Note on Triangle Anomalies and Assignment of Singlet in 331-like Model

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Abstract

It is pointed out that in the 331−like model which uses both fundamental and complex conjugate representations for an assignment of the representations to the left-handed quarks and the scalar representation to their corresponding right-handed counterparts, the nature of the scalar should be taken into account in order to make the fermion triangle anomalies in the theory anomaly-free, i.e. renormalizable in a sense with no anomalies, even after the spontaneous symmetry breaking.

In this note, the 331 model[1, 2, 3], in which the fundamental representation for the left-handed quarks in some families and complex conjugate representation for those in the other families are assigned together with the singlet representation for the corresponding right-handed counterparts, is discussed as an example though our discussion can be applied to any such model and the primary model[4] assigning only the fundamental representation to the left-handed fields is not treated here. The original Lagrangian of the 331 model is constructed to be anomaly-free, in the sense that total charge on all particles over three families vanishes, as well as gauge-invariant and thus it is expected that the results will be renormalizable. However, the renormalizability of the theory should hold not only for the original Lagrangian[5] but also for the Lagrangian written in terms of the mass eigenstate (physical) bases after the spontaneous symmetry breaking[6, 7]. If the anomaly coefficients for all possible triangle diagrams after the symmetry breaking can not be expressed in terms of the trace of the product of the representation matrices corresponding to those before the symmetry breaking, the anomalies associated with the triangle diagrams will remain without vanishing and then the renormalizability of the theory will be lost even though the starting Lagrangian is not so[6]. In what follows, it is shown that the result of the theory depends through the Yukawa interactions on whether the nature of the singlet is taken into account or not[8].

The 331 model[1, 2, 3] is studied by many authors since Pisano and Pleitez and many discussions on the lepton sector approach are made phenomenologically from various points of view[9, 10]. We have an interest in the anomalies which are singularities associated with the fermion triangle contributions to the vertex of three currents arisen from the fermion covariant kinetic energy terms and have no relation to the mass of the fermions. Thus, the following assignment of the representations to the basic particles in the three families is adopted in the case of the SU(3)_C ⊗ SU(3)_L ⊗ U(1)_N model as follows[2]

\[
\begin{align*}
\ell^0_{aL} &= \begin{pmatrix} \nu^0_a \\ e^0_a \\ E^0_a \end{pmatrix}_L \sim (1, 3, 0), \\
\nu^0_{aR} &\sim (1, 1, 0), \ e^0_{aR} \sim (1, 1, -1), \ E^0_{aR} \sim (1, 1, 1), \\

Q^0_{iL} &= \begin{pmatrix} u^0_i \\ d^0_i \\ J^0_i \end{pmatrix}_L \sim (3, 3, -1/3), \\
u^0_{iR} &\sim (3, \text{ singlet}, 2/3), \ d^0_{iR} \sim (3, \text{ singlet}, -1/3), \ J^0_{iR} \sim (3, \text{ singlet}, -4/3), \\
Q^0_{3L} &= \begin{pmatrix} u^0_3 \\ d^0_3 \\ J^0_3 \end{pmatrix}_L \sim (3, 3, 2/3), \\
u^0_{3R} &\sim (3, \text{ singlet}, 2/3), \ d^0_{3R} \sim (3, \text{ singlet}, -1/3), \ J^0_{3R} \sim (3, \text{ singlet}, 5/3),
\end{align*}
\]
where the suffices \( a (= 1, 2, 3) \) and \( i (= 1, 2) \) denote the family numbers.

And the word “singlet” for the \((u_0^i, d_0^i, J_0^i)_R\) and \((u_3^0, d_3^0, J_3^0)_R\) quarks are used instead of usual “1”, though “1” is used in the leptons without any subscript. This is because in the case of leptons only the fundamental representation is assigned to the left-handed fields and thus it is obvious that the singlet of the right-handed field accompanied with the left-handed one is a scalar with respect to the transformation of the fundamental representation. On the other hand, the fundamental and complex conjugate representations are used for the assignment of the left-handed quarks and then there are some possibilities about the singletness (scalar) in physics but not mathematics, i.e., bosonic, fermionic and antifermionic singlets such as the colorless singlets in the quark model. As is well known, the singlet appears in the representation theory of SU(3) as follows

\[
\begin{align*}
3 \otimes \bar{3} &= 1 \oplus 8, \quad \text{(i)} \\
3 \otimes 3 \otimes 3 &= 1 \oplus 8 \oplus 8 \oplus 10, \quad \text{(ii)} \\
\bar{3} \otimes \bar{3} \otimes \bar{3} &= 1 \oplus 8 \oplus 8 \oplus 10, \quad \text{(iii)}
\end{align*}
\]

The singlets “1” on the right side are equivalent mathematically or from the point view of the transformation of \( SU(3) \) with each other due to the relation \( 3 \otimes 3 = \bar{3} + 6 \) but physically should be considered nonequivalent because their configurations are different from each other in the form as it stands and if only the fundamental representation \( 3 \) is assigned to the basic particles such as in the color quark model, the “1” in (i) \( \sim \) (iii) will represent different scalars physically [7, 8], e.g., will be called “bosonic” singlet (“1”, meson) for (i), “fermionic” singlet (“1\( _3 \)”, baryon) for (ii) and “antifermionic” singlet (“1\( _\bar{3} \)”, antibaryon) for (iii). If the configurations for these scalars as it is are adopted, it may be considered that the bosonic singlet in (i) is the scalar with respect to the transformation of 3 and/or \( \bar{3} \), the fermionic singlet in (ii) is the scalar with respect to the transformation of only 3 but not \( \bar{3} \), and the antifermionic singlet in (iii) is the scalar with respect to the transformation of only \( \bar{3} \) but not 3. There is no problem when only the fundamental (or complex conjugate) representation is assigned to the three left-handed quarks as in the case of the leptons [4, 7]. The problem is that though the “singlet” in (1) invariant under the transformation of \( SU(3)_L \) should not be considered as the composite as in (2) and the assignment of the singlet will be free a priori, the singlets accompanied with the 3 and \( \bar{3} \) representations may be considered unique or non-unique. Usually, only the bosonic singlet “1” is adopted as far as we know [1, 2, 3]. It is important for us to distinguish the possibilities because the assignment 1\( _3 \) to the right-handed quarks accompanied with the 3 representation and 1\( _\bar{3} \) to those with the \( \bar{3} \) representation bring about the reasonable results than the assignment “1” of the bosonic singlet to all right-handed quarks as shown below and an adoption of such a transformation for the right-handed singlet counterparts seems reasonable from a physical point of view in the chiral theory with left-handed fundamental and complex conjugate representations.

The three scalar fields, \( \chi, \rho \) and \( \eta \), are introduced to break the symmetry spontaneously and then give the mass to the fields as follows [2]

\[
\begin{align*}
\chi &= \begin{pmatrix} \chi^- \\ \chi^- \\ \chi^0 \end{pmatrix} \sim (1, 3, -1), \\
\rho &= \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^{++} \end{pmatrix} \sim (1, 3, 1), \\
\eta &= \begin{pmatrix} \eta^- \\ \eta^- \\ \eta^+ \end{pmatrix} \sim (1, 3, 0).
\end{align*}
\]

The charge operator \( Q \) is defined by \( Q = T_3 - \sqrt{3} T_8 + N \) with the generators \( T_3, T_8 \) of \( SU(3)_L \) together with \( T_i \) and \( N \) of \( U(1)_N \). In what follows, the color symmetry is omitted because it has no direct relation to our discussion. The invariant Lagrangians for the fermions and the Yukawa interactions are given by
$$L_f = \bar{r}_a l_a i \gamma^\mu \eta a + (\bar{r}_a, e_a, E_a) R i \gamma^\mu (\nu^a_0 e^a_0 E^a_0) + \bar{Q}_a L e_{aR} + (\bar{u}_a, \bar{d}_a, f^a_0) R i \gamma^\mu (\nu^a_0 d^a_0 f^a_0)$$,

$$L_Y = (\bar{r}_1, \bar{r}_2, \bar{r}_3)_L \mu^\chi$$

$$+ (\bar{Q}_1, \bar{Q}_2)_L \left( \left( \begin{array}{c}
\Gamma^\chi_1 \\
\Gamma^\chi_2 \\
\Gamma^\chi_3
\end{array} \right) \left( \begin{array}{c}
J^\chi_1 \\
J^\chi_2 \\
J^\chi_3
\end{array} \right) \right) \chi^* + h.c.,$$

$$L_Y^\rho = (\bar{r}_1, \bar{r}_2, \bar{r}_3)_L \mu^\rho,$n + \bar{Q}_3 L \left( \left( \begin{array}{c}
\rho^0 \\
\rho^1 \\
\rho^2
\end{array} \right) \right) \rho^* + h.c.,$$

$$L_Y^\eta = (\bar{r}_1, \bar{r}_2, \bar{r}_3)_L \mu^\eta,$n + \bar{Q}_3 L \left( \left( \begin{array}{c}
\eta^0 \\
\eta^1 \\
\eta^2
\end{array} \right) \right) \eta^* + h.c.,$$

$$D^\mu Q^\mu_i = (\partial^\mu + i g_2 \chi^T \cdot A^\mu + \frac{1}{3} g_N B^\mu) Q^\mu_i,$$
\[ e = \frac{99N}{\sqrt{g^2 + 4g_N^2}} = \frac{99y}{\sqrt{g^2 + y_Y^2}}, \quad t_W \equiv \tan \theta_W = \frac{9N}{\sqrt{g^2 + 3g_N^2}}. \]

The gauge bosons can be expressed in terms of the original \( A_\mu \)'s, \( B_\mu \) but their explicit forms are omitted here because it is easily known and trivial[1, 2, 3]. It is noted that as in the case of the standard model (SM)[11] all the right-handed fields are absorbed into the \( J_{em}^d \) current together with the corresponding left-handed fields and only some left-handed fields remain in the expressions of the currents except for the \( J_{em}^u \) current, and the current parts of the left-handed fields are given in the form sandwiching the representation or complex conjugate representation matrices between the two left-handed fields given in (1). The representation matrices of 3 appear in the left-handed leptons and the quarks of the third family due to an assignment 3 to these fields in (1), while the assignment of \( \bar{3} \) to the left-handed fields in the first and second families in (1) leads to an introduction of the complex conjugate representation matrices, i.e., transposed matrices with a minus sign. Thus, it will be necessary that the expressions for the current are given in the forms similar to the above ones in order for the anomaly coefficients even after the symmetry breaking to disappear, otherwise the anomaly coefficients can not be expressed in the form with the trace of the product of the representation matrices [6]. It is, thus, expected that in order for the theory to be anomaly free and thus renormalizable the mass eigenstates (physical fields) after symmetry breaking be expressed in terms of a linear combination of the original fields in three families for the leptons, while in the case of the quarks those in the first and second families be given in terms of the original fields of the two families and those in the third family agree with the original fields corresponding to each. We will show that the anomaly coefficients for the possible fermion triangle diagrams can not be expressed in the form of the trace of only the product of the representation matrices in the case of the adoption of the bosonic singlet “1” for the right-handed quarks as usually assigned[1, 2, 3] and in the case of “1" and “1\( \bar{3} \)” the anomalies do not appear producing the renormalizable results in a sense with no anomaly[8].

The masses for the leptons are obtained from the Yukawa interactions in (4) in a similar way as in SM, and then the currents in (5) may be given by replacing the weak interaction bases \( l_{0L} \rightarrow \) the bases \( l_{4L} \) corresponding to the mass eigenstates. Thus, the discussion on the currents is the same as in SM and thus is omitted here [1, 2, 3]. Similarly, the quark masses are given from the Yukawa interactions in (4) by the vacuum expectation value (VEV) of \( \chi, \rho, \) and \( \eta \).

The VEV of \( \langle \chi \rangle = (0,0,\chi_v)^T/\sqrt{2} \) gives the mass to the \( J \)'s quarks as follows

\[ m_{J_3}J_3 + J^{M}J, \quad J_3 + J^{M_3} \]

where \( m_{J_3}(= \chi_v \Gamma_3/\sqrt{2}) \) denotes the mass of \( J_3(= J^0_3) \), and

\[ J_{L,R}^0 = A_{L,R}^0 J_{L,R}, \quad A_{L,R}^0 M^X A_{L,R}^0 = M_J = \begin{pmatrix} m_J & 0 \\ 0 & m_{J_2} \end{pmatrix}, \]

\[ J_{L,R}^0 = \begin{pmatrix} J_{L,R}^0 \\ \chi_v \end{pmatrix}, \quad M^X = \frac{1}{\sqrt{2}} \chi_v \begin{pmatrix} \Gamma_{11}^X & \Gamma_{12}^Y \\ \Gamma_{21}^1 & \Gamma_{22}^Y \end{pmatrix}, \]

with the \( 2 \times 2 \) unitary matrices \( A_{L,R}^0 \). It is noted that the mass eigenstate of the \( J_3 \) quark in the third family is the same as the weak interaction basis \( J_3^0 \) but those \( J_1, J_2 \) in the first and second families are given by a linear combination of the weak interaction bases \( J^0 \)'s in the families independently of an interpretation of the singlets. The result is due to the fact that the Yukawa interactions of the quarks with the \( \chi \) are common in the cases of “1” and “1\( \bar{3} \)”.

The VEV of \( \langle \rho \rangle = (0, \rho_v, 0)^T/\sqrt{2} \), together with that of \( \langle \eta \rangle = (\eta_v, 0, 0)^T/\sqrt{2} \), in (4) gives the masses to the \( u \) and \( d \) quarks in terms of the linear combinations of the quarks in the three families in order to forbid unphysical processes such as \( t \leftrightarrow (u,c) \), \( b \leftrightarrow (d,s) \) without interactions (or through interaction with vacuum) and then the mass terms of the \( u \) and \( d \) quarks are given from the Yukawa interactions in terms of a linear combination of the corresponding interaction bases in three families as follows

\[ U_L M_u U_R + D_L M_d D_R + h.c., \]

(7)
It is noted that the mass eigenstates, expressed in terms of product of the representation matrices.,
representations matrices after the symmetry breaking.
coefficients after the symmetry breaking to have expressions corresponding to those in terms of the
The result is desirable from a physical point of view as mentioned before and is necessary for the anomaly
the following relations between the interaction bases and the mass eigenstate ones hold
\[ J \]
\[ \text{combination of the corresponding interaction bases in three families in contrast with the case of } J' \text{'s and} \]
It is evident from the expressions that the relations corresponding to (8) for the mass eigenstates split
\[ \text{where} \]
\[ U_L^0, R = \begin{pmatrix} u^0_L & u^0_R \\ u^0_2 & u^0_3 \end{pmatrix}, \quad D_L^0 = \begin{pmatrix} d^0_1 \\ d^0_2 \\ d^0_3 \end{pmatrix}, \]
\[ U_L^0, R = B_L^u U_L, \quad D_L^0 = B_L^d D_L, \quad \]
\[ \frac{1}{\sqrt{2}} B_L^{u\dagger} (I_2 \Gamma_\rho^0 \rho_0 + I_1 \Gamma_\eta^0 \eta_0) B_R = M_u = \begin{pmatrix} m_{u_1} & 0 & 0 \\ 0 & m_{u_2} & 0 \\ 0 & 0 & m_{u_3} \end{pmatrix}, \]
\[ \frac{1}{\sqrt{2}} B_L^{d\dagger} (I_1 \Gamma_\rho^0 \rho_0 + I_2 \Gamma_\eta^0 \eta_0) B_R = M_d = \begin{pmatrix} m_{d_1} & 0 & 0 \\ 0 & m_{d_2} & 0 \\ 0 & 0 & m_{d_3} \end{pmatrix}, \]
\[ B_L^{u(d)\dagger} B_L^{u(d)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad I_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad I_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \]
It is noted that the mass eigenstates, \( U_L, R \) and \( D_L, R \), of the \( u \) and \( d \) quarks are given in terms of a linear combination of the corresponding interaction bases in three families in contrast with the case of \( J' \)’s and the following relations between the interaction bases and the mass eigenstate ones hold
\[ U_L^0, R U_L^0, R = U_L, R U_L, R \text{ and } D_L^0, R D_L^0, R = D_L, R D_L, R. \] (8)
As will be seen, (8) means that the anomaly coefficients after the symmetry breaking can not be expressed in terms of product of the representation matrices.
In the case of an adoption of \( 1_3 \) and \( 1_3 \) for the singlets, it follows that the mass eigenstates for the \( u \) and \( d \) quarks are given as in the case of \( J' \)’s because the mass matrices are given explicitly by putting \( \Gamma^{0, \eta} = \Gamma^{\rho, \eta} = 0 \) (\( i = 1, 2 \)) in the above expressions as follows
\[ \rho_v I_2 \Gamma_\rho^0 + \eta_v I_1 \Gamma_\eta^0 = \rho_v \begin{pmatrix} \Gamma_\rho^{11} & \Gamma_\rho^{12} & 0 \\ \Gamma_\rho^{12} & \Gamma_\rho^{22} & 0 \\ 0 & 0 & 0 \end{pmatrix} + \eta_v \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Gamma_\eta^{33} \end{pmatrix}, \]
\[ \rho_v I_1 \Gamma_\rho^0 + \eta_v I_2 \Gamma_\eta^0 = (\rho \leftrightarrow \eta \text{ in above}). \]
It is evident from the expressions that the relations corresponding to (8) for the mass eigenstates split into two parts, one for a linear combination of the interaction bases in the first and second families and only the other one in the third family as in the case of \( J' \)’s. Explicitly, the followings hold
\[ u^0_L = A_L^{uL} u^0_L, \quad d^0_L = A_L^{dL} d^0_L, \]
\[ \frac{1}{\sqrt{2}} \rho_v A_L^{u\dagger} \begin{pmatrix} \Gamma_\rho^{11} & \Gamma_\rho^{12} & 0 \\ \Gamma_\rho^{12} & \Gamma_\rho^{22} & 0 \\ 0 & 0 & 0 \end{pmatrix} A_R^u = m_u = \text{diagonal}, \quad u_3 = u_3^0, \]
\[ \frac{1}{\sqrt{2}} \eta_v A_L^{d\dagger} \begin{pmatrix} \Gamma_\eta^{11} & \Gamma_\eta^{12} & 0 \\ \Gamma_\eta^{12} & \Gamma_\eta^{22} & 0 \\ 0 & 0 & 0 \end{pmatrix} A_R^d = m_d = \text{diagonal}, \quad d_3 = d_3^0, \]
\[ u^{0\dagger}_L u^0_L = u^0_{L, R} u^0_{L, R}, \quad d^{0\dagger}_L d^0_L = d^0_{L, R} d^0_{L, R}, \]
\[ u^{0\dagger}_{L, R} u^0_{L, R} = u^0_{L, R} u^0_{L, R}, \quad d^{0\dagger}_{L, R} d^0_{L, R} = d^0_{L, R} d^0_{L, R}, \] (9)
The result is desirable from a physical point of view as mentioned before and is necessary for the anomaly coefficients after the symmetry breaking to have expressions corresponding to those in terms of the representations matrices after the symmetry breaking.
The currents in (5) in terms of the above mass eigenstates in (7) are expressed by replacing the weak interaction bases $l_{aL}$ → the bases containing the lepton mass eigenstate $l_{aL}$ for the leptons, while the expressions for the quark parts in the $J^\mu_W$, $J^\mu_Y$, $J^\mu_Y$, $J^\mu_Z$, $J^\mu_Z$, except for the current $J^\mu_{em}$ which is diagonal in the quark flavor and thus is given by the same form as the original one due to (8), can not be obtained only by replacing the weak interaction bases $Q^a_{aL}$ → the mass eigenstate bases consisting of $Q_{aL}$ because the quarks in the first and second families are mixed with the corresponding one in the third family and the relations (8) must hold in contrast with that in the case of $J$'s. That is, in the case of the bosonic singlet ("1") the expressions for the quark currents (except for $J^\mu_{em}$) after the spontaneous symmetry breaking can not be expressed in the forms similar to those before the symmetry breaking though the lepton parts have the similar form in all currents due to the assignment of only the fundamental representation to three families. Thus, it follows that the anomaly coefficients can not be expressed in terms of the trace of the products of the representation matrices for the possible triangle diagrams and then the anomalies will appear bringing about non-renormalizability of the theory in a sense with anomaly. Furthermore, it is noted that the flavor changing neutral current (FCNC)[12] as well as CP(T) violation[13] through the lepton and quark parts appears in this case. It is known that FCNC can be avoided by taking into account the horizontal symmetry in addition to the 331 model[14].

On the other hand, in the case of the adoption of the fermionic ("1") and antifermionic ("1") singlets, all currents for the quark parts in terms of the mass eigenstates are given in the similar form only with replacements by the weak interaction bases → the bases containing the mass eigenstates as in SM and thus it is obvious from the relations (9) in this case that the triangle anomalies for all possible replacements by the weak interaction bases instead of the weak interaction bases because the quarks in the first and second families are mixed with the corresponding one in the third family and the relations (8) must hold in contrast with that in the case of $J$’s. Furthermore, it is noted that the flavor changing neutral current (FCNC)[12] as well as CP(T) violation[13] through the lepton and quark parts appears in this case. It is known that FCNC can be avoided by taking into account the horizontal symmetry in addition to the 331 model[14].

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The other currents as well as the $J^\mu_{em}$ current are expressed in the same way by these bases. It is noted that the $3 \times 3$ unitary matrices $U_{vc}$ and $U_{vE}$ are the Cabibbo-Kobayashi-Maskawa matrices[15] for the leptons and the $2 \times 2$ unitary matrices $U_{dL}$ and $U_{dL}$ denote the Cabibbo matrices[16]. The $J^\mu_{em}$ current may be given in terms of the above bases and the right-handed one or only in terms of the mass eigenstate bases instead of the weak interaction bases because the $J^\mu_{em}$ is diagonal in the flavor. Thus, it is evident that the anomaly coefficients are given by the same form as in the case of the weak interaction bases due to the following relations

$$
\begin{align*}
J^\mu_W &= \bar{l}_{aL}^\gamma\mu(\lambda_1 + i\lambda_2)l_{aL} + \bar{Q}_{aL}^\gamma\mu(\lambda_1 + i\lambda_2)Q_{aL} + \bar{Q}_{aL}^\gamma\mu(\lambda_2 - i\lambda_3)Q_{aL}, \\
J^\mu_Y &= \bar{l}_{aL}^\gamma\mu(\lambda_2 - i\lambda_3)l_{aL} + \bar{Q}_{aL}^\gamma\mu(\lambda_2 - i\lambda_3)Q_{aL} + \bar{Q}_{aL}^\gamma\mu(\lambda_2 - i\lambda_3)Q_{aL} + \bar{Q}_{aL}^\gamma\mu(\lambda_2 - i\lambda_3)Q_{aL} - 2s_W^2 J^\mu_{em}, \\
\end{align*}
$$

where

$$
\begin{align*}
l_{aL} &= \begin{pmatrix} \nu_a \\ e^a \\ \ell_a \\ \ell_a \end{pmatrix} \hspace{1cm} Q_{aL} = \begin{pmatrix} u_3 \\ d_3 \\ J_3 \end{pmatrix} \hspace{1cm} Q_{aL} = \begin{pmatrix} d_L \\ u_L \end{pmatrix}, \\
e^a_L &= U_{ve}e^E_L, \hspace{1cm} E^E_L = U_{vE}E^L_L, \hspace{1cm} u^a_L = U_{ua}u^d_L, \hspace{1cm} J^L = U_{dl}J_L.
\end{align*}
$$

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$$
\begin{align*}
e^a_L e^a_L &= e^a_L e^a_L = \begin{pmatrix} e^0_L e^0_L \\ e^0_L e^0_L \\ e^0_L e^0_L \\ e^0_L e^0_L \end{pmatrix} = E^E_L E^E_L = E^E_L E^E_L, \\
\nu^0_L \nu^0_L &= \nu^0_L \nu^0_L
\end{align*}
$$

and the mass eigenstates of the quarks in the third family with the same bases as in the original weak interaction bases. The mass eigenstates corresponding to the neutral gauge bosons $Z_\mu$ and $Z'_\mu$ are given in terms of a linear combination of $Z_\mu$ and $Z'_\mu$ and thus the anomaly coefficients for the related processes in terms of the mass eigenstates become zero if those in the case of $Z_\mu$ and $Z'_\mu$ is zero. It is noted that the anomaly coefficients are not necessarily expressed in terms of the sum over the whole charges in three families in contrast with those in SM. For instance, for the process $Z' \rightarrow Y^{++}Y^{--}$ the anomaly

$$
\begin{align*}
\end{align*}
$$

6
coefficient becomes essentially
\[
\sum \text{tr} T_{6L} \{ T_{6L} - iT_{7L}, T_{6L} + iT_{7L} \} = \frac{3}{8\sqrt{3}} \left[ \text{tr} \lambda_8 \{ \lambda_6 + i\lambda_7, \lambda_6 - i\lambda_7 \} 
+ \text{tr} \lambda_8 \{ \lambda_6 + i\lambda_7, \lambda_6 - i\lambda_7 \} 
+ 2\text{tr}(-\lambda_8^T) \{ -\lambda_6^T - i\lambda_7^T, -\lambda_6^T + i\lambda_7^T \} \right],
\]
where the first term expresses a contribution from the 3 families of the leptons and the representation matrices \( \lambda_i \) \( (i = 6, 7, 8) \), the second term from the third family of the quark with 3 colors and the representation matrices \( \lambda_i \), and the third term from the first and second families of the quark with each 3 colors and the representation matrices \( -\lambda_i^T \). It, thus, follows that the result becomes zero due to the sum over not all charge on the three families. The FCNC does not appear here and CP(T) violation appears through the lepton parts only.

It may be concluded that in order for the 331 like-model to be renormalizable even after the spontaneous symmetry breaking as well as before that the Yukawa interactions must be given in the form with \( \Gamma_{3i} = \Gamma_{i3} = 0 \) \( (i = 1, 2) \) in (4) which requires the assignment of the singlet to the right-handed quarks accompanied with the left-handed ones should be distinguished whether the right-handed singlet (scalar) is a counterpart of the left-handed 3 or \( \bar{3} \). In this case, the singlet accompanied with the left-handed 3 is only “13” and that with \( \bar{3} \) is “1\bar{3}”, i.e. a scalar under a transformation of the fundamental representation 3 but not under 3 for “13” and vice versa for “1\bar{3}”. Then, the renormalizability of the theory will be guaranteed in a sense of no anomalies even after the gauge fixing and of course FCNC as well as the triangle anomalies does not appear in contrast with use of only the singlet “1” for the right-handed counterparts of the left-handed 3 and \( \bar{3} \).

The above is discussed without fixing the gauge and detailed analysis with \( R_\xi \) gauges will be given somewhere because only papers based on Higgs mechanism [1, 2, 3] seems to be existing as far as we know.

References


[4] For earlier 331 models see:


[9] On neutrino mass see:

[10] On phenomenology see:

   A. Salam, in Elementary Particle Theory, ed. N. Svartholm (1968).


