II. SIGNAL MODULATION

A decomposition approach to LISA data analysis is considered. In a separate treatment of the Doppler signal, the LISA observations for the gravitational wave signal provide a constraint on the signal. The data contain information on the signal and the Doppler effect. The data are decomposed into a combination of the signal and the Doppler effect. The data are decomposed into a combination of the signal and the Doppler effect.

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Here $J_n$ is a Bessel function of the first kind of order $n$ and $\beta$ is the modulation index

$$\beta = 2\pi f \frac{R}{c} \sin \tilde{\beta}_s.$$  \hspace{1cm} (7)

The bandwidth of the signal - defined to be the frequency interval that contains 98% of the total power - is given by

$$B = 2(1 + \beta)f_m.$$  \hspace{1cm} (8)

Sources in the equatorial plane have bandwidths ranging from $B = 2.6 \times 10^{-7} \text{ Hz}$ at $f = 10^{-3} \text{ Hz}$ to $B = 2.1 \times 10^{-5} \text{ Hz}$ at $f = 10^{-2} \text{ Hz}$.

### III. DEMODULATION

If LISA were at rest with respect to the sky, each monochromatic source would produce a spike in the power spectrum of $s(t)$. While this would make data analysis very easy, it would also severely limit the science that could be done as it would be impossible to determine the source location or orientation. Since LISA will move with respect to the sky, each source will have its own unique modulation pattern, and this pattern can be used to fix its location and orientation. On the other hand, the modulation makes the data analysis more complicated as the Fourier power of each source is spread over a wide bandwidth $B$. One approach to the data analysis problem is to demodulate the signal, thereby re-assembling all the power of a given source at one frequency. Since each source has a unique modulation, each source will also require a unique demodulation. Demodulating a particular source is not difficult if one happens to know its frequency, location, orientation and orbital phase. One could imagine searching through this parameter space and looking for spikes in the power spectrum corresponding to sources that have been properly demodulated. A more practical approach is to focus on the Doppler modulation as it causes the largest spreading of the Fourier power.

Consider the Doppler modulated phase

$$\Phi(t) = 2\pi f \left[ t + \frac{R}{c} \sin \tilde{\beta}_s \cos(2\pi f_m t - \tilde{\phi}_s) \right] + \varphi_0.$$  \hspace{1cm} (9)

We seek a new time coordinate $t'$ in which this phase is stationary:

$$\frac{d\Phi}{dt'} = 2\pi f = \frac{d\Phi}{dt} \frac{dt}{dt'}. \hspace{1cm} (10)$$

Thus,

$$t' = \int \left( 1 - \frac{v}{c} \sin \tilde{\beta}_s \sin(2\pi f_m t - \tilde{\phi}_s) \right) dt.$$  \hspace{1cm} (11)

Working to first order in $v/c$ we have

$$t = t' - \frac{R}{c} \sin \tilde{\beta}_s \cos(2\pi f_m t - \tilde{\phi}_s) \hspace{1cm} (12)$$

Taking the data stream from the detector over a one year period and performing a fast Fourier transform allows us to write

$$s(t) = \sum_n a_n e^{i\omega_n t + \gamma_n}.$$  \hspace{1cm} (13)

Performing the coordinate transformation (12) we arrive at the new Fourier expansion

$$s(t') = \sum_k c_k e^{i\omega_{m,k} t'},$$  \hspace{1cm} (14)

where the Fourier coefficients $c_k$ are given by

$$c_k = \sum_n a_n J_n(\beta) e^{i(n-k)(\tilde{\phi}_s - \tau/2)}.$$  \hspace{1cm} (15)

Since the modulation has a limited bandwidth, an excellent approximation to $c_k$ is given by

$$c_k \approx \sum_{n=k}^{k+\delta} a_n J_n(\beta) e^{i(n-k)(\tilde{\phi}_s - \tau/2)}.$$  \hspace{1cm} (16)

where

$$\alpha = 2\pi k f_m \frac{R}{c} \sin \tilde{\beta}_s.$$  \hspace{1cm} (17)

and

$$\delta = 1 + \lceil \alpha \rceil.$$  \hspace{1cm} (18)

The squares brackets imply that we have taken the integer part of $\alpha$. Consider the pure Doppler modulation described in (6) for the simple case where $f = q f_m$ and $q$ is an integer (the non-integer case is more involved, but the idea is the same). The source has Fourier components

$$a_n = A e^{i\varphi_n} J_n(\beta) e^{i(n-q)(\tau/2 - \tilde{\phi}_s)}, \hspace{1cm} (19)$$

so that

$$c_k \approx A e^{i\varphi_n} J_{n-k}(\beta) \sum_n J_{n-q}(\beta) J_{n-k}(\alpha).$$  \hspace{1cm} (20)

Using the Neumann addition theorem,

$$\sum_n J_{n-q}(\beta) J_{n-k}(\alpha) = J_{k-q}(\alpha - \beta),$$  \hspace{1cm} (21)

and the fact that $\alpha \approx \beta$, we find

$$c_k \approx A e^{i\varphi_n} J_{k-q}. \hspace{1cm} (22)$$

In other words, the Doppler demodulation procedure re-assembles all the power at the barycenter frequency $f = q f_m$. The demodulation is less effective when applied to a real LISA source because it does not correct for the amplitude and phase modulations.

Our Fourier space approach is very efficient if one is interested in a limited frequency range. It does not offer any great saving over a direct implementation in the
time domain if one wants to consider all frequencies at once. Hellings\cite{6} has recently implemented the Doppler demodulation procedure in the time domain and found similar results to ours.

To illustrate how the Doppler demodulation works we consider monochromatic, circular Newtonian binaries as described in Ref.\cite{4}. Figure 1 shows the power spectrum of $s(t)$ before and after Doppler demodulation for a source with $f = 5.000167 \times 10^{-3} \text{ Hz}$, $\bar{\theta}_s = 2.385366$ and $\bar{\varphi}_s = 4.462868$. We see that most of the power is collected into a spike of width $\sim 3f_m$ about the barycenter frequency.

Next we consider an example where there are two sources that are nearly in frequency, but nevertheless do not have overlapping bandwidths. The sources have $f = 5.001419 \times 10^{-3} \text{ Hz}$, $(\bar{\theta}_s, \bar{\varphi}_s) = (2.056685, 2.212044)$ and $f = 4.9988 \times 10^{-3} \text{ Hz}$, $(\bar{\theta}_s, \bar{\varphi}_s) = (1.280671, 1.446971)$. The higher frequency source has an amplitude 1.103 times larger than the lower frequency source. The effect of demodulating each source is shown in Figure 2.

The performance of the demodulation procedure is considerably less impressive when the two sources have overlapping bandwidths. Consider sources A and B described in Table 1. The amplitude of Source B is 1.59 times larger than the amplitude of Source A. Figure 3 shows what happens when we demodulate Source A and Source B in turn. Because the two sources overlap, their signals interfere with each other, and the demodulation is only able to detect the stronger of the two sources.

![Figure 1](image1.png)

**FIG. 1**: Power spectra, $P(f)$, before (dashed line) and after (solid line) Doppler demodulation.

<table>
<thead>
<tr>
<th>$f$ (Hz)</th>
<th>$\bar{\theta}_s$</th>
<th>$\bar{\varphi}_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$4.99994 \times 10^{-3}$</td>
<td>2.867424</td>
</tr>
<tr>
<td>B</td>
<td>$4.99729 \times 10^{-3}$</td>
<td>0.661738</td>
</tr>
</tbody>
</table>

**TABLE 1**: Description of Sources A and B

To find the sky location of a source by successively demodulating each point on the sky. In practice we use the HEALPix\cite{7} hierarchical, equal area pixelation scheme to provide a finite number of sky locations to demodulate. We search for the maximum power contained in three adjacent frequency bins, and record this value for each sky pixel. Rather than find the maxima across all frequencies, we produce separate sky maps for frequency intervals of width $4\pi f B / f_m$. There is little point in trying to use smaller frequency intervals due to the interference effect discussed earlier.

**A. Angular resolution**

The Doppler demodulation technique can be used to find the sky location of a source with overlapping bandwidths with $f = 5.00015 \times 10^{-3} \text{ Hz}$ and $(\bar{\theta}_s, \bar{\varphi}_s) = (1.851478, 4.934289)$. Using sky pixels of angular size $\sim 0.92^\circ$ we arrive at the graph shown in Figure 4. The demodulation code
produced the best fit values of $f = 5.00014 \times 10^{-3}$ Hz and $(\tilde{\theta}_s, \tilde{\phi}_s) = (1.253, 5.04)$ and $(\tilde{\theta}_s, \tilde{\phi}_s) = (1.889, 5.04)$. The degenerate fit for the sky location illustrates one of the drawbacks of the Doppler demodulation method - it is unable to distinguish between sources above or below the equator. The errors in the source location, $(\Delta \tilde{\theta}_s = 2.1^\circ, \Delta \tilde{\phi}_s = 6.3^\circ)$, are larger than the pixel size, and can be attributed to the amplitude and phase modulations which we have not corrected for. It should be noted that the error in the source location is not caused by instrument noise since we are working in the large signal-to-noise limit ($n(t) = 0$). Indeed, instrument noise has little effect on the demodulation procedure since the noise is incoherent. The noise gets moved about in frequency space, but there is little power accumulation at any one frequency. When noise was added to the previous example at a signal to noise ratio of 5, the frequency determination and $\tilde{\phi}_s$ determination were unaffected, while the error in $\tilde{\theta}_s$ grew to $\Delta \tilde{\theta}_s = 3.4^\circ$.

Finally, we illustrate how the source interference phenomena discussed earlier limits our ability to locate sources that overlap in frequency. Figures 5 and 6 show how the demodulation procedure is able to locate Sources A and B when one of the sources is turned off. Figure 7 shows what happens when both sources are present. We see that source B shows up clearly, while source A is washed out.

FIG. 3: Power spectra before (dashed line) and after (solid line) Doppler demodulation. The upper graph shows the demodulation when the detector is made stationary with respect to Source A, while the lower graph shows the same for Source B.

FIG. 4: Determining the sky location of a source with $(\tilde{\theta}_s, \tilde{\phi}_s) = (1.851478, 4.934289)$. The sky map uses the Mollweide projection in Ecliptic coordinates with $(\pi/2, 0)$ at the center of the map.

FIG. 5: Determining the sky location of source A without source B.

IV. DISCUSSION

The Doppler demodulation procedure provides a quick way of finding the frequencies and sky locations of the brightest sources detected by LISA. The method has a number of limitations, the most serious being its inability to locate more than one source per bandwidth, and its inability to determine if a source is in the northern or southern hemisphere. Despite these limitations, the Doppler demodulation procedure will be a useful tool in the LISA data analysis arsenal.
Acknowledgements

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