to reduce the octupole term is to increase the wiggler period \( \lambda \), and/or \( B_{o} \), as the dependence is a quadratic one. With increasing the wiggler period, however, at some moment the term with derivative comes to \( S \equiv \delta \beta B^{2} / ds^2 \rightarrow \delta B_{o} / A^2 \), where \( A \) is a vertical aperture of the wiggler. In this case the octupole integral becomes about two times higher, as the particle passes fringe field with higher angle. From the other hand if the damping time is fixed, its dependence on \( B_{o} \) what gives the factor 2 in extreme case back.

Sinusoidal field dependence and trajectory can be restored either by increasing the vertical gap (with increasing the feeding current) or by changing distribution of the winding, making the coil wider (and flatter).

### 4 PROCEDURE

Formulas represented above give an understanding of analytical dependencies on different parameters. For more realistic optimization the following sequence was used. First, 3D calculations were carried with real dimensions and multi-wired currents with all possible details with the help of numerical code [6].

Figure 2: The wigglers cold mass cross section. Room temperature aperture is \( 9 \times 5 \text{cm}^2 \).

Dense hatch in Fig. 2 related to the soft iron, rare –to Stainless Steel. Iron begins at 1.5in from the median plane. Upper and lower containers for liquid Helium are connected through the side tubing. This design drastically simplifies assembling, excluding, however, any magnet shunts between upper and lower parts inside a cryostat.

During the modeling, the magnet was plunged into a box with appropriate margins. In longitudinal direction the sides of this box coincide with the positions of the cryostat flanges, located at 20-cm distance from the yoke. Magnetic field was investigated everywhere in the box and forces acting to the coils calculated. Optimization of the iron thickness, pole shapes and tapering, shims, field profile above the iron and so on were done here. Energy stored in the field (and inductance) obtained as well. Influence of the stainless steel magnetic permeability was investigated also (Fig. 2), coming to restriction on \( \mu \leq 1.02 \). Possible screening of HTS leads with local iron cylinders was evaluated also.
After the field distribution obtained, the tracking through the box was carried with the help of another numerical code [7]. After post processing, all necessary parameters of the particles represented as a function of transverse phase space location at the entrance of the wiggler.

![Figure 3](image-url) Field in the gap, $kG$, as a function of the iron height, $cm$, measured from median plane on Fig.2.

![Figure 4](image-url) Top view on the wiggler poles.

Half of the wiggler is represented in Fig.4. Regular pole length is 20 cm. The end pole length is 15 cm. Radiiuses chosen for easy winding and to lower the local field strength. The smallest one in the Fig.4 is 2.9 cm. The coil width is 2.54 cm. The end pole contains main coil caring the same current as the central poles and the trimming SC coil with independent power supply. Poles in regular part have lowering of 5 mm deep, allowing alignment the field distribution across the pole, see Fig.5. End poles have the lowering depth of 3.5 mm only. The widths of the lowering are about $\pm 6$ cm. This allows having the integral $I_1 = \int B_z(s) ds$ variation across the aperture $I_1 \leq 0.05kG \cdot cm$, while integral along single pole (from zero to zero) is 318.8 $kG \cdot cm$.

![Figure 5](image-url) Vertical field, $kG$ as a function of transverse coordinate $x$, $cm$ for two different feeding currents and pole shapes—flat and with lowering. Vacuum chamber ends at $\pm 4.5$ cm.

Two upper curves in Fig.5 correspond to the feeding current 100 kA total, flat pole, up, and next lower curve to the modified ones. Two lower curves correspond to feeding current 80 kA. Upper among them for flat pole, lower—for the same modified one.

During modeling the steel 1010 properties were used, however we investigated the magnet field variation with other ones, such as 1006, Steel110 for example.

As one can see from Fig.6, oscillations sweep in $x$ direction area about 3.3 mm wide.

![Figure 6](image-url) Longitudinal $B_z$ profile (left, $kG$), and example of particle’s trajectory starting at $(x, y)=(1,1)$. Scales at the right are in cm.

![Figure 7](image-url) Deviation of vertical position of the particle from initial one as a function of initial vertical position for different horizontal initial $x$–positions.

Parameters of the wiggler are represented in the Table.

<table>
<thead>
<tr>
<th>Height of the cold iron</th>
<th>29 cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pole length</td>
<td>20/15 cm (end pole)</td>
</tr>
<tr>
<td>Number of poles</td>
<td>5+2</td>
</tr>
<tr>
<td>Current</td>
<td>150-195 A</td>
</tr>
<tr>
<td>Pole width</td>
<td>23.8 cm</td>
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<tr>
<td>Field</td>
<td>1.7-2.1T</td>
</tr>
<tr>
<td>Cubic /linear kick ratio</td>
<td>$\delta \leq 8% @2cm$ in $</td>
</tr>
<tr>
<td>Energy stored</td>
<td>47.3 kJ (at 2.2T)</td>
</tr>
</tbody>
</table>

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5 REFERENCES