Comment on “Damping of energetic gluons and quarks in high-temperature QCD”

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Burgess and Marini have recently pointed out that the leading contribution to the damping rate of energetic gluons and quarks in the QCD plasma, given by $\gamma = cg^2 \ln(1/g)T$, can be obtained by simple arguments obviating the need of a fully resummed perturbation theory as developed by Braaten and Pisarski. Their calculation confirmed previous results of Braaten and Pisarski, but contradicted those proposed by Lebedev and Smilga. While agreeing with the general considerations made by Burgess and Marini, I correct their actual calculation of the damping rates, which is based on a wrong expression for the static limit of the resummed gluon propagator. The effect of this, however, turns out to be canceled fortuitously by another mistake, so as to leave all of their conclusions unchanged. I also verify the gauge independence of the results, which in the corrected calculation arises in a less obvious manner.

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It has been established by Braaten and Pisarski [1] that a perturbation theory for the dispersion relations of quasiparticles in high-temperature QCD requires at least the resummation of the leading-order terms, called “hard thermal loops,” whose characteristic scale is given by $gT$, where $g$ is the coupling constant and $T$ the temperature. By now, a number of applications exist [2–5] which employ the resummation techniques developed in Ref. [1] to explore the physics of the hot QCD plasma at the scale $g^2 T$. Complete results can be obtained, if they are not sensitive to a further resummation of the corrections of order $g^2 T^2$, which would have to include the perturbatively incalculable screening of static magnetic fields [6].

Burgess and Marini [7] have recently discussed the case where the resummation of the hard thermal loops leaves logarithmic infrared divergences, and they have made precise the notion [2] that the resummation procedure still allows reliable extraction of terms $g^2 T \ln(m_{el}/m_{magn}) \sim g^2 \ln(1/g)T$, if not those of order $g^2 T^2$.

The particular example considered in Ref. [7] is the evaluation of the leading contributions to the damping rate of gluons or quarks with momenta $|p| \gg gT$. This kinematical region leads to an enormous simplification of the resummation program, because only the leading corrections to one internal propagator carrying soft integration momentum need be resummed, with no complications from the vertices. The similar case of very massive quarks has previously been discussed in Refs. [2] and [5]. Burgess and Marini further noticed that in such processes, which are dominated by the subleading scale $g^2 T$, only the static limit of the resummed gauge propagator is needed.

The calculation thus becomes technically similar to the well-known resummation of “ring diagrams” in thermodynamical potentials, which goes under the name of the “plasmon effect” [8]. However, this term is somewhat misleading, as only the static limit of internal lines with multiple self-energy insertions is relevant, which thus resums the electric Debye screening mass rather than the (different) plasmon mass corresponding to long-wavelength plasma oscillations. The latter is determined by the long-wavelength limit of the gluon self-energy

$$\lim_{q_0 \to 0} \Pi_{\mu\nu}(q_0, q) = m^2 (\eta_{\mu\nu} - \delta_{\mu\nu} \delta^0) + O(gm^2q_0),$$

with $m^2 = \frac{3}{8}(C_a + \frac{1}{2}n_q)(gT)^2$, whereas the static limit is

$$\lim_{q_0 \to 0} \Pi_{\mu\nu}(q_0, q) = m_{el}^2 \delta_{\mu\nu} + O(gm^2q_0),$$

with $m_{el}^2 = 3m^2$ [9]. Evidently, these limits do not commute.

In the calculation carried out in Ref. [7], Eq. (1) was used instead of Eq. (2) for the resummed gluon propagator at zero frequency, which led the authors of Ref. [7] to using

$$\Delta_{\mu\nu}^{\text{wrong}}(q_0=0) = -\frac{1}{q^2} \delta_{\mu\nu} q^0 + \frac{1}{q^2 - m^2} \left( \eta_{\mu\nu} - \delta_{\mu\nu} \delta^0 q_\nu \frac{q_\mu q_\nu}{q^2} \right) + \xi \frac{q_\mu q_\nu}{(q^2 - \xi m^2)q^2}$$

(3)
in place of the correct
\[
\Delta_{\mu\nu}^{*} \bigg|_{q_{0}=0} = -\left[\frac{1}{q^{2} - m_{el}^{2}} \eta_{\mu\nu}^{0} q_{0}^{0} + \frac{1}{q^{2}} \left(\eta_{\mu\nu}^{0} - \partial_{\mu}^{0} \partial_{\nu}^{0} - \frac{q_{\mu} q_{\nu}}{q^{2}}\right)\right] + \xi q_{\mu} q_{\nu} \left(\frac{q^{2}}{q^{2}}\right) \right].
\]
(4)

In the latter only the spatially longitudinal mode is screened, leaving both the spatially transverse mode and the (four-dimensional longitudinal) gauge mode massless.

Recall that the “hard” (|q| > \lambda \gg g^{2}T) contributions to the damping rate \(\gamma\) of energetic (|p| \gg gT) transverse gluons considered in Ref. [7] leads to
\[
\gamma_{\text{hard}} = g^{2}C_{A}T \left[\frac{1}{4\pi^{2}} \right] \int_{-1}^{1} dz \int_{\lambda}^{\infty} \frac{dq}{q} \left[\frac{1}{q^{2} - m_{el}^{2}} - (1 - \xi)(\frac{q^{2}}{q^{2} + \xi m_{el}^{2}})\right] + O(g^{2}T\lambda^{0}),
\]
(5)

where the terms in the large brackets correspond to the contributions of spatially transverse, spatially longitudinal, and gauge modes, respectively. In the case of quarks, it turns out that the only change consists in replacing \(C_{A}\) by \(C_{T}\).

On the other hand, with the wrong propagator of Eq. (3) used in Ref. [7], these terms would read
\[
\left[\frac{1}{q^{2} - m_{el}^{2}} - (1 - \xi)(\frac{q^{2}}{q^{2} + \xi m_{el}^{2}})\right].
\]
(6)

The leading contribution to \(\gamma\) can be extracted from the logarithmic dependence of \(\gamma_{\text{hard}}\) on the cutoff \(\lambda \ll gT\), together with the assumption that the inherent scale of the undetermined soft contribution is given by \(g^{2}T\) (through the nonperturbative magnetic mass or through dynamical screening at this scale). The spatially transverse and spatially longitudinal contributions in Eq. (5) thus lead to
\[
\gamma \approx g^{2}C_{A}T \frac{\ln m_{el}}{\lambda} + \ln \frac{\lambda}{g^{2}T} = g^{2}C_{A}T \frac{\ln m_{el}}{\lambda} + O(g^{2}T),
\]
(7)

with the transverse mode being responsible for the dominant term proportional to \(\ln(g^{2}T)\), and therefore for the positive sign of \(\gamma\). The latter is a consequence of the positivity of the transverse density in a spectral representation of the resummed gluon propagator [2].

The wrong result of Eq. (6), on the other hand, should have led to a result of equal magnitude, but with a reversed sign, as the roles of spatially longitudinal and transverse modes happen to be interchanged. [The difference between \(m_{el} = \sqrt{3}m_{el}^{0}\) only affects the terms of \(O(g^{2})\).] The fact that in Ref. [7] also a positive result was reported is due to the additional mistake of a reversed sign of \(\xi\) in their Eq. (11) compared with Eq. (5) above. With the usual sign convention \(\gamma = -\text{Im} E_{\text{pole}}\), the correct analytical continuation is given by \(k_{0} \rightarrow -k_{0} + \text{i}\xi\).

A more conspicuous difference between the correct and the wrong results, Eq. (5) and Eq. (6), respectively, concerns the contributions from the gauge modes. With the wrong expression for the static gluon propagator, Eq. (3), the gauge modes obviously would not contribute to the infrared singular part, whereas in the corrected result, Eq. (5), they seem to do so by superficial power counting. However, performing the angular integration
\[
\int_{-1}^{1} dz = \frac{z^{2}}{z + q/|p| - \text{i}\epsilon} = O\left(\frac{q}{|p|}\right)
\]
(8)

reveals that they indeed do not contribute to the leading logarithms in Eq. (7), as expected from general arguments [10] for the gauge independence of dispersion relations in finite-temperature QCD.

Thus, all the results on \(\gamma\) presented in Ref. [7], its magnitude, its sign, and its gauge independence, remain, somewhat fortuitously, unchanged, and continue to confirm the results by Braaten and Pisarski [11], while contradicting those proposed by Lebedev and Smilga [12].

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