Electromagnetic Form Factors and the Localization of Quark Orbital Angular Momentum in the Proton

JOHN P. RALSTON and ROMAN V. BUNIY

Department of Physics and Astronomy, University of Kansas, Lawrence, KS 66045, USA

PANKAJ JAIN

Department of Physics, IIT Kanpur, Kanpur-208 016, India

A new picture is given of generalized parton distributions probed in experiments in which the probe scale $Q^2$ and the momentum transfer $\Delta^2$ are well separated. Application of this picture to the $Q^2$ dependence of the form factors $F_1, F_2$ shows that gauge invariant quark orbital angular momentum can be measured and indeed localized in the transverse profile of the proton. A previous prediction that $\sqrt{Q^2 F_2(Q^2)/F_1(Q^2)} \sim \text{const.}$ is generalized to GPD language. This prediction appears to have been confirmed by recent CEBAF data.

1 A Physical Picture of Generalized Parton Distributions

Deeply inelastic scattering experiments contradict the notion that the proton’s spin is the sum of the spins of the quarks. The $s$-wave non-relativistic bound state picture of the proton has then been ruled out. Consequently there is great interest in the orbital angular momentum of quarks and the spin of gluons. While the spin of gluons is difficult to define gauge invariantly, the orbital angular momentum of quarks is well defined in terms of generalized parton distributions (GPD). One of the early motivations for introducing GPDs, \cite{1,2} predating the current focus on virtual Compton scattering, was the ability to express the role of orbital angular momentum in exclusive reactions. Rather than concentrating on the total orbital angular momentum, obtained via a sum rule that is probably unobservable, \cite{3} we discuss the localization of the orbital angular momentum region-by-region across the transverse profile of the proton, which not only contains more information, but is perfectly observable. Our method extends to a useful physical picture of the uses and purposes of GPDs.

Translational Symmetry: Consider the quark-proton scattering amplitude

$$\text{quark}(k) + \text{proton}(P) \rightarrow \text{quark}'(k') + \text{proton}'(P + \Delta).$$
The density matrix elements describing this scattering will be defined as
Φ_{ρσ}(k, k')_{P, P'} \equiv \int dzdz' e^{ik\cdot z - ik'\cdot z'} < P', s' | T \bar{\psi}_\rho(z') \psi_\sigma(z) | P, s >.

By momentum conservation \( k - k' = P' - P = \Delta \). The quark fields are evaluated at spatial coordinates \( z^\mu, z'^\mu \) and have Dirac indices \( \rho, \sigma \). We may rewrite the Fourier factors via
\[
e^{ik\cdot z - ik'\cdot z'} = e^{i\frac{k+k'}{2} \cdot (z - z')} + i\Delta \cdot \frac{z + z'}{2}.\]

The proton state can be expressed as
\[
| P, s > = \int dY \exp(-iPY)| Y, s >,
\]
where \( Y \) is a center of momentum (CM) coordinate. The coordinate \( Y \) parameterizes collective variables of the state without specifying more about the internal coordinates. With a similar step for \( | P', s' > \), matrix elements depend on \( e^{iP\cdot Y - iP'\cdot Y'} = e^{i\tilde{b}\cdot \tilde{Y} - i\Delta \cdot \frac{z + z'}{2}} \). This isolates all dependence on the variable \( \Delta \) in
\[
Φ_{ρσ}(k, k')_{P, P'} = \int dY dY' dzdz' \Phi e^{i\frac{P+P'}{2} \cdot (Y - Y')} + i\tilde{b} \cdot \tilde{Y} e^{-i\Delta \cdot \frac{z + z'}{2}}, \quad (1)
\]
where \( \Phi = | Y', s' | \bar{\psi}_\rho(z') \psi_\sigma(z) | Y, s > \), and
\[
\tilde{Y} = (Y + Y')/2, \quad \tilde{b} = (z + z')/2.
\]

A fundamental difference between ordinary parton distributions, and the GPD, lies in the dependence on \( \Delta \). We may interpret \( \Delta \) as probing the dependence of the quark 2-point correlation on \( \tilde{b} - \tilde{Y} \), namely the offset of the quark CM coordinate relative to the hadron CM coordinate. The transverse component of \( \tilde{b} \) is the average impact parameter of the two quark fields (explaining symbol "\( b \)".) This concept cannot be formulated with ordinary (diagonal) parton distributions evaluated at \( \Delta = 0 \). In ordinary parton formalism, the quark 2-point correlation may only depend on the difference between the locations of the two fields. This follows immediately from translational symmetry; as \( \Delta \to 0 \), translational invariance states that the dependence on \( \tilde{b} - \tilde{Y} \) cannot be conceived. No wonder the GPD’s pose such a puzzle when starting from the usual parton distribution.
The past few years have seen an explosion of interest in GPD’s and an overkill in short-distance expansions. The meaning of these expansions is that large virtual momenta $Q$ select small spatial separations, namely the region of $z - z' \to O(1/Q)$. Here $Q$ is a large scale such as a virtual photon’s momentum serving as a pointlike probe of the GPD. The short distance expansions themselves contain no information about the target, and the mild $Q^2$ dependence of logarithmic scaling violations is understood. This is old physics; we take as established that the quark operators in a large $Q$ reaction can be viewed as nearly local and “touching one another”. This concept is gauge-invariant. When renormalizing the operators as a function of $Q^2$, we may expect mixing to be diagonalized in $\vec{b}$ space, because physics is local. Meanwhile the spatial location of the event inside the proton is absolutely not addressed by the $Q^2$ dependence, should not be expanded via the operator product expansion, and to reiterate, cannot be conceived or observed with forward matrix elements.

**New Partonic Amplitudes at Each Impact Parameter:** The $\Delta^+$ dependence of reactions, usually denoted “skewness” in virtual Compton parlance, is observable in particular frames; it is conjugate to $(\vec{z} - vt)$, the time dependence induced by a pulse of moving fields in the Lorentz-boosted pancake. Unfortunately the $\Delta^+$ dependence describes a vicinity in a pancake which is not resolved better than the thickness of the pancake. Localization in the longitudinal direction is difficult. Our view of the reactions focuses on the $b_T$ dependence. Measurement of the $\Delta_T$ dependence of amplitudes, by Fourier transform, can be inverted to find the spatial $b_T$ location of the partons. The transverse structure is directly observable when amplitudes are measured by interference. The transverse structure is remarkably decoupled from the longitudinal smearing. Thus the region of $0 < \Delta_T < GeV$, in which the handbag approximation is consistent, encodes the localization of various types, flavors, momenta, transverse location and spin of partons, informing us of these new quantities at each transverse locale. The average over the transverse plane of the imaginary parts of these partonic amplitudes is the usual parton distributions at $\Delta_T = 0$. Attention to the $\Delta_T$ dependence of reactions with GPDs at large $Q^2$ can then reveal and localize the orbital angular momentum of the quarks.

### 2 The $F_2$ Form Factor and Orbital Angular Momentum

Experiments on the proton’s Pauli electromagnetic form factor $F_2$, more than illustrate these ideas. We have developed a new approach to $F_2$, keeping in mind that the form factors have a restrictive feature of $Q^\mu = \Delta^\mu$. This means that the spatial resolution via form factors is never better than the region of
Figure 1: Cartoon of the natural interpretation of $GD$. The fast struck quark (stack of plane waves) is located at the transverse offset position $b_T$, while being spatially localized to within order $1/Q$ in the transverse directions. The quark’s longitudinal coordinate is not so well localized, but is spread along the light-cone in the direction conjugate to $xP$.

the object being measured, a limitation easily taken into account.

We go to a $(+,T,−)$ Lorentz frame in which the photon momentum $Q^\mu = (0,\Delta T,0)$. The initial and final proton momenta are $P^\mu(P') = (P, ±\Delta T/2, m^2 + \Delta^2 T/4)$. Physically $F_2$ represents a chirality (helicity) flip amplitude $< −|J|+ >$ where $|±⟩ = |x⟩ ± i|y⟩$ are chirality eigenstates. Letting $x,y$ be two transverse orientations of the chirality (“transchirality”), nearly equal to transverse spin in the high energy limit, then $F_2 ∼ <x|J|x⟩ − <y|J|y⟩$. Thus $F_2$ distinguishes the two possible transverse spin orientations relative to the scattering plane $\Delta_T$.

We write the form factor as

$$<P',s'|J^\mu|P,s> = \int dk^−dxP^+d^2b_T e^{i\Delta T\cdot b_T/x} \bar{u}_i(p',s')V^\mu_{ij}u_j(p,s).$$

We make the ansatz

$$V^\mu_{ij} = (A\gamma^\mu + B\sigma^{\nu\rho}z_\nu + C\sigma^{\lambda\nu}Q_\nu + D\frac{(P + P')^{\mu}}{m}\gamma_5)_{ij} + \ldots.$$  

The coefficients $A, B, C, D$ are scalar functions of $\Delta^2, \Delta b^2, \Delta \cdot b$, which under dimensional scaling behave like $Q^2, 1/Q^2, 1$, respectively. At large $Q^2$ we impose
a factorization scheme forcing all diagrams into the handbag: this unconventional step avoids the assumption that all processes are necessarily factored into wave functions times hard scattering. Perhaps that assumption can be justified later; if so, one relates the single unit of orbital angular momentum of the localized pair of quarks to the interference of one and zero units in wave functions, as we assumed elsewhere.\footnote{5} We avoid asserting that all constituents of the proton are localized at short distances relative to one another due to any asymptotic limit; our argument only requires the far more general property that $b_T$ of one pair of operators scales like $1/Q$.

We postponed defining the operator $\omega_{ij} = \Lambda_{\alpha\beta\rho\sigma}^{\gamma\delta} \sigma^{\gamma\delta}_{ij} b^\rho P^\sigma / m$ in Eq. [2]. This operator selects orbital SO(2) harmonics $b_x \pm i b_y = |b_T| e^{\pm i \phi}$ which are good under Lorentz boosts. The operator also contains the non-relativistic scalar $b_T \times \vec{p}$ contracted with the Lorentz generator $\sigma^{\alpha\beta}$. Spinors are ($\pm 1/2$) eigenstates of the Pauli-Lubanski operator $W^\mu s_\mu \gamma_5 = \epsilon^{\mu \alpha \beta \rho} \sigma_{\alpha \beta} P^\rho s_\mu \gamma_5 / m$. Separate the component of $\bar{b}$ parallel to $s$ for this reason, writing

$$\bar{b}^\mu = \bar{b} \cdot s s^\mu / (s^2) + \bar{b}^\mu.$$ 

Set aside $\bar{b}^\mu$ orthogonal to $s$ for now; this makes $\omega_5 = \Lambda_{ST} \bar{b}_T$ on transversely polarized states, and the $D$ term is recognized as directly probing $b_T \times \vec{p} \cdot \vec{s}_T$. Note these coordinates apply to the pair of operators and are gauge invariant.

In the limit of large enough $Q^2$ perturbation theory for three quarks scattering yields a power series expansion of the form factors. We apply that information to the handbag description to find bounds on $A_{D}$. In particular, the helicity-flip amplitudes are power-suppressed compared to the helicity conserving ones, up to Sudakov-corrections. On this basis $D$ and $A$ are comparable, although the numerical values depend on the relative strength of the orbital $m = \pm 1$ terms compared to the $m = 0$ terms. The reasonable and difficult-to-avoid asymptotic scaling rule $b_T \rightarrow Q_T / Q^2$ then gives

$$F_2 / F_1 \sim \frac{Q_T \cdot s_T}{Q^2}.$$ 

Of course $s_T$ appears in the amplitude using the Pauli-Lubanski trick with the frames indicated, and a direct calculation will give $F_2 / F_1 \sim 1 / \sqrt{Q^2}$.

This result contradicts conventional wisdom, but it appears that conventional wisdom overlooked orbital angular momentum. The distribution amplitude formalism of Brodsky and Lepage excludes orbital angular momentum in the first steps, leading to hadronic helicity conservation as an exact dynamical selection rule, and $F_2 = 0$. Our calculation here parallels earlier work,\footnote{5} but is much more general. After the meeting we learned that Diehl, Feldman,
Jacob and Kroll \(^8\) used GPDs to study \(F_2\). The interpretation of interference between zero and one unit of orbital angular momentum is explicit: see also Kroll’s report at this meeting\(^{10}\).

Following an earlier CEBAF study \(^7\) we corrected a math error in our 1993 paper \(^1\) introducing GPDs and relating them to quark orbital angular momentum for \(F_2\). This led to a prediction \(^5\) for \(F_2/F_1 \sim 1/\sqrt{Q^2}\). Recent data from CEBAF extending to \(Q^2 = 6\text{ GeV}^2\) appears to support this prediction. The normalization of \(QF_2/F_1\) indicates that the relative amplitudes for one and zero units of orbital angular momentum are comparable. Future studies accessible to GPDs should show that the transversely polarized proton is not a small round dot, but is flattened by the correlation of \(s_T\) with the scattering plane. The power-law dependence predicted by quark counting corresponds to a quadratic hole in the middle. These features should be more clearly visible when \(Q^2\) and \(\Delta_T\) are decoupled with DVCS experiments. \(^{11}\): having \(Q^2 >> \Delta_T\) corresponds to examining the target on a resolution small compared to the target size.

**Acknowledgments:** Work supported by DOE grant number DE-FGO2-98ER41079.

**References**

10. P. Kroll, these Proceedings.