On the formation of inner vacuum gaps in radio pulsars

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ABSTRACT

The problem of formation of the Ruderman-Sutherland type inner vacuum gap in neutron stars with $\Omega \cdot B < 0$ is considered. It is argued by means of the condition $T_i/T_s > 1$ (where $T_i$ is the critical temperature above which $^{56}$Fe ions will not be bound at the surface and $T_s$ is the actual temperature of the polar cap surface heated by the back-flow of relativistic electrons) that the inner vacuum gap can form, provided that the actual surface magnetic field is extremely strong ($B_s > 10^{13}$ G) and curved ($R < 10^6$ cm), irrespective of the value of dipolar component measured from the pulsar spin down rate. Calculations are carried out for pulsars with drifting subpulses and/or periodic intensity modulations, in which the existence of the quasi-steady vacuum gap discharging via $E \times B$ drifting sparks is almost unavoidable. Using different pair-production mechanisms and different estimates of the cohesive energies of surface iron ions, we show that it is easier to form the vacuum gap controlled by the resonant inverse Compton scattering seed photons than by the curvature radiation seed photons.

1. Introduction

The consecutive subpulses in a sequence of single pulses of a number of pulsars change phase systematically between adjacent pulses, forming apparent driftbands of the duration from several to a few tenths of pulse periods. The subpulse intensity is also systematically modulated along driftbands, typically increasing towards the pulse centre. In some pulsars, which can be associated with the central cut of the line-of-sight trajectory, only the periodic intensity modulation is observed, without any systematic change of subpulse phase. On the other hand, the clear subpulse driftbands are found in pulsars associated with grazing line-of-sight trajectories. These characteristics strongly suggest an interpretation of this phenomenon as a system of subpulse-associated beams rotating slowly around the magnetic axis. \[ \text{[hereafter RS75]} \] proposed a natural explanation of the origin of subpulse drift, which involved a number of isolated $E \times B$ drifting sparks discharging the quasi steady

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vacuum gap formed above the polar cap of the pulsar with \( \Omega \cdot \mathbf{B} < 0 \), in which \(^{56}\text{Fe}\) ions were strongly bound at the surface. Although the original idea of RS75 associating the rotating sub-beams with the circulating sparks is still regarded as the best model of drifting subpulse phenomenon, their vacuum gap was later demonstrated to suffer from the so-called binding energy problem ([?](#), for review see)\(^{39,91,um95,xqz99}\). In fact, the cohesive energies of \(^{56}\text{Fe}\) ions used by RS75 proved largely overestimated and the inner vacuum gap envisioned by RS75 was impossible to form. However, it is worth emphasizing that RS75 considered the canonical surface dipolar magnetic fields with values determined from the pulsar spindown rate, although they implicitly assumed small radii of curvature \( R \sim 10^6 \) cm required by the pair creation conditions, which is inconsistent with a purely dipolar field. Recently Gil & Mitra (2001; hereafter GM01) revisited the binding energy problem and found that the formation of a vacuum gap is in principle possible, although it requires an extremely strong non-dipolar surface magnetic field \( B_s = bB_d \), where the coefficient \( b \gg 1 \) in a typical pulsar, \( B_d = 6.4 \times 10^{19}(P\dot{P})^{0.5}G = 2 \times 10^{12}(P\dot{P}_{-15})^{0.5}G \) is the dipolar field at the pole, \( P \) is the pulsar period in seconds, \( \dot{P} \) is the period derivative and \( \dot{P}_{-15} = \dot{P}/10^{-15} \).

In a superstrong surface magnetic field \( B_s > 0.1B_q \), where \( B_q = 4.414 \times 10^{13} \) G, the asymptotic approximation of Erber (1966) used by RS75 in derivation of the height of quasi steady vacuum gap is no longer valid. In fact, in such strong field the high energy \( E_f = \hbar \omega \) photons produce electron-positron pairs at or near the kinematic threshold \( \hbar \omega = 2m\gamma c^2/\sin \theta \), where \( \sin \theta = h/R \), \( h \) is the gap height, and \( R = R_610^6 \) cm is the radius of curvature of surface magnetic field lines ([?](#) dh83), \( \hbar \) is the Planck constant, \( c \) is the speed of light, \( m \) and \( e \) are the electron mass and charge, respectively. The vacuum gap formed under such conditions was called the Near Threshold Vacuum Gap (hereafter NTVG) by GM01. They considered two kinds of high energy seed photons dominating the \( e^-e^+ \) pair production: the Curvature Radiation (CR) photons with energy \( \hbar \omega = (3/2)\hbar \gamma c^2/R \) ([?](#) RS75), and resonant Inverse Compton Scattering (ICS) photons with energy \( \hbar \omega = 2\gamma \hbar eB_s/mc \) (Zhang & Qiao 1996; Zhang et al. 1997), where \( \gamma \) is a typical Lorentz factor of particles within the gap. The corresponding vacuum gap is called the Curvature Radiation dominated (CR-NTVG) and the Inverse Compton Scattering dominated (ICS-NTVG), respectively. GM01 estimated the characteristic heights of both CR-NTVG and ICS-NTVG. In this paper we further refine these estimates by including the general relativistic (GR) effects of inertial frame dragging (IFD) and considering the heat flow conditions within the thin uppermost surface layer of the polar cap. Moreover, we use a broader range of cohesive energies of surface \(^{56}\text{Fe}\) ions. The obtained VG models are applied to pulsars with drifting subpulses and/or periodic intensity modulations, in which the presence of \( \mathbf{E} \times \mathbf{B} \) drifting spark discharges seems almost unavoidable (Deshpande & Rankin 1999, 2001; Vivekanand & Joshi 1999, and Gil & Sendyk 2000).
2. Near threshold vacuum gap formation

If the cohesive energy of $^{56}_{26}$Fe ions is large enough to prevent them from thermionic (this section) or field emission (Appendix A), a vacuum gap forms right above the polar cap with a characteristic radius $r_p = b^{-0.5}10^4P^{-0.5}$ cm (? , e.g.)]. Jessner et al. (2001) pointed out that GR effect of IFD (Muslimov & Tsygan 1992; Muslimov & Harding 1997) should affect the RS75 type models with a vacuum gap above the polar cap. Although Jessner et al. (2001) did not investigate the problem, they implicitly suggested that the electric fields distorted by GR effects make formation of “starved” inner magnetospheric regions even more difficult than in the flat space case. However, Zhang et al. (2000) demonstrated that although GR-IFD effect is small, it nevertheless slightly helps formation of VG above the polar caps. In other words, GR modified potential drop within VG is slightly lower than in the flat space case. Below we confirm this finding for NTVG conditions, with a very strong and complicated surface magnetic field.

The gap electric field $E_\parallel$ along $B_s$ results from a deviation of the local charge density $\rho \approx 0$ from the corotational charge density $\rho_c = (\zeta/\alpha)\rho_{GJ}$, where $\rho_{GJ} = -\Omega \cdot B_s/(2\pi c)$ is the flat space-time Goldreich-Julian (1969) charge density, $\Omega = 2\pi/P$, $\zeta = 1 - \kappa_g$, $\kappa_g \sim (r_g/R)(I/MR^2)$, $\alpha = (1 - r_g/R)^{1/2}$ is the redshift factor, $r_g$ is the gravitational radius, $M$ is the neutron star mass and $I$ is the neutron star moment of inertia. The potential $V$ and electric fields $E_\parallel$ within a gap are determined by GR-analog (Muslimov & Tsygan 1992) of the one dimensional the Poisson equation $(1/\alpha)d^2V/dz^2 = -4\pi(\rho - \rho_c) = 2\Omega B_s\zeta/(c\alpha)$, with a boundary condition $E_\parallel(z = h) = 0$, where $h$ is the height of an infinitesimal gap. The solution of the Poisson equation gives

$$E_\parallel(z) = \zeta(2\Omega \frac{B_s}{c})(h - z),$$

and

$$\Delta V = \zeta \frac{\Omega B_s}{c} h^2.$$  

In further calculations we will adopt a typical value of the correction factor $\zeta \sim 0.85$ (corresponding to $M = 1M_\odot$, $R = 10^6$ cm and $I = 10^{45}$g cm$^2$), although its value can be as low as 0.73 (Zhang et al. 2000). Thus, the potential drop within the actual gap can be 15 to 27 percent lower than in the conventional RS75 model.

The polar cap surface of the pulsar with $\Omega \cdot B < 0$ is heated by a back-flow of relativistic electrons (accelerated in the parallel electric field $E_\parallel$) to the temperature $T_s = k^{1/4}(e\Delta V \dot{N}/\sigma r_p^2)^{1/4}$, where $\Delta V$ is described by equation (2), $\dot{N} = \pi r_p^2 B_s/eP$ is the Goldreich-Julian kinematic flux and the heat flow coefficient $0.1 < k < 1$ is described in Appendix B. The thermal condition for the vacuum gap formation can be written in the form $T_i/T_s > 1$, where $T_i$ is the temperature of the inner polar cap.
where $T_s$ is the actual surface temperature described above, and $T_i = \Delta \varepsilon_c/30k$ is the iron critical temperature above which $^{56}_{26}\text{Fe}$ ions are not bound on the surface (that is, a copious amounts of $^{56}_{26}\text{Fe}$ ions at about Goldreich-Julian density are available for thermionic ejection from the surface), where $\Delta \varepsilon_c$ is the cohesive energy of condensed $^{56}_{26}\text{Fe}$ matter in the neutron star surface and $k = 1.38 \times 10^{-23}\text{JK}^{-1}$ is the Boltzman constant (Cheng & Ruderman 1980; Usov & Melrose 1995). The properties of condensed matter in very strong magnetic fields characteristic of a neutron star surface have been investigated by many authors using different examination methods (?, for review see)um95. There exist many discrepancies in determination of the cohesive energy $\Delta \varepsilon_c$ and in this paper we refer to the two papers, representing the limiting extreme cases. Abrahams & Shapiro (1991; AS91 henceforth) estimated $\Delta \varepsilon_c = 0.91\text{keV}, 2.9\text{keV}$ and $4.9\text{keV}$ for $B_s = 10^{12}\text{G}, 5 \times 10^{13}\text{G}$ and $10^{13}\text{G}$, respectively. These values were approximated by Usov & Melrose (1995) in the form $\Delta \varepsilon_c \simeq (0.9\text{keV})(B_s/10^{12}\text{G})^{0.73}$, which leads to critical temperatures

$$T_i = (3.5 \times 10^5\text{K})(B_s/10^{12}\text{G})^{0.73} = (6 \times 10^5)b^{0.73}(P \dot{P}_{-15})^{0.36}\text{K}. \quad (3)$$

On the other hand, Jones (1986; J86 henceforth) obtained much lower cohesive energies $\Delta \varepsilon_c = 0.29\text{keV}, 0.60\text{keV}$ and $0.92\text{keV}$ for $B_s = 2 \times 10^{12}\text{G}, 5 \times 10^{12}\text{G}$ and $10^{13}\text{G}$, respectively. They can be approximated by $\Delta \varepsilon_c \simeq (0.18\text{keV})(B_s/10^{12}\text{G})^{0.7}$ and converted to critical temperatures

$$T_i = (0.7 \times 10^5\text{K})(B_s/10^{12}\text{G})^{0.7} = (1.2 \times 10^5)b^{0.7}(P \cdot \dot{P}_{-15})^{0.36}\text{K}. \quad (4)$$

Below we consider the condition $T_i/T_s > 1$, using both expressions for critical temperatures described by equations (3) and (4), for CR- and ICS-dominated NTVG models, separately.

### 2.1. CR-NTVG

In this case the gap height $h = h_{CR}$ is determined by the condition that $h = l_{ph}$, where $l_{ph} \approx \sin \theta \mathcal{R} = (B_\perp/B_s)\mathcal{R}$ is the mean free path for pair production by a photon propagating at an angle $\theta$ to the local surface magnetic field (RS75). The CR-NTVG model is described by the following parameters: the height of a quasi steady gap

$$h_{CR} = (3 \times 10^3)\zeta^{-3/7} \mathcal{R}_{6}^{2/7}b^{-3/7} P_{-14}^{3/14} \dot{P}_{-15}^{-3/14} \text{cm}, \quad (5)$$

(notice typographical errors in eq. [6] of GM01, which are corrected here), the gap potential drop

$$\Delta V_{CR} = (1.2 \times 10^{12})\zeta^{1/7} \mathcal{R}_{6}^{4/7}b^{1/7} P_{-14}^{-1/14} \dot{P}_{-15}^{1/14} \text{V}, \quad (6)$$
and the surface temperature

\[ T_s = (3.4 \times 10^6) \zeta^{1/28} k^{1/4} R_0^{1/7} b^{2/7} P^{-1/7} \dot{P}_{-15}^{1/7} \text{K.} \]  

(7)

The thermal condition \( T_i/T_s > 1 \) for the formation of CR-NTVG leads to a family of critical lines on the \( P - \dot{P} \) diagram (see Fig. 1 in GM01)

\[ \dot{P}_{-15} \geq A^2 \zeta^{0.16} k^{1.14} R_0^{0.64} b^{-2} P^{-2.3}, \]  

(8)

where \( A = (2.7 \times 10^3)^{1/2} = 52 \) for AS91 case (eq. [3]) and \( A = (3.96 \times 10^6)^{1/2} = 1990 \) for J86 case (eq. [4]). Alternatively, one can find a minimum required surface magnetic field \( B_s = b B_d \) expressed by the coefficient \( b \) in the form

\[ B_{min}^{CR} = A^2 \zeta^{0.08} k^{0.57} R_0^{0.32} P^{-1.15} \dot{P}_{-15}^{0.5}. \]  

(9)

### 2.2. ICS-NTVG

In this case the gap height \( h = h_{ICS} \) is determined by the condition \( h = l_{ph} \sim l_e \), where \( l_e \) is the mean free path of the electron to emit a photon with energy \( \hbar \omega = 2 \gamma \hbar e B_s / mc \) (? GM01zhm00. The ICS-NTVG model is described by the following parameters: the height of a quasi steady gap

\[ h_{ICS} = (5 \times 10^3) \zeta^{0.14} k^{-0.07} R_0^{0.57} b^{-1} P^{-0.36} \dot{P}_{-15}^{0.5} \text{cm}, \]  

(10)

the gap potential drop

\[ \Delta V_{ICS} = (5.2 \times 10^{12}) \zeta^{0.72} k^{-0.14} R_0^{1.14} b^{-1} P^{-1.22} \dot{P}_{-15}^{0.5} \text{V}, \]  

(11)

and the surface temperature

\[ T_s = (4 \times 10^6) \zeta^{0.18} k^{0.21} R_0^{0.28} P^{-0.43} \text{K.} \]  

(12)

The thermal condition \( T_i/T_s > 1 \) for the formation of ICS-NTVG leads to a family of critical lines on the \( P - \dot{P} \) diagram (see Fig. 1 in GM01)

\[ \dot{P}_{-15} \geq B^2 \zeta^{0.5} k^{0.7} R_0^{0.8} b^{-2} P^{-2.2}, \]  

(13)

where \( B = (2 \times 10^2)^{1/2} = 14 \) for AS91 case (eq. [3]) and \( B = (1.69 \times 10^4)^{1/2} = 130 \) for J86 case (eq. [4]). Alternatively, one can find a minimum required surface magnetic field \( B_s = b B_d \) expressed by the coefficient \( b \) in the form

\[ B_{min}^{ICS} = B \zeta^{0.25} k^{0.34} R_0^{0.39} P^{-1.1} \dot{P}_{-15}^{0.5}. \]  

(14)

A comparison of spark developing time scales in ICS- and CR-dominated vacuum gaps is presented in Appendix C.
2.3. NTVG development in pulsars with drifting subpulses

GM01 examined the $P - \dot{P}$ diagram (their Fig. 1) with 538 pulsars from the Pulsar Catalog (Taylor et al. 1993) with respect to the possibility of VG formation by means of the condition $T_i/T_s$, with $T_i$ determined according to AS91 (eq. [3]). They concluded that formation of VG is in principle possible, although it requires a very strong and curved surface magnetic field $B_s = b \cdot B_d > 10^{13}$ G, irrespective of the value of $B_d$. Here we reexamine this problem by means of equations (9) and (14), with critical temperatures corresponding to both AS91 case ($A = 52$ and $B = 14$) and J86 case ($A = 1990$ and $B = 130$), for CR and ICS seed photons, respectively. Therefore, we cover practically almost the whole range of critical temperatures between the two limiting cases, which differ by a factor of five between each other. We adopt the GR-IFD correction factor $\zeta = 0.85$ (Harding & Muslimov 1998; Zhang et al. 2000), the normalized radius of curvature of surface magnetic field lines $0.01 \leq R_6 \leq 1.0$ (GM01 and references therein) and the heat flow coefficient $0.1 \leq k < 1.0$ (Appendix B).

The results of calculations of NTVG models for 42 pulsars with drifting subpulses and/or periodic intensity modulations (after Rankin 1986) are presented in Fig. 1. We calculated the ratio $B_s/B_q = 0.0453 b (P \dot{P}_{-15})^{0.5}$, where $B_q = 4.414 \times 10^{13}$ G and the coefficient $b$ is determined by equation (9) and equation (14) for CR and ICS seed photons, respectively. Noting that the functional dependence on $P$ and $\dot{P}_{-15}$ in both equations is almost identical, we sorted the pulsar names (shown on the horizontal axes) according to the increasing value of the ratio $B_s/B_q$ (shown on the vertical axes). Calculations were carried out for $B_s/B_q \geq 0.1$, since below this value the near-threshold treatment is no longer relevant. On the other hand, the values of $B_s/B_q$ are limited by the photon splitting threshold, which is roughly about $10^{14}$ G in typical pulsars (see also astro-ph/0102097) bh98,bh01,z01. Therefore, the physically plausible NTVG models lie within the shaded areas, with the upper boundary determined by the photon-splitting level and the lower boundary determined by the lowest ICS line calculated for $k = 0.1$ (see below). The left hand side of Fig. 1 corresponds to AS91 case ($A = 52$ and $B = 14$ in eqs. [9] and [14], respectively) and the right hand side of Fig. 1 corresponds to J86 case ($A = 1930$ and $B = 130$ in eqs. [9] and [14], respectively). Four panels in each side of the figure correspond to different values of the radius of curvature $R_6 = 1.0$, 0.1, 0.05 and 0.01 from top to bottom, respectively (indicated in the upper corner of each panel). Two sets of curved lines in each panel correspond to the CR (thin upper lines) and ICS (thick lower lines) seed photons, respectively. Three different lines within each set correspond to different values of the heat flow coefficient $k = 1.0$ (dotted), $k = 0.6$ (dashed) and $k = 0.2$ (long dashed), and lower boundary of shaded areas corresponds to $k = 0.1$. 
A visual inspection of the model curves within shaded areas in Fig. 1 shows that in AS91 case ICS-NTVG is favored for larger radii of curvature $R_6 > 0.05$, while CR-NTVG requires lower values of $R_6 < 0.1$. In J86 case, only ICS-NTVG corresponding to $R_6 < 0.1$ can develop. If the actual cohesive energies correspond to some intermediate case between AS91 and J86 cases, they will also be associated with ICS-NTVG. Therefore, we can generally conclude that ICS-NTVG model is apparently favored in pulsars with drifting subpulses. CR-NTVG is also possible, although it requires a relatively low cohesive energies, as well as an extremely strong ($B_s/B_q \sim 1$) and/or curved ($R_6 \sim 0.01$) surface magnetic field at the polar cap.

3. Conclusions

There is a growing evidence that the radio emission of pulsars with systematically drifting subpulses (grazing cuts of the line-of-sight) or periodic intensity modulations (central cuts of the line-of-sight) is based on the inner vacuum gap developed just above the polar cap (Deshpande & Rankin 1999, 2001; Vivekanand & Joshi 1999; Gil & Sendyk 2000). To overcome the binding energy problem Xu et al. (1999, 2001) put forward an attractive but exotic conjecture that pulsars showing the drifting subpulses represent bare polar cap strange stars (BPCSS) rather than neutron stars. However, as demonstrated in this paper, invoking the BPCSS conjecture is not necessary to explain the drifting subpulse phenomenon. The quasi steady vacuum gap, with either curvature radiation or inverse Compton scattering seed photons, can form in pulsars with $\Omega \cdot B < 0$, provided that the actual surface magnetic field at the polar cap is extremely strong $B_s \sim 10^{13}$ G and curved $R < 10^6$ cm, irrespective of the value of dipolar component measured from the pulsar spindown rate. We have used two sets of the cohesive (bounding) energies of the surface iron ions: higher values obtained by AS91 and about five times lower values obtained by J86. If the actual cohesive energies are close to J86 values, then only ICS controlled VG can form and CR controlled VG never forms even under the most extreme conditions (see Fig. 1).

It is worth noting that ICS discussed in this paper is the so-called ”resonant” scattering. The choice of this mode of ICS is justified by the superstrong magnetic fields relevant for the near-threshold regime. However, it is clear that the thermal photons also contribute some pair-producing gamma-rays ($\gamma$, e.g.)zetal97. This process cannot dominate the breakdown of the gap, since it requires higher surface temperatures, which may inhibit the formation of VG in the first place. One can only speculate that the dominating thermal ICS mode is associated with the pulse nulling phenomenon. These issues require further investigation.

The pulsars with drifting subpulses and/or periodic intensity modulation do not seem
to occupy any particular region of the $P - \dot{P}$ diagram (see Fig. 1 in GM01). Rather, they are spread uniformly all over the $P - \dot{P}$ space, at least for typical pulsars (excluding young and millisecond pulsars, in which observations of single pulses are difficult in the first place). Therefore, it seems tempting to propose that radio emission of all pulsars should be driven by vacuum gap activities. An attractive property of such proposition is that the nonstationary sparking discharges induce the two-stream instabilities that develop at relatively low altitudes (Usov 1987; Asseo & Melikidze 1998) where the pulsars radio emission is expected to originate (Cordes 1978; Kijak & Gil 1997, 1998). It is generally believed that the high frequency plasma waves generated by the two-stream instabilities can be converted into coherent electromagnetic radiation at pulsar radio wavelengths (?, e.g.)]mgp00. With such scenario, all radio pulsars would require a strong, non-dipolar surface magnetic field at their polar caps (e.g. Gil et al. 2002 a, b).

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A. Field emission

The vacuum gap can form in pulsars with $\Omega \cdot B < 0$ if the actual surface temperature $T_s$ is not high enough to liberate $^{56}\text{Fe}$ ions from the polar cap surface by means of thermal emission. Now we examine the field (cold cathode) emission, which is possibly important when thermionic emission is negligible (?, e.g.)zh00. The maximum electric field (along $B_s$) at the NS surface $E_{\parallel}(max) = \zeta(4\pi/eP)B_s h_{ICS} = (1.25 \times 10^9)\zeta^{-0.66}b^{-1}R_6^{0.57}P_0^{0.14}$ V/cm, or taking into account that $\zeta = 0.85$ and $b \sim 2(P \cdot \dot{P}_{-15})^{-0.5}$, $E_{\parallel}(max) \leq (5 \times 10^8)R_6^{0.57}P_0^{0.64}\dot{P}_{-15}^{0.5}$ V/cm. The critical electric field needed to pull $^{56}\text{Fe}$ ions from the NS surface is $E_{\parallel}(crit) = (8 \times 10^{12})(\Delta \epsilon_c/26 \text{keV})^{3/2}$ V/cm (Abrahams & Shapiro 1991; Usov & Melrose 1995, 1996), where the cohesive (binding) energy of iron ions in a strong surface magnetic field $B_s \sim 10^{13}$ G is $\Delta \epsilon_c = 4.85$ keV (Abrahams & Shapiro 1991). Thus, $E_{\parallel}(crit) = 6.4 \times 10^{11}$ V/cm $> E_{\parallel} \sim 10^{9}$ V/cm and no field emission occurs. It is possible that the cohesive energy is largely over-estimated and $\Delta \epsilon_c$ can be much smaller than about 4 keV even at $B_s > 10^{13}$ G. We can thus ask about the minimum value of $\Delta \epsilon_c$ at which the field emission is still negligible. By direct comparison of $E_{\parallel}(max)$ and $E_{\parallel}(crit)$ we obtain $\Delta \epsilon_c > 40x^{0.67}$ eV, where $x = R_6^{0.57}P_0^{0.64}\dot{P}_{-15}^{0.5}$ is of the order of unity. Therefore, contrary to the conclusion of Jessner et al. (2001), no field emission is expected under any circumstances.
B. Heat flow conditions at the polar cap surface

Let us consider the heat flow conditions within the uppermost surface layer of the pulsar polar cap above which NTVG can develop. The basic heat flow equation is (\(\frac{\partial T}{\partial t}\)) = \(C\frac{\partial T}{\partial x}\) (\(\kappa\)),

\[C\frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(\kappa \frac{\partial T}{\partial x}\right), \tag{B1}\]

where \(C\) is the heat capacity (per unit volume) and \(\kappa|| \gg \kappa\perp \approx 0\) is the thermal conductivity along (\(||\)) surface magnetic field lines (which are assumed to be perpendicular (\(\perp\)) to the polar cap surface). The heating of the surface layer of thickness \(\Delta x\) is due to sparking avalanche with a characteristic development time scale \(\Delta t\). We can write approximately \(\partial T/\partial t \approx T/\Delta t\), \(\partial T/\partial x \approx T/\Delta x\), \(\partial x/\partial x(\partial T/\partial x) \approx T/\Delta x^2\) and thus \(C/\Delta t \approx \kappa|| \Delta x^2\). Therefore, the crust thickness \(\Delta x\) heated during \(\Delta t\) is approximately \(\Delta x \approx (\kappa|| \Delta t/C)^{\frac{1}{2}}\) (within an uncertainty factor \(\delta x\) of 2 or so), where the time scale corresponds to the spark development time \(\Delta t \approx 10\mu\text{s}\) (RS75). The energy balance equation is \(Q_{\text{heat}} = Q_{\text{rad}} + Q_{\text{cond}}\), where \(Q_{\text{heat}} = e\eta G J V_{\text{max}}\) (e.g. RS75), \(Q_{\text{rad}} = \sigma T^4\) \(\left(T_s\right)\) is the actual surface temperature and \(\sigma = 5.67 \times 10^{-5}\) erg cm\(^{-2}\)K\(^{-4}\)s\(^{-1}\), and \(Q_{\text{cond}} = -\kappa|| \partial T/\partial x \approx -\kappa|| T_s/\Delta x\). We can now define the heat flow coefficient \(k = Q_{\text{rad}}/Q_{\text{heat}} < 1\), which describes deviations from the black-body conditions on the surface of the sparking polar cap. In other words, the value of \(k\) describes the amount of heat conducted beneath the polar cap which cannot be transferred back to the surface from the penetration depth \(\Delta x\) during the time-scale \(\Delta t\) (see Appendix C). The coefficient \(k\) can be written in the form

\[k = \frac{1}{1 + \kappa|| / (\sigma T^3_\text{s} \Delta x)} = \frac{1}{1 + (\kappa|| / C)^{1/2} / (\sigma T^3_\text{s} \Delta t^{1/2})}, \tag{B2}\]

whose value can be estimated once the parameters \(C\), \(\kappa||\) and \(T_s\) as well as \(\Delta x\) or \(\Delta t\) are known.

The matter density at the neutron star surface composed mainly of \(^{56}\text{Fe}\) ions (\(?\), e.g.) is \(\rho(B_s) \approx 4 \times 10^3(B_s/10^{12}\ \text{G})^{6/5}\) g cm\(^{-3}\) (Flowers et al. 1977). Thus for \(B_s \approx (1 \div 3) \times 10^{13}\) G we have \(\rho \approx (0.6 \div 2.4) \times 10^5\) g cm\(^{-3}\). The thermal energy density of \(^{56}\text{Fe}\) is \(U_{Fe} \approx 2.2 \times 10^{19} \rho_5 T_s\) erg cm\(^{-3}\) (Eichler & Cheng 1989), where \(\rho_5 = \rho/(10^5\) g cm\(^{-3}\)) and \(T_s = T_\text{s}/(10^6\) K). Thus, the heat capacity \(C = dU_{Fe}/dT = 10^{-8} dE_{Fe}/dT_s = (1 \div 5) \times 10^{11} \rho_5\) erg cm\(^{-3}\)K\(^{-1}\). For \(\rho_5 \approx 1\) we have \(C \approx 2 \times 10^{14}\) erg cm\(^{-3}\)K\(^{-1}\). The longitudinal thermal conductivity can be estimated as \(\kappa\perp \approx (2 \div 4) \times 10^{13}\) erg s\(^{-1}\)cm\(^{-1}\)K\(^{-1}\) (\(?\), see Fig. 5 in)\(p99\). Thus \(\kappa||/C \approx 1.5 \times 10^2\) cm\(^2\) and the penetration depth \(\Delta x \approx 10 \Delta t^{1/2}\) s\(^{-1/2}\) cm. Since the characteristic spark development time scale \(\Delta t \approx 10\mu\text{s} = 10^{-5}\) s (see Appendix C), then \(\Delta x \approx 0.03\) cm. More generally, the penetration depth can be written as

\[\Delta x = 0.03 \delta x \left(\frac{\kappa_{13}}{C_{11}}\right)^{1/2}, \tag{B3}\]
where $\kappa_{13} = \kappa_{1}/10^{13} \approx 2/4$ and $C_{11} = C/10^{11} \approx 1/5$ and the uncertainty factor $\delta x \approx 0.5/2$. Now the heat flow coefficient can be written as

$$k = \frac{1}{1 + 5.6\Delta/T_{6}^{3}}, \quad (B4)$$

where $T_{6} = T_{s}/10^{6}$ and $\Delta = (\kappa_{13}C_{11})^{1/2}/\delta x \approx 0.7 \div 6.3$. Figure 2 shows variations of the heat flow coefficient $k$ versus the surface temperature $T_{6}$ (in $10^{6}$ K) for three values of $\Delta = 1$ (upper curve), $\Delta = 3$ (middle curve) and $\Delta = 6$ (lower curve). As one can see, for realistic surface temperatures of a few times $10^{6}$ K, the values of the heat flow coefficient $k$ are in the range $0.2 \div 0.9$. Gil et al. (2002a) found from independent considerations that in PSR B0943+10 the heat flow coefficient $k < 0.8$, in consistency with the results obtained in this paper.

The above treatment of the heat transportation corresponds to one spark event or a relatively short sequence of sparks reappearing at the same place of the polar cap (modulo the slow $E \times B$ drift). If a longer sequence of consecutive sparks can operate at the same place, deep-layer photons can also diffuse out of the surface, although after a considerably longer time (Eichler & Cheng 1989). The contribution from this component would make $k$ closer to unity. Even if this is the case, ICS driven VG is still possible to form for all pulsars from our sample, although an extremely strong and curved surface magnetic field would be required (the right hand-side panels of Fig. 1).

## C. Characteristic spark development time scale

The characteristic spark development time scale is defined as the time interval after which the density of electron-positron spark plasma grows from essentially zero to the Goldreich-Julian (1969) value. In the case of CR-induced sparks this time scale is

$$\tau_{CR} = (30 - 40)h_{CR}/c \approx 10\mu s.$$  

The time structure of the ICS-induced spark avalanche may be different. This follows from the fact that in the CR-induced avalanche the mean free path $l_{e}$ of electron/positron to generate one pair-producing gamma photon is much smaller than the mean free path $l_{ph}$ of gamma photon to produce an electron-positron pair, while in the ICS-induced avalanche $l_{e}$ is comparable with $l_{ph}$ (Zhang et al. 2000). Let us introduce two time scales: the time $t_{p} = l_{e}/c$ for a charged particle (electron or positron) to emit a gamma-quantum, and the time $t_{q} = l_{ph}/c$ for a gamma-quantum to create an electron-positron pair. In both CR- and ICS-induced gap discharges the condition $t_{p} \leq t_{q}$ holds ($t_{p} \ll t_{q}$ in the CR case and $t_{p} \sim t_{q}$ in the ICS case), which suggests that the avalanche time scale is determined by $t_{q}$. In fact, one can assume that the cascade consists of a sequence of $n(t) \approx t/t_{q}$ generations, occurring between the initial time $t = 0$
and the current time \( t \). Thus, the number of particles created during the n-th generation is \( N(t) = (2t_q/t_p)^n(t) = (2t_q/t_p)^{t/t_\tau} \). This estimate represents the following scheme of generation-by-generation cascade development: \((2t_q/t_p) \rightarrow (2t_q/t_p)^2 \rightarrow ... \rightarrow (2t_q/t_p)^{t/t_\tau}\). For the ICS-induced cascade we have \( t_q \approx t_q \approx h_{ICS}/c \), while for the CR-induced case \( t_q = h_{CR}/c \) and \( t_p \approx 2\pi \nu_m h/L_{CR} \approx 3 \times 10^{-3} R_6/\gamma \), where \( L_{CR} = (2/3)e^2\gamma^4/R \) is the power of the curvature radiation at the characteristic frequency \( \nu_m = 0.4\gamma^3 c/R \approx 2 \times 10^9 \gamma^3/R_s^{-1}, \gamma = e\Delta V/m c^2 = 10^7 \zeta^{0.14} R_6^{0.57} b^{0.14} \) is the Lorentz factor of relativistic charged particles, \( \Delta V = \Delta V_{CR} \) is the NTVG potential drop described by eq.(6), and \( R = R_610^6 \) cm is the radius of curvature of the surface magnetic field \( B_s = bB_d \) (we assumed \( P = \dot{P}_{-15} = 1 \) for simplicity). Here \( e \) is the electron charge, \( c \) is the speed of light and \( h \) is the Planck constant.

The total number of particles created during the time interval \( t \) is \( N = \int_0^t N(x)dx \), where \( x = t/t_q \). Thus, for the CR-induced case in which \( t_q/t_p \approx h_{CR}/(t_p c) \)

\[
N_{CR} \sim \left( \frac{2 h_{CR}}{t_p c} \right)^{\frac{t}{t_{CR}}} \ln \left( \frac{2 h_{CR}}{t_p c} \right), \tag{C1}
\]

and for the ICS-induced case in which \( t_q/t_p \approx 1 \)

\[
N_{ICS} \sim \left( \frac{2 h_{ICS}}{t_p c} \right)^{\frac{t}{t_{ICS}}} \ln \frac{2 h_{ICS}}{t_p c}, \tag{C2}
\]

where \( h_{CR} \) and \( h_{ICS} \) are described in eqs.(5) and (10), respectively.

The cascade terminates at the time \( t = \tau \) when the spark plasma density reaches the Goldreich-Julian density \( n_{GJ} \), that is \( N_{ICS}/h_{ICS}^3 = N_{CR}/h_{CR}^3 \approx n_{GJ} \). Using this condition and eqs.(C1) and (C2) we obtain the ICS-induced spark development time scale \( \tau_{ICS} \) as a function of CR-induced spark development time scale \( \tau_{CR} \) in the form

\[
\tau_{ICS} = \frac{h_{ICS}}{h_{CR} \ln 2} \ln \left( \frac{2 h_{CR}}{t_p c} \right) \tau_{CR} + \frac{h_{ICS}}{c \ln 2} \ln \left( \frac{\ln 2}{\ln \left( \frac{2 h_{CR}}{t_p c} \right)} \right) \left( \frac{h_{ICS}}{h_{CR}} \right)^3. \tag{C3}
\]

This time scale depends also on other pulsar parameters, mostly on \( b \) and \( R_6 \) (and weakly on \( P, \dot{P}, k \) and \( \zeta \)). The second term in eq. (C3) is always negative and much smaller than the first term (for \( \tau_{CR} > 20h_{CR}/c \); e.g. RS75). Therefore we obtain an upper limit for the ratio

\[
\frac{\tau_{ICS}}{\tau_{CR}} = 2.4 \zeta^{0.57} b^{0.07} R_6^{0.28} b^{-0.79} \left( \ln \frac{6.67 \times 10^2}{\zeta^{0.28} R_6^{0.14} b^{0.07}} \right). \tag{C4}
\]

Fig. 3 shows the upper limits (equality in eq. [C4]) of the ratio \( \tau_{ICS}/\tau_{CR} \) as a function of \( B_s/B_q = 0.0453b \) (see section 2.3 and Fig. 1 for explanation), for different values of \( R_6 \).
and $k$ (for the clarity of presentation we adopted $P = \dot{P}_{-15} = 1$ and $\zeta = 0.85$). As one can see from this figure, the characteristic spark development time scales of CR- and ICS-induced discharges are comparable. This is a consequence of the fact that the stronger the magnetic field the smaller the $l_e$ for ICS, thus the larger the pair-production multiplicity and the smaller the spark development time scale $\tau_{ICS}$. We can thus generally state that in the near-threshold regime ICS-driven spark avalanche looks very similar to the conventional RS75 CR-driven avalanche.

**REFERENCES**


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Figure Captions

Fig. 1.— Models of NTVG for 42 pulsar with drifting subpulses and/or periodic intensity modulations for AS91 and J86 cohesive energy calculations. Both curvature radiation (CR) and resonant inverse Compton radiation (ICS) seed photons generating electron-positron pairs in a superstrong surface magnetic field $B_s$ are considered. The ratio of $B_s/B_q \geq 0.1$ (near threshold treatment), where $B_s$ is the actual surface magnetic field and $B_q = 4.4 \times 10^{13}$ G is the quantum critical magnetic field, is presented on the vertical axis. The horizontal axis corresponds to 42 pulsars with drifting subpulses and/or periodic intensity modulations. Three different lines correspond to different values of the heat flow coefficient $k = 1.0$ (dotted), $k = 0.6$ (dashed) and $k = 0.2$ (long dashed). Four different panels correspond to different values of curvature radii from $R_6 = 1.0$ (top) to $R_6 = 0.01$ (bottom). The physically plausible models correspond to shaded areas limited by the photon splitting threshold $B_s \sim 10^{14}$ G from above and by the lowest ICS curve (calculated for $k = 0.1$) from below.

Fig. 2.— Heat flow coefficient $k$ versus surface temperature $T_6 = T_s/10^6$ K (see text for explanations).

Fig. 3.— Upper limits for the ratio of development time scales in ICS- and CR- induced spark discharges of NTVG, as a function of the surface magnetic field $B_s/B_q$, ranging from 0.1 (near threshold treatment) to 2.25 (photon splitting limit). Four values of $R_6 = 1.0$, 0.1, 0.05 and 0.01 (from the top to the bottom) and two values of $k = 1.0$ (solid line) and 0.2 (dashed line) were used.
Surface temperature $T_6$ vs. Heat flow coefficient $k$
\[ \frac{\tau_{ICS}}{\tau_{CR}} \] vs. \[ \frac{B_s}{B_q} \]