A Solution to the Coincidence Puzzle of $\Omega_B$ and $\Omega_{DM}$

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Abstract

We show that a class of Affleck–Dine baryogenesis directly relates the observed mass density of baryons, $\Omega_B$, to that of dark matter, $\Omega_{DM}$. In this scenario, the ratio of baryon to dark matter mass density is solely determined by the low energy parameters, except for an $O(0.1)$ effective CP-violating phase. We find that $\Omega_B/\Omega_{DM} = O(0.1)$ with reasonable parameters, which lies surprisingly just in the range of observation. This scenario is totally free from the cosmological gravitino problem, and independent of the detailed history of the Universe as long as it satisfies quite weak constraints.

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1 Introduction

The searches for both the origin of the observed baryon asymmetry and that of the dark matter in the present Universe have been attractive subjects for many physicists. The great success of the Big Bang nucleosynthesis (BBN) predicts an abundance of the baryon asymmetry in the present Universe as $\Omega_B h^2 \simeq 0.02$. Here, $h$ is the present Hubble parameter in units of 100 km sec$^{-1}$ Mpc$^{-1}$, and $\Omega_B \equiv \rho_B/\rho_c$. ($\rho_B$ and $\rho_c$ are the energy density of the baryon and the critical energy density in the present Universe.) As for the dark matter, various observations of the dynamics of galaxies and their clusters, as well as theoretical analyses of the large-scale structure formation indicate that its cosmological abundance lies in the range, $\Omega_{DM} h^2 \simeq 0.1$–0.2.

There exist a number of scenarios of “baryogenesis”, which can generate the required baryon asymmetry. Also for dark matter, several particles have been proposed as candidates, such as axion, neutrino and, among others, the lightest supersymmetric (SUSY) particle (LSP) (especially the lightest neutralino, $\chi$).

However, there remains one prominent problem. The matter density of baryons is roughly $\mathcal{O}$($10$–$20$)% of that of dark matter, i.e. $\Omega_B/\Omega_{DM} \simeq 0.1$–0.2. Why are they so close to each other? Most of the models of baryogenesis use baryon $B$- and/or lepton $L$-number-violating operators in high energy physics. On the other hand, the abundance of dark matter does not seem to be at all related with such high energy physics relevant to baryogenesis. In the standard SUSY dark matter scenario, for example, the mass density of the dark matter is solely determined by the weak scale properties of the neutralino LSP, such as the mass and the annihilation cross section. As long as dark matter is composed of the thermal relics of the LSPs, it is completely independent of other details in the history of the Universe. It must be very exciting if we can find a simple mechanism to directly relate the two independent physical parameters, the baryon asymmetry and the dark matter density.

One interesting solution to this puzzle was proposed by using the Affleck–Dine (AD) baryogenesis [1] in the context of the minimal supergravity (mSUGRA) scenario [2]. In this scenario, non-topological solitons called Q-balls [4] play the role of the common source of the baryon asymmetry and the LSP. In the AD baryogenesis, a complex scalar

\footnote{The possibility to explain both the baryon asymmetry and dark matter by a single mechanism was first considered in the context of gauge-mediated SUSY breaking scenarios [3].}
field $\phi$, which is a linear combination of squark fields, obtains a large expectation value along one of the flat directions in the scalar potential during inflation. The subsequent coherent oscillation of the $\phi$ field obtains a phase rotational motion and begins to carry non-zero baryon number in the presence of $B$-violating operators. A crucial point is that this coherent oscillation of the $\phi$ field is unstable, with spatial perturbations and fragments into the Q-balls after dozens of oscillations. Recent detailed lattice simulations have revealed that almost all the baryon asymmetry initially carried by the coherent $\phi$ field is absorbed into Q-balls [5, 6].

The large amplitude of the $\phi$ field inside the Q-ball protects it from being thermalized. The typical decay temperature of Q-balls is given by $T_d \lesssim \text{(a few)} \text{ GeV}$, which is well below the freeze-out temperature of the LSP. As a result, the LSPs produced by the Q-ball decay are not thermalized and retain the initial abundance. Therefore, the mass density of the non-thermal neutralino dark matter $\Omega_\chi$ is directly connected to that of baryons [2]:

$$\Omega_\chi \simeq \left( \frac{m_\chi}{m_p} \right) \left( \frac{n_B}{n_\phi} \right)^{-1} \times \Omega_B,$$

where $m_\chi$ and $m_p$ are the LSP and nucleon masses, respectively; $n_B$ and $n_\phi$ are the baryon and $\phi$ field number densities. One can easily understand this relation by considering that the decay of a single $\phi$ field produces $\gtrsim 1$ neutralino LSP in the final state. Here, the ratio of $n_B$ to $n_\phi$ is fixed when the $\phi$ field starts coherently oscillating and remains constant until it eventually decays. Under the assumption of $R$-parity conservation, the possible maximum value of this ratio is $(n_B/n_\phi) = 1/3$. The typical value of this ratio, which naturally appears in models of AD baryogenesis in the mSUGRA scenario is $n_B/n_\phi \lesssim 0.1$.

Unfortunately, this simple model is not cosmologically viable. We need an extremely light bino $m_\chi \lesssim 1 \text{ GeV}$ to explain the required mass density of dark matter. This fact indicates that the formation of a Q-ball is a serious obstacle for the AD baryogenesis rather than a solution to the $\Omega_B-\Omega_{DM}$ coincidence puzzle.\(^2\)

There are several ways to reconcile this situation. The obvious one is to make the Q-balls small enough that they can evaporate well above the freeze-out temperature of the LSP. This can be done by gauging the $U(1)_{B-L}$ symmetry [7] or assuming a relatively strong three-point coupling of the inflaton to the gluino [8]. In both cases, it is clear that the relation between $\Omega_B$ and $\Omega_\chi$ is lost completely. Another possibility is to adopt the LSP\(^2\)

\(^2\)In the original work, the baryon absorption into the produced Q-balls was assumed not to be so efficient, which conflicts with the detailed lattice simulations [5, 6].
with a large annihilation cross section, such as higgsino $\tilde{H}$ or wino $\tilde{W}$. In Refs. [9, 10], it is shown that the subsequent pair annihilations of these LSPs can naturally lead to the desired mass density of dark matter. However, in this case, one expects that the resultant mass density of baryons and that of dark matter are completely independent of each other, since the number of produced LSPs should be drastically reduced via annihilation processes so as not to overclose the Universe. However, astonishingly, this is not always the case.

In this letter, we will show that a class of AD baryogenesis scenarios allow us to connect the two crucial quantities in our Universe. Our final goal is to show the following relation between the mass density of baryons and that of dark matter:

$$\frac{\Omega_B}{\Omega_\chi} \approx 10^{3-4} \left( \frac{m_\phi^2}{\langle \sigma v \rangle_\chi^{-1}} \right) \left( \frac{m_p}{m_\chi} \right) \delta_{\text{eff}},$$

where $m_\phi$ is the mass of the flat direction field $\phi$, which is approximately given by the squark mass; $\langle \sigma v \rangle_\chi$ is the $s$-wave component of the annihilation cross section of the LSP; $\delta_{\text{eff}} = \mathcal{O}(0.1)$ is the effective CP-violating phase of the $\phi$ field. A numerical factor in the right-hand side depends on the details of the produced Q-balls, but basically they are calculable. If we take the typical annihilation cross section of the $\tilde{H}$- or $\tilde{W}$-like LSP, $\langle \sigma v \rangle_\chi \sim 10^{-7-8}$ GeV$^{-2}$, one can easily see that this is just the desired relation. In the remainder of this paper, after a brief review of the AD baryogenesis, we will derive this interesting relation and discuss the conditions for it to hold.

## The model

Our model of the AD baryogenesis is basically the same as the one presented in Section II B of Ref. [10]. For self-consistency, we briefly review the mechanism with particular attention to the late-time decays of Q-balls. The derivation of the relation Eq. (2) and some conditions for it to hold will be discussed in subsequent sections.

We consider the situation that there exist some chiral symmetries, such as $R$-symmetry,

\[^3\]The LSP is not necessarily a pure $\tilde{H}$ or a pure $\tilde{W}$. A significant mixing of the bino component in the LSP is quite possible. In the case of a large $\tan\beta$, the acceptable mass density of dark matter can be obtained even with the LSP whose dominant component is a bino.

\[^4\]As we will see, when the mass of the gravitino $m_{3/2}$ is larger than $m_\phi$, $m_\phi$ in Eq. (2) should be replaced by $m_{3/2}$.\]
which forbid non-renormalizable operators in the superpotential that lift the relevant flat directions for baryogenesis. $B$-violating operators needed for AD baryogenesis are supplied in the Kähler potential. We take the following operators, for example, which are consistent with the $R$-symmetry:

$$\delta \mathcal{L} = \int d^4 \theta \left( \lambda_1 \frac{I^I J^J}{M_*} Q \bar{U}^{\dagger} \bar{D}^{\dagger} L + \lambda_2 \frac{Z^I Z}{M_*} Q \bar{U}^{\dagger} \bar{D}^{\dagger} L + \text{h.c.} \right),$$  

(3)

where $M_* = 2.4 \times 10^{18}$ GeV is the reduced Planck scale, and $\lambda_1, \lambda_2$ are the coupling constants of the order of 1, which are generally complex numbers. Here, $Q, \bar{U}, \bar{D}, L, I$ and $Z$ denote superfields (and their scalar components) of left-handed quark doublets, right-handed up-type and down-type quarks, left-handed lepton doublets, inflaton and the one relevant to SUSY breaking in the true vacuum, respectively. These interactions induce the following terms in the scalar potential:

$$\delta V = - \left( \lambda_1 \frac{3H^2}{M_*^2} Q \bar{U}^{\dagger} \bar{D}^{\dagger} L + \lambda_2 \frac{3m_{3/2}^2}{M_*^2} Q \bar{U}^{\dagger} \bar{D}^{\dagger} L + \text{h.c.} \right),$$  

(4)

where $m_{3/2}$ is the mass of the gravitino, and $H$ is the Hubble parameter of the expanding Universe.

If the expectation value $\langle Q \bar{U}^{\dagger} \bar{D}^{\dagger} L \rangle$ is large enough after the inflation, the terms in Eq. (4) give the phase rotational motion to this scalar condensate and generate net baryon asymmetry. In fact, in the scalar potential of the minimal supersymmetric standard model (MSSM), there are many $D$- and $F$-flat directions [11] that give a non-zero expectation value of $\langle Q \bar{U}^{\dagger} \bar{D}^{\dagger} L \rangle$. In the remaining discussion, we adopt the flat direction labelled by a linear combination of the monomials of the chiral superfields: $UDD, QDL$. An explicit parametrization of this direction, as suggested in the original work [1], is given by

$$Q^{(1)} = \begin{pmatrix} \phi_1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad L^{(1)} = \begin{pmatrix} 0 \\ \phi_1 \end{pmatrix},$$

$$\bar{D}^{(2)} = \begin{pmatrix} \phi_3 & 0 & 0 \end{pmatrix}, \quad \bar{U}^{(1)} = \begin{pmatrix} 0 & \phi_2 & 0 \end{pmatrix}, \quad \bar{D}^{(1)} = \begin{pmatrix} 0 & 0 & \phi_2 \end{pmatrix},$$

$$|\phi_3|^2 = |\phi_1|^2 + |\phi_2|^2,$$  

(5)

where superscripts (1)–(3) denote the generations of the chiral superfields. Here, the column and low vectors denote the isospin of the SU(2)$_L$ and the colour of the SU(3)$_C$ gauge groups of the MSSM, respectively. Hereafter, we use this simple parametrization.

$^5$The $B$-violating operator $\propto QQU^{\dagger}E^{\dagger}$ can also be applied to the following arguments.
Although there are many other parametrizations of the flat direction, the main arguments in the following discussion do not change much.

To give large expectation values to the flat-direction fields, we assume the four-point couplings of these fields to the inflaton in the Kähler potential [12]:

\[
\delta K = \frac{I^†I}{M_*^2} \left( b_1 \phi_1^† \phi_1 + b_2 \phi_2^† \phi_2 + b_3 \phi_3^† \phi_3 \right),
\]

(6)

where \(b_i\)’s are real coupling constants of the order of 1. Here we use the same symbols for the chiral superfields as for the corresponding scalar components in Eq. (5). By calculating the scalar potential of the supergravity, we can find that the following terms are induced at leading order:

\[
\delta V = 3 \left[ 1 - (b_1 + b_3) \right] H^2 |\phi_1|^2 + 3 \left[ 1 - (b_2 + b_3) \right] H^2 |\phi_2|^2.
\]

(7)

Therefore, if both \((b_1 + b_3)\) and \((b_2 + b_3)\) are somewhat larger than 1, \(\phi_1\) and \(\phi_2\) (and so \(\phi_3\)) obtain large expectation values during inflation. Furthermore, if \((b_1 + b_3) \approx (b_2 + b_3)\), we can naturally expect that \(|\phi_1| \approx |\phi_2| (\equiv |\phi|)\). We assume that this is the case in the remainder of this paper.

Now, we can approximately write down the relevant scalar potential of the \(\phi\) field as follows:

\[
V = (m_\phi^2 - c_H H^2) |\phi|^2 + \frac{H^2}{4M_*^2} (a_H \phi^4 + \text{h.c.}) + \frac{m_\phi^2}{4M_*^2} (a_m \phi^4 + \text{h.c.}) + \ldots,
\]

(8)

where the ellipsis denotes the higher-order terms coming from the Kähler potential; \(a_H\), \(a_m\) are complex coupling constants; \(c_H = O(1)\) is a real coefficient; \(m_\phi\) denotes the soft SUSY-breaking mass of the \(\phi\) field, which is roughly given by the average value of the soft SUSY-breaking masses of the \(Q, \bar{U}, \bar{D}\) and \(L\). Here, we have neglected the terms induced by the interactions with thermal backgrounds, which will be justified later.

We are, now, at the point where we want to discuss the evolution of the \(\phi\) field. During inflation, the large negative Hubble-induced mass term causes an instability around the origin, and the \(\phi\) field develops a large expectation value. The amplitude of the \(\phi\) field just after the inflation, \(|\phi_0|\), is determined by the balance point between the Hubble-induced mass term and non-renormalizable operators in the Kähler potential. In the following discussion, we treat it as a free parameter and assume \(|\phi|_0 \lesssim M_*\). (Actually, there exists an interesting method to fix \(|\phi|_0\) below the Planck scale. This can be done by gauging
the U(1)$_{B-L}$ symmetry. In this case, we can fix the amplitude of the $\phi$ field at the $B - L$ breaking scale, $|\phi|_0 \simeq v_{B-L}$.  The details on this point are discussed in Ref. [7].)

After the end of inflation, the amplitude of the Hubble parameter gradually decreases. When $m_\phi$ exceeds the Hubble parameter, the $\phi$ field starts coherent oscillation around the origin. At this time $H = H_{\mathrm{osc}} \simeq m_\phi$, a huge baryon asymmetry is produced because of the second and third operators in Eq. (8). As long as $|\phi|_0 \lesssim M_*$, the curvature along the phase direction is smaller than the Hubble parameter during the inflation. Therefore, the initial phase of the $\phi$ field is generally displaced from the bottom of the valley of the scalar potential. This is the reason why the second and third operators give the phase rotational motion to the $\phi$ field.

It is not difficult to estimate the ratio of baryon to $\phi$-number density, which is fixed at $H = H_{\mathrm{osc}}$, as

$$\left(\frac{n_B}{n_\phi}\right) \simeq \max \left[ |a_H| \left( \frac{|\phi|_0}{M_*} \right)^2 \delta_H, \, |a_m| \left( \frac{m_{3/2}}{m_\phi} \right)^2 \left( \frac{|\phi|_0}{M_*} \right)^2 \delta_G \right],$$

(9)

where $\delta_H \equiv \sin(\text{arg}a_H + 4\text{arg}(\phi_0))$, and $\delta_G \equiv \sin(\text{arg}a_m + 4\text{arg}\phi_0)$. Note that this ratio remains constant until the $\phi$ field eventually decays into the SM particles and the LSPs.

After dozens of oscillations, the scalar condensate of the $\phi$ field fragments into the Q-balls. Almost all the baryonic charge and energy density carried by the coherent $\phi$ field are absorbed into the Q-balls [5, 6]. As for the details about the Q-ball, see Ref. [10]. The produced Q-balls behave as ordinary matter, and their energy density decreases as $\rho_Q = \rho_\phi \propto R^{-3}$, where $R$ is the scale factor of the expanding Universe. On the other hand, the energy of the inflaton is converted into radiation through reheating. After completion of the reheating process, the energy density of the radiation decreases as $\rho_R \propto R^{-4}$. Then, the energy density of the Universe is dominated by the produced Q-balls before their decays, which take place at $T = T_d$, when the following condition is satisfied:

$$T_R > 3T_d \left( \frac{M_*}{|\phi|_0} \right)^2,$$

(10)

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6 Although the operator conserves the $B - L$ symmetry, the flat directions carry non-zero $B - L$ charges with the same sign; then, they can be lifted at the $B - L$ breaking scale by the potential induced by the U(1)$_{B-L}$ D-term [7].

7 This ratio cannot exceed 1/6 for our choice of the flat direction. If $m_{3/2} \gg m_\phi$, as in the case of anomaly-mediated SUSY-breaking models, the condition $|\phi|_0 \lesssim m_\phi M_* / m_{3/2}$ is necessary for the $\phi$ field not to be trapped at the local (or global) minimum located near the Planck scale [13].
where \( T_R \) is the reheating temperature of the inflation. Although it depends on the size of the Q-ball charge, the typical decay temperature of the Q-ball lies in the range \( 10 \text{ MeV} \lesssim T_d \lesssim (\text{a few}) \text{ GeV} \), and hence the relation Eq. (10) can be easily satisfied. In the following, we assume this to be the case. We will come back to this condition after the derivation of Eq. (2).

The resultant baryon asymmetry after the decays of the Q-balls is now given by the following simple formula:

\[
\frac{n_B}{s} = \frac{\rho_Q}{\rho_Q} \left( \frac{n_B}{n} \right) = \frac{3}{4} \frac{T_d}{m_\phi} \left( \frac{n_B}{n} \right),
\]

where \( s \) is the entropy density of the Universe. Here, in the second equality, we have used the fact that the effective mass of the Q-ball per \( \phi \)-number is given by \( m_\phi \) in a good approximation. Note that the resultant baryon asymmetry is completely independent of the detailed history of the Universe, since the Q-balls dominate the energy density of the Universe before they decay.

If the LSP \( \chi \) has a large \( s \)-wave annihilation cross section, which is the case for the \( \tilde{H} \)- or \( \tilde{W} \)-like LSP, \(^8\) the resultant relic abundance of \( \chi \) is given by the following simple expression to quite good accuracy \([9, 10]\):

\[
\frac{n_\chi}{s} = \sqrt{\frac{45}{8\pi^2 g_*(T_d) M_* T_d}} \frac{\langle \sigma v \rangle_\chi^{-1}}{M_* T_d},
\]

where \( n_\chi \) is the number density of the LSP, and \( g_*(T_d) \) denotes the relativistic degrees of freedom at \( T = T_d \). In terms of the density parameter, the resultant matter density of baryons and that of dark matter can be expressed as follows:

\[
\Omega_B h^2 \approx 0.02 \left( \frac{1 \text{ TeV}}{m_\phi} \right) \left( \frac{T_d}{100 \text{ MeV}} \right) \left[ \left| a_H \right| \left( \frac{|\phi|_0}{M_*} \right)^2 \frac{\delta_H}{\delta_{\text{eff}}} \right] \times 10^6, \tag{12}
\]

\[
\Omega_\chi h^2 \approx 0.3 \left( \frac{10}{g_*(T_d)} \right)^{1/2} \left( \frac{m_\chi}{100 \text{ GeV}} \right) \left( \frac{T_d}{100 \text{ MeV}} \right) \left( \frac{10^{-7} \text{ GeV}^{-2}}{\langle \sigma v \rangle_\chi} \right). \tag{13}
\]

\(^8\)The sneutrino \( \tilde{\nu} \) is also an interesting candidate. Even the bino-like LSP can have a large \( s \)-wave component in the case of very large \( \tan \beta \) via \( H \) contamination.
Therefore, if either $\tilde{H}$ or $\tilde{W}$ is the LSP, the baryon asymmetry and the dark matter may be explained simultaneously in the following range of parameters:

$$
10^{15} \text{ GeV} \lesssim |\phi|_0 \lesssim 10^{17} \text{ GeV}, \quad 10 \text{ MeV} \lesssim T_d \lesssim (\text{a few}) \text{ GeV}, \\
10^{-8} \text{ GeV}^{-2} \lesssim \langle \sigma v \rangle \chi \lesssim 10^{-7} \text{ GeV}^{-2}, \quad 10^{-2} \lesssim |a| \delta_{\text{eff}} \lesssim 10^{-1}, 
$$

(14)

where $a$ and $\delta_{\text{eff}}$ denote either $a_H$ or $a_m$, and either $\delta_{\text{eff}}^H$ or $\delta_{\text{eff}}^G$, respectively.

3 Derivation of the relation of $\Omega_B$ and $\Omega_{DM}$

Now, we come to deriving the relation in Eq. (2). For this purpose, we have to know the $|\phi|_0$ dependence on the decay temperature $T_d$ of the Q-ball. The terms $|\phi|_0$ and $T_d$ are related to each other through the size of the Q-ball charge “$Q$”.

The size of the Q-ball charge crucially depends on the initial amplitude of the $\phi$ field and its scalar potential. The scalar potential relevant at the time of the Q-ball formation can be written as

$$
V(\phi) = m_\phi^2 \left( 1 + K \log \left( \frac{|\phi|^2}{M_G^2} \right) \right) |\phi|^2,
$$

(15)

where $M_G$ is the renormalization scale at which the soft mass $m_\phi$ is defined, and the $K \log(|\phi|^2)$ term represents the one-loop correction. This mainly comes from the gluino loops and the typical value of $K$ is found to be in the range $-0.1 \lesssim K \lesssim -0.01$ \cite{14}. This negativeness of the $K$-factor makes the potential a little bit flatter than the quadratic one, which is a necessary and sufficient condition for the Q-ball to be formed in the present baryogenesis.

Recently, the typical size of the Q-ball charge has been calculated by detailed lattice simulations \cite{5, 6}. Applying their result to our model, the size of the Q-ball charge can be estimated to

$$
Q = \bar{\beta} \left( \frac{|\phi|_0}{m_\phi} \right)^2 \epsilon,
$$

(16)

where $\bar{\beta} \simeq 6 \times 10^{-3}$, which depends on the $K$-factor and the fluctuations of the $\phi$ field after inflation, and $\epsilon \simeq 0.01$ is a constant independent of $|\phi|_0$. \footnote{This is the case for $(n_B/n_\phi) \lesssim 0.01$, which is naturally satisfied in the present model with $|\phi|_0 \lesssim M_*$.
If this is not the case, $\epsilon$ should be replaced by $(n_B/n_\phi)$.}
The size of the charge that evaporates from the surface of a single Q-ball through interactions with thermal backgrounds is about $\Delta Q \sim 10^{18}$ [16]. Therefore, as long as $|\phi|_0 \gtrsim 10^{14} \text{GeV}$, the Q-ball survives thermal evaporation. The remaining charges of the Q-ball are emitted through its decay into light fermions. The decay rate was calculated in Ref. [17] as

$$\Gamma_Q = -\frac{dQ}{dt} \sim \frac{\omega^3 A}{192\pi^2}.$$  \hspace{1cm} (17)

Here, $A = 4\pi R_Q^2$ is the surface area of the Q-ball, where $R_Q \simeq \sqrt{2/(m_\phi \sqrt{|K|})}$ is the Q-ball radius; $\omega \simeq m_\phi$ is the effective mass of the Q-ball per $\phi$-number. This upper bound is likely to be saturated for $\phi(0) \gg m_\phi$, where $\phi(0)$ is the field value of $\phi$ at the centre of the Q-ball. This is the case in the present model.

Then, the decay temperature of the Q-ball is given by

$$T_d = \frac{\eta}{\sqrt{48|K|\pi}} \left( \frac{90}{\pi^2 g_*(T_d)} \right)^{1/4} \left( \frac{m_\phi M_*}{Q} \right)^{1/2},$$  \hspace{1cm} (18)

$$\simeq 2 \text{ GeV} \times \eta \left( \frac{0.03}{|K|} \right)^{1/2} \left( \frac{m_\phi}{1 \text{ TeV}} \right)^{1/2} \left( \frac{10^{20}}{Q} \right)^{1/2},$$  \hspace{1cm} (19)

where $\eta \lesssim 1$ denotes the ambiguity coming from an inequality of the decay rate in Eq. (17). \hspace{1cm} 10

From Eqs. (12), (13) and (20), we can see that both the matter density of baryons and that of dark matter are linearly proportional to $|\phi|_0$, and hence we can easily derive the wanted relation:

$$\frac{\Omega_B}{\Omega_\chi} = \frac{\eta^2}{16\pi|K|\beta e} \left( \frac{m_p}{m_\chi} \right) \left( \frac{m_\phi^2}{\langle |\phi|_0 |^2 \rangle} \right) \times \max \left\{ \left| a_H \delta_H^H \right|, \left| a_m \left( \frac{m_{3/2}}{m_\phi} \right)^2 \delta \bar{\chi}^H \right| \right\}. \hspace{1cm} (21)$$

\hspace{1cm} 10This factor is, in principle, numerically calculable by determining the accurate profile of the Q-ball, which requires the details of the scalar potential. There might also exist an ambiguity from the decay channels induced by loop diagrams.
If we assume \(|a_H|, |a_m|, \eta \approx 1\) and the typical values for the parameters, \(K, \bar{\beta}\) and \(\epsilon\), the above equation is written as

\[
\frac{\Omega_B}{\Omega_\chi} \approx 10^{3-4} \left(\frac{m_p}{m_\chi}\right) \times \max \left[\left(\frac{m_\phi^2}{\langle \sigma v \rangle_{\chi}^{-1}}\right) \delta^H_{\text{eff}}, \left(\frac{m_{3/2}^2}{\langle \sigma v \rangle_{\chi}^{-1}}\right) \delta_{\text{eff}}^G\right].
\] (22)

By using the typical annihilation cross section for \(\tilde{H}\)- or \(\tilde{W}\)-like LSP, \(\langle \sigma v \rangle_{\chi} \approx 5 \times 10^{-8} \text{ GeV}^{-2}\), we derive

\[
\frac{\Omega_B}{\Omega_\chi} \approx 0.15 \times \left(\frac{100 \text{ GeV}}{m_\chi}\right)^2 \left(\frac{\langle \sigma v \rangle_{\chi}}{5 \times 10^{-8} \text{ GeV}^{-2}}\right)^2 \left(\frac{\delta_{\text{eff}}}{0.1}\right),
\] (23)

where \(m\) and \(\delta_{\text{eff}}\) denote either \(m_\phi\) or \(m_{3/2}\), and either \(\delta^H_{\text{eff}}\) or \(\delta^G_{\text{eff}}\), respectively. This is perfectly consistent with observations.

We can obtain another interesting piece of information by “multiplying” the mass density of baryons and that of dark matter together. In this case, we can remove the ambiguity associated with the decay temperature of the Q-ball. It is easy to show that the following relation holds:

\[
\Omega_B h^2 \times \Omega_\chi h^2 = 1.76 \times 10^{-2} \frac{m_\chi}{m_\phi} \left(\frac{\langle \sigma v \rangle_{\chi}^{-1}}{\text{GeV}^2}\right) \left(\frac{1}{\sqrt{g_*(T_d)}}\right) \left(\frac{n_B}{n_\phi}\right).
\] (24)

This relation gives us information on \(|\phi|_0\), the initial amplitude of the \(\phi\) field. If we take the typical values for parameters, we can see that the present mass density of baryons and that of dark matter suggest \(10^{15} \text{ GeV} \lesssim |\phi|_0 \lesssim 10^{16} \text{ GeV}\), which surprisingly coincides with the \(B-L\) breaking scale suggested from the see-saw neutrino masses [18]. Note that this value does not affect the prediction in Eq. (21).

4 Conditions for the relation to hold

In this section, we discuss the conditions for relation (21) to hold. We have to examine the thermal effects, on the scalar potential, of the \(\phi\) field given in Eq. (8). If the thermal effects dominate the scalar potential when the \(\phi\) field starts to oscillate, they cause the evaporation of the \(\phi\) field before the formation of the Q-balls, or drastically change the \(|\phi|_0\) dependence on the Q-ball charge.

First, let us discuss the effects related with the thermal mass terms. If the cosmic temperature \(T\) satisfies \(f|\phi|_0 < T\), the field coupled to the \(\phi\) field through the coupling
constant $f$ induces the thermal mass term $c^2 f^2 T^2 |\phi|^2$, where $c$ is a real constant of the order of 1. Therefore, if the two conditions, $f|\phi_0| < T$ and $c f T > H$, are satisfied simultaneously when $H \gtrsim m_\phi$, the thermal mass term causes the early oscillation of the $\phi$ field [12, 19, 20]. In this case, the $\phi$ field is thermalized and Q-balls are not formed. The condition for the reheating temperature of inflation, $T_R$, to avoid this early oscillation is given by

$$T_R \lesssim \max \left\{ \frac{f}{\sqrt{c}} |\phi_0| \left( \frac{|\phi_0|}{M_*} \right)^{1/2}, \frac{m_\phi}{c^2 f^2} \left( \frac{m_\phi}{M_*} \right)^{1/2} \right\}$$

(25)

This condition should be satisfied in all the gauge and Yukawa coupling constants associated with the $\phi$ field. The most stringent constraint comes from the first term with Yukawa coupling of an up quark, $Y_u \simeq 10^{-5}$. This constraint becomes significantly weak if we adopt the second and third generations of squarks for the flat direction.

A much more stringent constraint comes from the thermal logarithmic potential [21]:

$$\delta V \supset a_g \alpha T^4 \log \left( \frac{|\phi|^2}{M_*^2} \right),$$

(26)

where $|a_g| = \mathcal{O}(1)$, and $\alpha$ is a constant given by the fourth power of gauge and/or Yukawa coupling constants. This leads to the following constraint on the reheating temperature [7, 22]:

$$T_R \lesssim \frac{1}{\sqrt{|a_g| \alpha}} \left( \frac{m_\phi}{M_*} \right)^{1/2} |\phi_0|.$$  

(27)

From Eqs. (25) and (27) combined with the Q-ball dominance condition in Eq. (10), we obtain the allowed region of the reheating temperature where the relation Eq. (21) holds. We show this region in Fig. 1.

From this figure, we see that there is a wide allowed region for the reheating temperature if $|\phi_0| \gtrsim 10^{15}$ GeV. Note that, even if we use $T_R \gg 10^8$ GeV, there is no "cosmological gravitino problem" [23] as emphasized in Ref. [10]. This is because the large entropy production associated with the decays of Q-balls dilutes the gravitino number density substantially. As long as the reheating temperature and the initial amplitude of the $\phi$ field are within this allowed region, $\Omega_B/\Omega_\chi$ is fixed by the low-energy parameters, and is totally independent of $T_R$ and $|\phi_0|$.

\textsuperscript{11}Here, we have used the fact that the cosmic temperature behaves as $T \simeq (HT_R^2 M_\gamma)^{1/4}$ before the completion of the reheating process of inflation.
Figure 1: The allowed region of reheating temperature for relation (21) to hold. The red (solid) line denotes the upper bound from Eq. (27) with $|a_g| = 1$ and $\sqrt{\alpha} = 1/20$. The purple (dotted) line denotes the upper bound from the thermal mass term, Eq. (25). Here, we take $c = 1$, $f = 10^{-5}$. This constraint becomes significantly weak if we do not adopt the first generation of up-type squark in the flat direction. The blue (dashed) lines represent the lower bounds from the Q-ball dominance condition given in Eq. (10) with $\eta = 0.3$, $|K| = 0.1$ and $\eta = 1$, $|K| = 0.03$ from the bottom up, respectively. Throughout this calculation, we have used $m_{\phi} = 1$ TeV.

5 Conclusions and discussion

In this paper, we have proposed a solution to the $\Omega_B - \Omega_{DM}$ coincidence puzzle. We have pointed out that a class of the AD baryogenesis directly relates the observed mass density of baryons to that of dark matter with the help of the late-time decays of Q-balls, if the LSP has a large s-wave annihilation cross section, as in the case of $\tilde{H}$- or $\tilde{W}$-like LSPs. The relation we have shown is $^{12}$

$$\frac{\Omega_B}{\Omega_\chi} \approx 10^{3-4} \left( \frac{m_{\phi}^2}{\langle \sigma v \rangle} \right) \left( \frac{m_p}{m_\chi} \right) \delta_{\text{eff}}.$$  \hspace{1cm} (28)

A beautiful point of this scenario is that the ratio is totally independent of the reheating temperature of inflation and the initial amplitude of the $\phi$ field, as well as from a detailed

$^{12}$In the case of $m_{3/2} > m_{\phi}$, we should replace $m_{\phi}$ by $m_{3/2}$.
history of the Universe, once the relatively weak constraints shown in Fig. 1 are satisfied. We should stress that the ratio is solely determined by the low-energy parameters, except for the $\mathcal{O}(0.1)$ effective CP phase, which makes this scenario quite testable in the future experiments. Our scenario is free from the “cosmological gravitino problem”, since the large entropy production associated with the Q-ball decay sufficiently dilutes the number density of gravitinos.

In addition to the above beautiful relation, there are many interesting implications of the present scenario. The large $\tilde{H}$ or $\tilde{W}$ component of the LSP significantly enhances its detection rates in both direct and indirect dark matter searches [10]. Furthermore, such non-thermal dark matter may naturally explain the observed excess of positron flux in cosmic rays [24].

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