The Failure of Self-Interacting Dark Matter to solve the Overabundance of Dark Satellites and the Soft Core Question

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\textbf{ABSTRACT}

Self-interacting dark matter was proposed by Spergel & Steinhardt (2000) to alleviate a number of conflicts in Cold Dark Matter (CDM) models: the overabundance of subhalos in the Galactic halo predicted by N-body simulations and constant density cores observed in late-type dwarf and Low Surface Brightness (LSB) galaxies. Using a simple semi-analytical argument we show that a weak self-interacting dark matter capable of reproducing the halo cores of sizes observed in dark matter dominated galaxies, is unable to reconcile the number of satellites in the Galactic halo with the observed number of dwarf galaxies in the Local Group.

\textit{Subject headings:} dark matter–galaxies: satellites - galaxies – cosmology: theory

1. Introduction

Recent improvements in observational and numerical techniques have allowed a comparison between predictions of the CDM scenario and observational data on galactic scales. The results point out discrepancies between predictions and observations. High-resolution N-body simulations have shown that on scales of the Local Group the predicted number of subhalos is at least a factor of ten higher than the observed number of dwarf galaxies (Klypin et al. 1999, Moore et al. 1999a). This disagreement, usually called “the satellite question”, can be attributed to the high core densities of satellite dark halos found in cosmological models (Navarro, Frenk, White 1997). These densities combined with a small central velocity dispersion (Fukushige & Makino 1997) tend to stabilize the satellites against tidal disruption on galactic scales. A lack of a luminous component associated with most subhalos was also found by semi-analytic models of galaxy formation (Kauffmann, White & Guiderdoni 1993). Another discrepancy emerges when comparing the density profiles of dark matter halos predicted by numerical simulations with observations of HI rotation curves in dwarf galaxies (Moore 1994; Flores & Primack 1994; Burkert 1995). Whereas observations show linearly
rising rotation curves out to radii larger than 1 $h^{-1}$ kpc, indicating that the dark matter has a constant density core (soft cores), cosmological simulations predict dark halo density profiles with $\rho \propto r^{-1.5}$ in the central parts (Moore et al. 1999b; Fukushige & Makino 2001). Other N-body simulations seem to converge to halo density profiles described by $\rho \propto r^{-1}$ (Power et al. 2002). Further evidence for density cores comes from the robustness of rapidly rotating bars in high surface brightness spiral galaxies (de Battista & Sellwood 1998).

Each single piece of evidence taken individually may not be sufficient to claim that CDM fails on galactic scales. Results derived from observed density profiles of the inner regions in galaxies are controversial, due to beam smearing effects in HI rotation curves (van den Bosch & Swaters 2001), even though high-resolution observations of Hα observations also show shallower density profiles than those predicted by CDM numerical simulations (e.g. de Blok & McGaugh & Rubin 2001; Marchesini et al. 2002). Several authors have attempted to reconcile the number of observed Local Group dwarf galaxies with the number predicted by CDM theory through conservative solutions within the framework of the current theory. Energetic mechanisms, more efficient in low mass systems, such as feedback from evolving stars as well as heating by ionizing UV background were proposed to explain a decoupling of luminous and dark components for small mass dwarfs (Efstathiou 1992, Bullock et al. 2000, Gelato & Sommer-Larsen 1999, Thacker & Couchman 2000). Another solution proposed by Klypin (1999) suggested an identification of the missing satellites seen in numerical simulations with observed compact high-velocity clouds (Blitz et al. 1999). This solution may be premature, since it is unclear whether the high-velocity clouds are galactic or extragalactic in nature. Comparisons of the dark satellite halos in CDM dominated simulations to the distribution of observed neutral hydrogen high-velocity clouds and compact high-velocity clouds were investigated by Putman & Moore (2001). Recently, Stoehr et al. (2002) and Hayashi et al. (2002) demonstrated that the Galactic satellites could be identified with the most massive subhalos of CDM simulations. This would be in favour of the scenario where baryons are lost preferentially from low-mass halos for yet unknown reasons.

Still, the disagreement between observations and predictions might indicate that a revision to the CDM scenario is required. Self-interacting dark matter was proposed by Spergel & Steinhardt (2000) to overcome the satellite question and the soft core question. In this model, dark matter particles experience weak, not dissipative, collisions on scales of kpc to Mpc for typical galactic densities. These collisions thermalize the inner regions of the dark halos, producing a soft core. In addition, the excess of subhalos predicted by the CDM models should be reduced. The model has attracted great attention. Numerical simulations (e.g. Burkert 2000; Yoshida et al. 2000; Moore et al. 2000; Firmani, D’Onghia & Chincarini 2001; Davé et al. 2001; D’Onghia, Firmani & Chincarini 2002) demonstrated that soft cores would form naturally by energy transport into the cold inner regions. Ostriker (2000) and
Hennawi & Ostriker (2001) pointed out that self-interacting dark matter in a very weak cross section regime in the centers of galaxies reproduces supermassive black hole masses and their observed correlation with the velocity dispersion of the host bulges. However, they point out a possible inconsistency of the collisional scenario: indeed the model would cause an exorbitant growth of supermassive black holes that imposes a very strict upper limit on the collision cross section.

Using an analytical approach, this Letter explores whether a weak self-interacting dark matter is likely to reconcile the apparent overabundance of subhalos with the small number of visible satellites in the Local Group. Two different disrupting processes are explored: collisions and tidal stripping. This work assumes \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \).

2. Disruption by Collisions

For a satellite dwarf galaxy orbiting the Milky Way, we define \( \tau \) to be the time for the dark satellite halo to be destroyed by collisions with the self-interacting dark matter within the halo of the Milky Way:

\[
\tau = \frac{1}{\rho_{MW} \sigma v} \tag{1}
\]

where \( \rho_{MW} \) is the Galactic dark halo density (local density), \( \sigma \equiv \sigma_{si}/m_x \) is the self-interacting cross section per unit mass and \( v \) is the relative velocity satellite-Milky Way. We identify \( v \) as the typical Milky Way halo velocity dispersion and assume that one collision for each particle of the satellite is enough to disrupt the satellite within a Hubble time.

Let us assume a cross section inversely proportional to the halo velocity dispersion. This choice for the cross section produces smaller, less spherical cores in clusters of galaxies and large cores in dwarf galaxies (Yoshida et al. 2000, Firmani et al. 2001, Wyithe, Turner & Spergel 2001), consistent with observations of cluster cores like Cl 0024+1654. It also implies that the product of the cross section times halo dispersion velocity is constant and independent of the mass. Thus, in the satellite the cross section \( \sigma' \) times the satellite dispersion velocity \( v_0 \) has the same value as the product \( \sigma \cdot v \) in the Milky Way.

Let us suppose that the self-interacting dark matter of the satellite is acting with itself in the satellite and at the same time with the Milky Way halo. In the satellite, the effect will be a halo central density decreasing due to the collisions between dark matter particles. The expected number of collisions \( N_{coll} \) per Hubble time \( t_H \) is:

\[
\frac{t_H}{N_{coll}} = \frac{1}{\rho_0 \sigma v}, \tag{2}
\]
Substituting eq.(2) in (1), under the hypothesis that $\sigma \cdot v$ is constant, $\tau$ is given by:

$$\tau = \left( \frac{\rho_0}{\rho_{MW}} \right) \frac{t_H}{N_{coll}}. \quad (3)$$

We assume for the satellite halo central density the same value observed in late-type dwarf galaxies: $\rho_0 = 0.02 \, M_\odot pc^{-3}$ (de Blok, McGaugh & Rubin 2001, Marchesini et al. 2002), since the choice of $\sigma \propto 1/v$ predicts halo central densities independent of the mass. The average collision rate is a function of the halo central density, the cross section and the halo velocity dispersion:

$$\frac{N_{coll}}{t_H} = \rho_0 \sigma v \quad (4)$$

Since in this case $\sigma \cdot v$ is constant, the average collision rate is a function only of the satellite central density: $N_{coll}/t_H \propto \rho_0$. Cosmological N-body simulations in which the cross section is assumed to be inversely proportional to the halo velocity dispersion estimate $N_{coll} \approx 3 - 4$ for each particle in the core in order to reproduce the central densities observed in late-type dwarf galaxies over a Hubble time (D’Onghia, Firmani & Chincarini 2002). The same estimates are found using a dynamical code based on the integration of the Boltzmann equation (Firmani, D’Onghia & Chincarini 2001).

Let us consider a dwarf halo placed at a distance of $25 \, h^{-1} \, kpc$ from the centre of the Galactic halo and assume that the self-interacting dark matter is acting simultaneously on the Galactic halo and the satellite. At this radius the Milky Way density is predicted to be $\rho_{MW} \approx 4 \cdot 10^{-3} \, M_\odot pc^{-3}$ (Moore et al. 2001). Hence $\rho_0$ is nearly 5 times larger than the Galactic halo density $\rho_{MW}$. For $N_{coll} = 4$ the satellite disruption time at $25 \, h^{-1} \, kpc$ is $\tau \approx t_H$. However, already at $30 \, h^{-1} \, kpc$ from the centre, $\rho_0$ is 10 times larger than the Milky Way density, producing $\tau \approx 2t_H$.

In Figure 1 the time required to destroy satellites in the Galactic halo is shown as a function of the distance from the centre of the Milky Way as predicted by our analytic consideration. The filled circles show the disruption time when $\sigma \propto 1/v$ is assumed. Note that, at a distance of $50 \, h^{-1} \, kpc$, 10 Hubble times are required to destroy the satellites, if the self-interaction is working to produce the central density we observe in late-type dwarf galaxies. Thus, for a cross section decreasing with velocity the radius inside which the self-interacting dark matter is effective in disrupting subhalos is within $\sim 25 \, h^{-1} \, kpc$.

Let us analyse now the case in which the cross section is independent of velocity: $\sigma \approx \text{const}$. This case has interesting implications for the supermassive black hole formation. Ostriker (2000) showed that for constant cross sections of unit mass lower than $0.02 \, cm^2 g^{-1}$ a collisional CDM theory promotes at the centre of galaxies the formation of supermassive black holes (SMBH) having the observed scaling relation $M_{BH} \propto v^{4-5}$ (Magorrian et al.
For \( \sigma \approx \text{const} \) eq. (3) becomes:

\[
\tau = \left( \frac{\rho_0}{\rho_{MW}} \right) \left( \frac{t_H}{N_{\text{coll}}} \right) \left( \frac{v_0}{\bar{v}} \right),
\]

with \( v_0 \) the satellite velocity dispersion. In this case, the halo central densities are not independent of the halo mass anymore and \( \rho_0 v_0 = \rho_{MW}^0 \bar{v} \) with \( \rho_{MW}^0 \) the central density of the Milky Way. Since the halo central density times the halo dispersion velocity is constant, the average collision rate is a function only of the cross section: \( N_{\text{coll}}/t_H \propto \sigma \). Cosmological N-body simulations with cross sections independent of velocity were carried out by Davè et al. (2001) who suggest for satellites of \( M = 9 \cdot 10^8 M_\odot \) a central density: \( \rho_0 \approx 4 \cdot 10^{-3} M_\odot \text{pc}^{-3} \), for cross sections per unit mass required to produce soft cores of sizes comparable to the observations of dwarf galaxies (\( \sigma \sim 0.6 \text{ cm}^2 \text{ GeV}^{-1} \)). In high-resolution N-body simulations carried out by Yoshida et al. (2000), the typical number of collisions for each particle in the halo core is \( N_{\text{coll}} \approx 3 - 4 \), for constant cross section with value \( \sigma = 0.1 \text{ cm}^2 \text{ GeV}^{-1} \). Thus, even in this case we assume \( N_{\text{coll}} = 4 \).

For the case of a constant cross section the radius in which the self-interacting dark matter is efficient in destroying subhalos is larger with respect to the previous case. In Figure 1 the disruption time of satellites at different radii from the centre of the Milky Way is shown, when the cross section is assumed to be constant (open circles). Now, for a satellite placed at a distance of 25 \( h^{-1}\text{kpc} \) from the centre the halo central density \( \rho_0 \) is of the same order as the Milky Way density, producing a suppression time much smaller than the Hubble time. Note that for dwarf galaxies placed at a distance larger than 100 \( h^{-1}\text{kpc} \), collisions between dark particles are inefficient in destroying satellites.

Satellite orbits are in general eccentric. As a result, subhalos can be destroyed efficiently when their pericentric distances are within 25 \( h^{-1}\text{kpc} \) or 100 \( h^{-1}\text{kpc} \), depending on whether \( \sigma \) is proportional to \( 1/\bar{v} \) or not. Let us concentrate on the case of cross section decreasing with the halo velocity dispersion, since it is in better agreement with the observed size of soft cores in dwarf galaxies. Using a Monte Carlo method we have computed the pericentric distance distribution of the dark satellites, assuming orbits with the same eccentricity distribution function found for the halo orbits of CDM N-body simulations (Ghigna et al. 1998). These simulations show that orbits of satellites are very eccentric, with pericentric over apocentric distance ratios of \( R_{\text{peri}}/R_{\text{apo}} \approx 0.2 \), whereas observational evidence of dwarf satellite orbits in the Local Group are more circular: \( R_{\text{peri}}/R_{\text{apo}} = 0.5 \) (Schweitzer et al. 1995). Assuming eccentric orbits predicted by our Monte Carlo realization, the chances are high for dark satellites to move through the inner regions with pericentric distances of 25 \( h^{-1}\text{kpc} \) and less and to be destroyed. Our Monte Carlo model assumes orbits with a distribution of semi-major axis as predicted by CDM models. In order to check the validity of our model we have compared the final pericentric distance distribution resulting from our Monte Carlo
realization to that found in CDM models by Font and co-workers (2001). In Figure 2 the masses and pericentric distances of the subhalos within twice the virial radius are shown, resulting from our Monte Carlo realization. The good agreement with the same plot shown in Font et al. (2001) is very encouraging.

What is the percentage of disrupted satellites when the orbital eccentricity distribution is taken into account? In Figure 3 the dashed region shows the cumulative number of dwarf galaxies observed within $250 \, h^{-1}\text{kpc}$ from the Milky Way centre (Grebel 2000)\textsuperscript{1}. The solid line indicates the cumulative number of satellites predicted by CDM models at the same radii\textsuperscript{2}. Note that the N-body simulations predict an excess of subhalos at all radii. The dashed line represents the cumulative number of substructures which should survive collisions, as their pericentric distances are larger than $\approx 25 \, h^{-1}\text{kpc}$. Still too many satellites are found outside of $25 \, h^{-1}\text{kpc}$. It is clear from Figure 3 that, for $\sigma \approx 1/v$, collisional processes are not efficient enough in destroying substructures at any radii.

If the cross section is assumed to be independent of the relative velocity of the particles, then satellites with pericentric distances within $100 \, h^{-1}\text{kpc}$ will be destroyed by collisions. Hence, no satellites should exist at radii smaller than $100 \, h^{-1}\text{kpc}$ in conflict with observations, while the overabundance of satellites is unsolved at larger radii (triangles in Figure 3). Note that we neglect those satellites that are spending a very short time within $100 \, h^{-1}\text{kpc}$. We do not have a good chance to detect these satellites at these pericentric distances.

### 3. Disruption by Tidal Stripping

In a self-interacting scenario the halo central densities are expected to be lower than in CDM models. As a result, tidal stripping should be more rapid and efficient and substructure halos orbiting in the tidal field of the Milky Way are expected to lose continuously mass and to be destroyed as a result of tidal forces. Thus, tidal stripping could be the main process in destroying subhalos in a weak self-interacting scenario. We roughly account of this process assuming that the orbiting satellite is tidally truncated at some radius $r_t$ where the differential tidal force of the Milky Way is equal to the gravitational attraction of the

\textsuperscript{1}The original distances of dwarf galaxies from the Sun in Grebel (2000) are corrected here for galactocentric distances.

\textsuperscript{2}The number of subhalos predicted by Standard CDM models is from B. Moore. High-resolution N-body simulations of ΛCDM models predict the same excess of dark satellites in the Local Group (B. Moore, private communication).
satellite:

$$\frac{m(r_t)}{r_t^3} = \frac{M(R)}{R^3} \left[ 2 - \frac{d\ln M}{d\ln R} \right]$$  \hspace{1cm} (6)

with \(R\) the distance satellite-Milky Way, \(m(r_t)\) the substructure mass within the tidal radius and \(M\) the host galaxy mass. For non circular orbits, the equation is holding taking for \(R\) the pericentric distance (Tormen, Diaferio & Syer 1998). Therefore the tidal radius is such that the mean density of the satellite within \(r_t\) is of the order of the mean density of the main halo at pericentric distance: \(\rho_{MW}(R_{peri}) \approx \rho_{sat}(r_t)\). In the CDM satellites, when the tidally imposed radius approaches a value smaller than the scale radius \(r_s\), substructures become unstable. Using N-body simulations of tidal stripping, the evolution of substructure halos described by a Hernquist or NFW profile within a static host potential have been explored by Mayer et al. (2001) and Hayashi et al. (2002).

In the case of self-interacting scenario, both dark satellites and the Milky Way have lower central densities than CDM counterparts and have core radii \(r_c\). When the tidally imposed radius approaches a value smaller than their core radius \(r_c\), substructures can be tidally stripped. A detailed study of this mechanism requires numerical N-body simulations and is beyond the scope of this work. However we note that, choosing a self-interacting cross section capable of reproducing the central density of 0.02 \(M_\odot pc^{-3}\) and \(r_c \approx 2 h^{-1} kpc\) for dark halos as observed in dwarf galaxies, the condition \(\rho_{MW}(R_{peri}) \approx \rho_{sat}(r_c) \approx 0.02 M_\odot pc^{-3}\) is satisfied for substructures with pericentric distances \(R_{peri} \leq 50 h^{-1} kpc\). Using our Monte Carlo realization for the pericentric distribution we determine the percentage of substructures with \(R_{peri} \leq 50 h^{-1} kpc\) that could be affected by tidal stripping and disrupted. Filled points in Figure 3 show the cumulative number of substructures surviving the disruption by tidal stripping. Note that in a weak self-interacting scenario capable to solve the soft core question, tidal stripping is more efficient than collisions in destroying subhalos with pericentric distances within 50 \(h^{-1} kpc\), reconciling the predictions to the observations at small scales. However, tidal forces are again unable to solve the problem at larger radii.

4. CONCLUSION

In a hierarchical universe, high-resolution N-body simulations of Standard CDM models predict an excess of subhalos with respect to the number of dwarf galaxies observed in the Local Group (Klypin et al. 1999, Moore et al. 1999a).

To solve this conflict between predictions and observations a successful theory should reduce the abundance of substructures at all radii. However, if the ‘satellite question’ remains a problem of CDM models, than our semi-analytical argument proves that self-interacting
dark matter is unable to solve it. If the value of the cross section is chosen such that it reproduces the soft cores with sizes observed in late-type dwarf and LSB galaxies, we can distinguish between two different cases. When the cross section is independent of the relative velocity of the particles, then the radius inside which self-interaction is effective in suppressing the substructure due to collisions is within $100 \, h^{-1} \text{kpc}$. In this case the collisions are more efficient than tidal forces in destroying subhalos, destroying too many satellites within $100 \, h^{-1} \text{kpc}$, in disagreement with the few dwarf spheroidals observed close to the Milky Way. In addition, at radii larger than $100 \, h^{-1} \text{kpc}$ the overabundance of subhalos is still a problem.

If, on the other hand, the cross section is assumed to decrease with the halo velocity dispersion, which is better in agreement with the observed sizes of soft cores in dwarf galaxies, then collisions between particles are effective in disrupting subhalos only within $\approx 25 \, h^{-1} \text{kpc}$ from the Milky Way centre. As a result, only a few percentage of all substructures is destroyed and the overabundance is only slightly reduced, but is unsolved at all radii. In this case, tidal stripping is more efficient than collisions in destroying subhalos within $50 \, h^{-1} \text{kpc}$ from the Galactic centre, reconciling the overabundance to the observed one at small radii. However, the question remains still unsolved at larger distances.

In summary, finding a process that is able to decrease the halo central density seems not to be sufficient in reducing the excess of dark satellite halos, especially for substructures placed at a large distances from the Milky Way centre. Thus, weak self-interaction, which was originally proposed to solve the soft core question in centres of dark matter dominated galaxies and the overabundance of subhalos in the Local Groups fails to solve both questions simultaneously.

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Fig. 1.— The time required to destroy satellites in the Galactic halo by collisions, normalized to the Hubble time, is shown as a function of the distance from the centre of the Milky Way as predicted by our analytic argument. The filled circles show the suppression time if the cross section for self-interacting dark matter is assumed to depend on the halo dispersion velocity: $\sigma \propto 1/v$. The open circles represent the time required to destroy satellites when the cross section is assumed to be velocity independent: $\sigma \approx \text{const.}$
Fig. 2.— Pericentric radii distribution vs. mass as a result of the our Monte Carlo realization for subhalos identified in SCDM cosmological simulations.
Fig. 3.— The abundance of dark satellite halos predicted by CDM N-body simulations at different distances from the centre of the Milky Way halo are represented by the solid line (courtesy B. Moore). The dashed region is the cumulative number of dwarf galaxies observed in the Local Group at different Galactocentric distances (Grebel 2000). The dashed line is the abundance of dark satellite halos, predicted for a cross section dependent on the halo velocity dispersion: $\sigma \propto 1/v$. The filled circles are the cumulative number of subhalos that survive tidal stripping in a self-interacting scenario. Triangles show that if the cross section is assumed to be independent of the relative velocity overkilling happens at radii smaller than $100 \, h^{-1} \, \text{kpc}$, while the overabundance is unsolved at larger radii.