Exact Standard Model Compactifications from Intersecting Branes

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ABSTRACT

We construct six stack D6-brane vacua (non-supersymmetric) that have at low energy exactly the standard model (with right handed neutrinos). The construction is based on D6-branes intersecting at angles in $D = 4$ type toroidal orientifolds of type I strings. Three $U(1)$'s become massive through their couplings to RR fields and from the three surviving massless $U(1)$'s at low energies, one is the standard model hypercharge generator. The two extra massless $U(1)$'s get broken, as suggested recently (hep-th/0205147), by requiring some intersections to respect $N = 1$ supersymmetry thus supporting the appearance of massless charged singlets. Proton and lepton number are gauged symmetries and their anomalies are cancelled through a generalized Green-Schwarz mechanism that gives masses to the corresponding gauge bosons through couplings to RR fields. Thus proton is stable and neutrinos are of Dirac type with small masses as a result of a PQ like-symmetry. The models predict the existence of only two supersymmetric particles, superpartners of $\nu_R$'s.
1 Introduction

Obtaining chiral string constructions where the low energy particle content may be the observable chiral spectrum of the standard model (SM) and gauge interactions, is one of the important directions of string theory research. In this respect semirealistic models have been pursued along those directions, both in the context of 4D $N = 1$ heterotic compactifications and in orientifold constructions [1].

In this work, we will examine standard model compactifications in the context of recent constructions [2, 3], which use intersecting branes and give 4D non-SUSY models. So why we have to resort to non-supersymmetric models in our search for realistic string models? In $N = 1$ heterotic orbifold compactifications (HOC) the semirealistic models derived were supersymmetric (SUSY) and included at low energy the MSSM particle content, together with a variety of exotic matter and/or gauge group factors. However, in order to reconcile the observed discrepancy between the unification of gauge couplings in the MSSM at $10^{16}$ GeV [4] and the string scale at HOC which is of order $10^{17} - 10^{18}$ GeV, it was assumed that the observed difference should be attributed to the presence of the string one loop corrections to the $N = 1$ gauge coupling constants [5]. Thus even though it was not possible for a single model to be found which realized at low energy only the MSSM it was assumed that this will become possible after an extensive study of the different compactification vacua was performed. However, the goal of obtaining a particular string compactification with only the MSSM content was not realized. On the other hand in type I compactifications (IC) the string scale is a free parameter. In addition, recent results suggest that it is possible in IC to lower the string scale, by having some compact directions transverse to all stacks of branes, in the TeV region even without SUSY [6]. Thus non-SUSY models with a string scale in the TeV region provide us with a viable alternative to SUSY models.

For models based on intersecting branes the main picture involves localization of fermions in the intersections between branes [7]. In these constructions the introduction of a quantized NS-NS B field [8] effectively produces semirealistic models with three generations [3]. Under T-duality these backgrounds transform to models with magnetic deformations [9, 10]. For additional developments with non-SUSY constructions in the context of intersecting branes, see [11, 12, 13, 14, 15, 16, 17, 18]. For a $N = 1$ SUSY construction in the context of intersecting branes and its phenomenology see [19].

Recently, an interesting class of models was found that uses four-stacks of branes
and gives at low energies exactly the SM content at low energies [20]. The models were based on D6-branes intersecting at angles on an orientifolded six-torus compactification [2, 3]. These models have an interesting generalization to classes of models with five-stacks of branes [21]. The features of the latter models are quite similar to those that constitute the main part of this work. The models of [20, 21] share some common features as proton and lepton number are gauged symmetries surviving as global symmetries to low energies, small neutrino masses and a remarkable Higgs sector that in cases is of the MSSM. On the contrary, while the four stack model is a non-SUSY model and has a variety of sectors with non-SUSY chiral fields, models that have five-stacks have one additional unusual feature. They have some sectors that preserve $N = 1$ SUSY, thus a $N = 1$ hypermupliplet remains initially massless $^1$, even though the model is overall a non-SUSY one. The latter feature, as we will see, is maintained in the six-stack classes of models presented in this work.

The models of [20] have been extended to describe the first examples of classes of GUT structured models, that are based on the Pati-Salam gauge group $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ - making use of four-stacks of D6 branes, and giving exactly the SM at low energies [22]. These classes of models maintain essential features of [20, 21] and in particular the fact that proton is stable, as the baryon number is an unbroken gauged symmetry, and small neutrino masses. Also the GUT four-stack classes of models share the unusual features of the five-, six-stack classes of SM’s of [21, 22] respectively, that is even though the models are non-SUSY they do allow for some sectors to preserve $N = 1$ SUSY. It is quite interesting to note that, even though it was generally believed that in D6 brane orientifolded six torus models it was not possible to find an apparent explanation for lowering the string scale in the TeV region $^2$, in the classes of GUT models of [22] this issue was solved. In particular, the models predict the existence of light weak doublets with mass of order $v^2/M_s$, that necessarily needs the string scale to be less than 650 GeV. The latter results are particularly encouraging as they represent strong predictions in the context of intersecting D-brane scenarios and are directly testable at present or future accelerators.

In this work, we will discuss compactifications of intersecting D6 branes, with exactly the SM at low energies that use six-stacks of D-branes on an orientifolded six-torus. Thus we practically extending the models of [21] by one more $U(1)$ stack. The classes of SM’s that we discuss in this work possess some general features:

$^1$with the scalar component receiving eventually a mass

$^2$since there were no torus directions transverse to all branes.
• Even if the classes of models are non-SUSY overall they have sectors that preserve $N = 1$ supersymmetry. This unusual feature has never appeared in the context of string theory before. It appears that the imposition of SUSY at particular sectors, of the non-SUSY models, creates the necessary singlets that break the two extra $^3 U(1)$’s symmetries leaving at low energy exactly the SM.

• Baryon number (B) is an unbroken gauged $U(1)$ symmetry, with the corresponding gauge boson to receive a mass and the baryon number to survive as a global symmetry to low energies. Thus proton is stable.

• Lepton number (L) is a gauged $U(1)$ symmetry, thus neutrinos have Dirac masses. Lepton number remains as a global symmetry to low energies. The small masses for the neutrinos will come from the existence of chiral condensates breaking the PQ-like symmetry $^4 U(1)_b$. The gauge bosons corresponding to the gauging of B, L get a mass through their couplings to RR fields that are being involved in a generalized Green-Schwarz mechanism.

The paper is organized as follows. In the next section we describe the rules for constructing the six-stack models for the six-torus orientifolded, with D6 branes intersecting at angles, constructions. We present the representation content and the general solutions to RR tadpole cancellation conditions for the classes of models giving rise exactly to the SM at low energies. In section 3 we describe the cancellation of the mixed $U(1)$ gauge anomalies by a dimensional reduction scheme which is equivalent to cancellation of the field theory anomaly by its Green-Schwarz amplitude [20]. This mechanism has been used in the context of toroidal models with branes at angles in [13, 20]. In section 4 we describe the electroweak Higgs sector of the models giving our emphasis on the definition of the geometrical quantities that characterize the geometry of the Higgs sector of the model. In section 5 we describe the remarkable effect of how by imposing the condition that $N = 1$ SUSY may be preserved by some sectors breaks the gauge symmetry to the SM itself. In section 6, we present a case by case analysis of the possible Higgs fields realized in the models as well describing the problem of neutrino masses. Our conclusions together with some comments are presented in section 7.

$^3$Beyond the SM gauge symmetries.

$^4$The same PS like symmetry was responsible for giving small neutrino masses in the models of [20, 21].
2 Exact Standard model compactifications from Intersecting branes

The formalism that we will make use in this work is based on type I strings with D9-branes compactified on a six-dimensional orientifolded torus $T^6$, where internal background gauge fluxes on the branes are turned on [2, 3]. By performing a T-duality transformation on the $x^4, x^5, x^6$, directions the D9-branes with fluxes are translated into D6-branes intersecting at angles. Also in this framework we note that the branes are not paralled to the orientifold planes. Under the T-duality the $\Omega$ symmetry, where $\Omega$ is the worldvolume parity is transformed into $\Omega R$, where $R$ is the reflection on the T-dualized coordinates,

$$T(\Omega R)T^{-1} = \Omega R,$$  \hspace{1cm} (2.1)

We assume that the D6$_a$-branes wrap 1-cycles $(n^i_a, m^i_a)$, $i = 1, 2, 3$ along each of the $i$th-$T^2$ torus of the factorized $T^6$ torus, namely $T^6 = T^2 \times T^2 \times T^2$. Thus we allow the six-torus to wrap factorized 3-cycles, so we can unwrap the 3-cycle into products of three 1-cycles, one for each $T^2$. Defining the homology of the 3-cycles as

$$[\Pi_a] = \prod_{i=1}^{3} (n^i_a[a_i] + m^i_a[b_i])$$  \hspace{1cm} (2.2)

defines consequently the 3-cycle of the orientifold images as

$$[\Pi_a^*] = \prod_{i=1}^{3} (n^i_a[a_i] - m^i_a[b_i])$$  \hspace{1cm} (2.3)

We note that in the presence of $\Omega R$ symmetry, each D6$_a$-brane 1-cycle, must be accompanied by its $\Omega R$ orientifold image partner $(n^i_a, -m^i_a)$.

The six-stack SM model structure that we consider in this work is based on the stack structure $U(3) \otimes U(2) \otimes U(1)_c \otimes U(1)_d \otimes U(1)_e \otimes U(1)_f$ or $SU(3) \otimes SU(2) \otimes U(1)_a \otimes U(1)_b \otimes U(1)_c \otimes U(1)_d \otimes U(1)_e \otimes U(1)_f$ at the string scale.

In addition, the presence of discrete NS B-flux [8] is assumed. Thus, when the B-flux is present, the tori involved are not orthogonal but tilted. In this way the wrapping numbers become the effective tilted wrapping numbers,

$$(n^i, m = \tilde{m}^i + n^i/2); \ n, \tilde{m} \in Z.$$  \hspace{1cm} (2.4)

Thus we allow semi-integer values for the m-wrapping numbers.
The chiral sector is computed from a number of different sectors. As usual in these constructions the chiral fermions get localized in the intersections between branes. The possible sectors are:

- The $ab + ba$ sector: involves open strings stretching between the D6$_a$ and D6$_b$ branes. Under the $\Omega R$ symmetry this sector is mapped to $a^*b^* + b^*a^*$ sector. The number, $I_{ab}$, of chiral fermions in this sector, transform in the bifundamental representation $(N_a, \bar{N}_a)$ of $U(N_a) \times U(N_b)$, and reads
  \begin{equation}
  I_{ab} = [\Pi_a] \cdot [\Pi_b] = (n_a^1m_b^1 - m_a^1n_b^1)(n_a^2m_b^2 - m_a^2n_b^2)(n_a^3m_b^3 - m_a^3n_b^3),
  \end{equation}
  where $I_{ab}$ is the intersection number of the wrapped cycles. Note that we denote the chirality of the fermions as being associated to the sign of $I_{ab}$ intersection, where $I_{ab} > 0$ denotes left handed fermions. Moreover, with negative multiplicity we denote the opposite chirality.

- The $ab^* + b^*a$ sector: It involves chiral fermions transforming into the $(N_a, N_b)$ representation with multiplicity given by
  \begin{equation}
  I_{ab^*} = [\Pi_a] \cdot [\Pi_{a^*}] = -(n_a^1m_b^1 + m_a^1n_b^1)(n_a^2m_b^2 + m_a^2n_b^2)(n_a^3m_b^3 + m_a^3n_b^3).
  \end{equation}
  The $\Omega R$ symmetry transforms this sector to itself.

- The $aa^*$ sector: under the $\Omega R$ symmetry it transforms to itself. In this sector the invariant intersections will give $8m_a^1m_a^2m_a^3$ fermions in the antisymmetric representation and the non-invariant intersections that come in pairs provide us with $4m_a^1m_a^2m_a^3(n_a^1n_a^2n_a^3 - 1)$ additional fermions in the symmetric and antisymmetric representation of the $U(N_a)$ gauge group. As it will be explained later, these sectors will be absent from our models.

Any vacuum derived from the previous intersection constraints is subject in addition to constraints coming from RR tadpole cancellation conditions [2, 3]. That demands cancellation of D6-branes charges
\footnote{We associate the action of $\Omega R$ on a sector $a, b$, as being associated to its images $a^*, b^*$, respectively.}, wrapping on three cycles with homology $[\Pi_a]$ and the O6-plane 7-form charges wrapping on 3-cycles with homology $[\Pi_{O_6}]$. Note that the RR tadpole cancellation conditions can be expressed in terms of cancellations of RR charges in homology as
\begin{equation}
\sum_a N_a[\Pi_a] + \sum_{a^*} N_{a^*}[\Pi_{a^*}] - 32[\Pi_{O_6}] = 0.
\end{equation}
In explicit form, the RR tadpole conditions read
\[
\sum_a n_a n_a n_a = 16,
\]
\[
\sum_a n_a m_a n_a = 0,
\]
\[
\sum_a n_a m_a m_a = 0,
\]
\[
\sum_a n_a m_a m_a = 0.
\]
That guarantees absence of non-abelian gauge anomalies.

The complete accommodation of the fermion structure of the six-stack SM model can be seen in table (1). Several comments are in order:

<table>
<thead>
<tr>
<th>Matter Fields</th>
<th>Intersection</th>
<th>(Q_a)</th>
<th>(Q_b)</th>
<th>(Q_c)</th>
<th>(Q_d)</th>
<th>(Q_e)</th>
<th>(Q_f)</th>
<th>(Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Q_L)</td>
<td>(3, 2)</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/6</td>
<td></td>
</tr>
<tr>
<td>(Q^L)</td>
<td>2(3, 2)</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/6</td>
<td></td>
</tr>
<tr>
<td>(U_R)</td>
<td>3(3, 1)</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-2/3</td>
<td></td>
</tr>
<tr>
<td>(D_R)</td>
<td>3(3, 1)</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/3</td>
</tr>
<tr>
<td>(L^1)</td>
<td>(1, 2)</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1/2</td>
</tr>
<tr>
<td>(L^2)</td>
<td>(1, 2)</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1/2</td>
</tr>
<tr>
<td>(L^3)</td>
<td>(1, 2)</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-1/2</td>
<td></td>
</tr>
<tr>
<td>(N^1_R)</td>
<td>(1, 1)</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>(E^1_R)</td>
<td>(1, 1)</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>(N^2_R)</td>
<td>(1, 1)</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
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<tr>
<td>(E^2_R)</td>
<td>(1, 1)</td>
<td>0</td>
<td>0</td>
<td>-1</td>
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<td>0</td>
<td>1</td>
</tr>
<tr>
<td>(N^3_R)</td>
<td>(1, 1)</td>
<td>0</td>
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<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>(E^3_R)</td>
<td>(1, 1)</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1: Low energy fermionic spectrum of the six stack string scale \(SU(3)_C \otimes SU(2)_L \otimes U(1)_a \otimes U(1)_b \otimes U(1)_c \otimes U(1)_d \otimes U(1)_e \otimes U(1)_f\), type I D6-brane model together with its \(U(1)_Y\) charges. Note that at low energies only the SM gauge group \(SU(3) \otimes SU(2)_L \otimes U(1)_Y\) survives.

- The models accommodate various known low energy gauged symmetries. The
latter can be expressed in terms of the $U(1)$ symmetries $Q_a, Q_b, Q_c, Q_d, Q_e, Q_f$. We find the following identifications

\begin{align*}
\text{Baryon number} & \rightarrow Q_a = 3B, \\
\text{Lepton number} & \rightarrow L = Q_d + Q_e + Q_f, \\
3(B - L) & \rightarrow Q_a - 3Q_d - 3Q_e - 3Q_f.
\end{align*}

Moreover, $Q_c = 2I_{3R}$, where $I_{3R}$ being the third component of weak isospin. Also, $3(B - L)$ and $Q_c$ are free of triangle anomalies. The $U(1)_b$ symmetry plays the role of a Peccei-Quinn symmetry in the sense of having mixed SU(3) anomalies. This symmetry appears to be a general feature, of the model building based orientifolded six-torus constructions with D6 branes intersecting at angles, in the models based on the four- [20] and five-stack SM's [21].

- The study of Green-Schwarz mechanism will show us that Baryon and Lepton number are unbroken gauged symmetries and the corresponding gauge bosons are massive. It is important to notice that baryon and lepton numbers remain as global symmetries to low energies. Thus proton should be stable. Also Majorana masses for right handed neutrinos are not allowed in the models, that is mass terms for neutrinos should be of Dirac type. In the SM only the diagonal combination

$$L_{\text{diag}} = L_e + L_\mu + L_\tau$$

is an exact symmetry, that means $L_{\text{diag}}$ is preserved in each SM interaction. Thus it appears that the six stack SM's offer a very logical explanation for the existence of the various global symmetries that exist in the SM, in particular the fact the $U(1)_{B-L}$ is an exact global symmetry.

- The mixed anomalies $A_{ij}$ of the six surplus $U(1)$'s with the non-abelian gauge groups $SU(N_a)$ of the theory cancel through a generalized GS mechanism [27, 26], involving close string modes couplings to worldsheet gauge fields. Two combinations of the $U(1)$'s are anomalous and become massive, their orthogonal non-anomalous combinations survive, combining to a single $U(1)$ that remains massless, the latter to be identified with the hypercharge generator.

- The structure of intersection numbers which give the parametric form of tadpole solutions is unique for a certain level of stacks of branes. Another choice of intersection numbers at this level of stacks neither produces the correct hypercharge assignments for the SM chiral particles nor is able to produce a general class of solutions like those
presented here or in the four-, five- stack SM’s [20, 21] respectively.

- The models make use of the constraint

\[ \Pi_{i=1}^{3} m^i = 0. \]  
\[ (2.11) \]

The latter constraint is essential to cancel the appearance of exotic representations in the model, appearing from sectors in the form \( \alpha \alpha^* \), in antisymmetric and symmetric representations of the \( U(N_a) \) group.

- The solutions satisfying simultaneously the intersection constraints and the cancellation of the RR crosscap tadpole constraints are given in parametric form in table (2). These solutions represent the most general solution of the RR tadpoles as they depend on six integer parameters \( n_a^2, n_d^2, n_e^2, n_f^1, n_b^1, n_c^1 \), the phase parameter \( \epsilon = \pm 1 \), the NS-background parameter \( \beta_i = 1 - b_i \), which is associated to the presence of the NS B-field by \( b_i = 0, 1/2 \), and the interpolating parameter \( \tilde{\epsilon} = \pm 1 \) which gives two different classes of RR tadpole solutions.

The multiparameter tadpole solutions appearing in table (2) represent deformations of the D6-brane branes, of table (1), intersecting at angles, within the same homology class of the factorizable three-cycles. The solutions of table (2) in (2.8) satisfy all tadpole equations but the first. The later becomes \(^8\):

\[ \frac{9n_a^2}{\beta^1} + 2\frac{n_b^1}{\beta^2} + \frac{n_d^2}{\beta^1} + \frac{n_e^2}{\beta^1} + \frac{n_f^2}{\beta^1} + N_D \beta^1 \beta^2 = 16. \]  
\[ (2.12) \]

Note that we had added the presence of extra \( N_D \) branes. Their contribution to the RR tadpole conditions is best described by placing them in the three-factorizable cycle

\[ N_D(1/\beta^1, 0)(1/\beta^2, 0)(2, m_D^3) \]  
\[ (2.13) \]

where we have set \( m_D^3 = 0 \). The cancellation of tadpoles is better seen, if we choose a numerical set of wrappings, e.g.

\[ n_a^2 = 1, \ n_b^1 = 1, \ n_c^1 \in Z, \ n_d^2 = -1, \ n_e = -1, \ n_f^2 = -1, \ \beta^1 = 1, \ \beta^2 = 1. \]  
\[ (2.14) \]

Within the above choise, all tadpole conditions but the first are satisfied, the latter is satisfied when we add four \( D6 \) branes, e.g. \( N_D = 4 \) positioned at \( (1, 0)(1, 0)(2, 0) \). Thus the tadpole structure \(^9\) becomes

\(^7\)We have added an arbitrary number of \( N_D \) branes which don’t contribute to the rest of the tadpoles and intersection number constraints. Thus in terms of the low energy theory they don’t have no effect.

\(^8\)We have set for simplicity \( \epsilon = \tilde{\epsilon} = 1 \).

\(^9\)Note that the parameter \( n_c^1 \) should be defined such that its choise is consistent with a tilted tori, e.g. \( n_c^1 = 1 \).
Table 2: Tadpole solutions for six-stacks of D6-branes giving rise to, exactly, the standard model gauge group and observable chiral spectrum, at low energies. The solutions depend on six integer parameters, $n^2_a$, $n^2_d$, $n^2_e$, $n^2_f$, $n^1_b$, $n^1_c$, the NS-background $\beta^i$ and the phase parameter $\epsilon = \pm 1$ and the extra interpolating parameter $\tilde{\epsilon} = \pm 1$. The $\tilde{\epsilon}$ parameter distinguishes the two different classes of tadpole solutions.

$$
\begin{array}{|c|c|c|c|}
\hline
N_i & (n^1_i, m^1_i) & (n^2_i, m^2_i) & (n^3_i, m^3_i) \\
\hline
N_a = 3 & (1/\beta^1, 0) & (n^2_a, \epsilon \beta^2) & (3, \tilde{\epsilon}/2) \\
N_b = 2 & (n^1_b, -\epsilon \beta^1) & (1/\beta^2, 0) & (\tilde{\epsilon}, 1/2) \\
N_c = 1 & (n^1_c, \epsilon \beta^1) & (1/\beta^2, 0) & (0, 1) \\
N_d = 1 & (1/\beta^1, 0) & (n^2_d, \epsilon \beta^2) & (1, -\tilde{\epsilon}/2) \\
N_e = 1 & (1/\beta^1, 0) & (n^2_e, \epsilon \beta^2) & (1, -\tilde{\epsilon}/2) \\
N_f = 1 & (1/\beta^1, 0) & (n^2_f, \epsilon \beta^2) & (1, -\tilde{\epsilon}/2) \\
\hline
\end{array}
$$

Actually, the satisfaction of the tadpole conditions is independent of $n^1_c$. Thus, when all other parameters are fixed, $n^1_c$ is a global parameter that can vary according to if
the first tori is, or not, tilted. Its precise value will be fixed in terms of the remaining
tadpole parameters when we determine the tadpole subclass that is associated with
the hypercharge embedding of the standard model.
Note that there are always choices of wrapping numbers that satisfy the RR tadpole
constraints without the need of adding extra parallel branes, e.g. the following choice
satisfies all RR tadpoles

\[ n_a^2 = 1, \ n_b^1 = -1, \ n_c^1 \in 2Z + 1, \ n_d^2 = 0, \]
\[ n_e^2 = 0, \ n_f^2 = 0, \ \beta^1 = 1/2, \ \beta^2 = 1. \]  (2.16)
with cycle wrapping numbers

\[ N_a = 3 \quad (2, 0)(1, 1)(3, 1/2) \]
\[ N_b = 2 \quad (-1, -1/2)(1, 0)(1, 1/2) \]
\[ N_c = 1 \quad (n_c^1, 1/2)(1, 0)(0, 1) \]
\[ N_d = 1 \quad (2, 0)(0, 1)(1, -1/2) \]
\[ N_e = 1 \quad (2, 0)(0, 1)(1, -1/2) \]
\[ N_f = 1 \quad (2, 0)(0, 1)(1, -1/2) \]  (2.17)

- The hypercharge operator corresponding to the spectrum of table (1), is defined
as a linear combination of the $U(1)$ gauge groups, $U(1)_a, U(1)_c, U(1)_d, U(1)_e, U(1)_f$, as

\[ Y = \frac{1}{6} U(1)_a - \frac{1}{2} U(1)_c - \frac{1}{2} U(1)_d - \frac{1}{2} U(1)_e - \frac{1}{2} U(1)_f. \]  (2.18)

## 3 U(1) anomaly cancelation

The general form the mixed anomalies $A_{ij}$ of the six $U(1)$’s with the non-Abelian
gauge groups are given by

\[ A_{ij} = \frac{1}{2} (I_{ij} - I_{ij'}) N_i. \]  (3.1)

From the mixed anomalies of the $U(1)$’s with the non-abelian gauge groups $SU(3)_c,$ $SU(2)_b,$ we conclude that there are two anomaly free combinations $Q_c, Q_a - 3Q_d - 3Q_e - 3Q_f$. Also the gravitational anomalies cancel since D6-branes never intersect

\[ \text{Another consistent choice will be } \beta^1 = 1, \beta^2 = 1/2, n_d^2 = n_e^2 = n_f^2 = 1, n_a^1 = 1, n_b^1 = 1. \] We have set for simplicity $\bar{\epsilon} = 1.$
O6-planes. Gauge anomaly cancellation [27] in the orientifolderd type I torus models is guaranteed through a generalized GS mechanism [13] that uses the 10-dimensional RR gauge fields $C_2$ and $C_6$ and gives at four dimensions the following couplings to gauge fields

$$N_a m_a^1 m_a^2 m_a^3 \int_{M_4} B_2^a \land F_a \ ; \ n_b^1 n_b^2 n_b^3 \int_{M_4} C^b \land F_b \land F_b, \quad (3.2)$$

$$N_a n^J n^K m^I \int_{M_4} B_2^I \land F_a \ ; \ n_b^I n_b^J n_b^K \int_{M_4} C^I \land F_b \land F_b, \quad (3.3)$$

where $C_2 \equiv B_2^o$ and $B_2^I \equiv \int_{(T^2)_o \times (T^2)_o} C_6$ with $I = 1, 2, 3$ and $I \neq J \neq K$. We notice that the four dimensional duals of $B_2^o$, $B_2^I$ are defined as:

$$C^o \equiv \int_{(T^2)_o \times (T^2)_o} \times (T^2)_o \times J_2, C_2, \quad (3.4)$$

where $dC^o = - \ast dB_2^o$, $dC^I = - \ast dB_2^I$.

The cancellation of triangle anomalies (3.1) derives from the existence of the string amplitude involved in the GS mechanism [26] in four dimensions [27]. The latter amplitude, where the $U(1)_a$ gauge field couples to one of the propagating $B_2$ fields, that couples to dual scalars, and couples in turn to two $SU(N)$ gauge bosons, is proportional [20] to

$$-N_a m_a^1 m_a^2 m_a^3 n_a^2 n_b^3 - N_a \sum_{I} n_a^I n_a^J n_b^K m_a^I m_a^K, I \neq J, K \quad (3.5)$$

Taking into account the constraint of (2.11) the RR couplings $B_2^1$ of (3.3) then appear into the following three terms $^{11}$:

$$B_2^1 \land \left( \frac{-2 \epsilon \beta^1}{\beta^2} \right) F^b,$$

$$B_2^2 \land \left( \frac{\epsilon \beta^2}{\beta^3} \right) \left( 9 F^a + F^d + F^e + \bar{F}^f \right),$$

$$B_2^3 \land \left( \frac{3 \tilde{\epsilon} n_b^2}{2 \beta^1} F^a + \frac{n^1_a}{\beta^2} F^b + \frac{n^1_b}{\beta^2} F^c - \frac{\tilde{\epsilon} n_b^2}{2 \beta^1} F^d - \frac{\tilde{\epsilon} n_b^2}{2 \beta^1} F^e - \frac{\epsilon \beta^1}{\beta^2} F^f \right). \quad (3.6)$$

Also the couplings of the dual scalars $C^I$ of $B_2^I$ that are required to cancel the mixed anomalies of the six $U(1)$’s with the non-abelian gauge groups $SU(N_a)$ are given by

$$C^1 \land \left[ \frac{\epsilon \beta^2}{2 \beta^1} (F^a \land F^a) - \frac{\epsilon \beta^2}{\beta^1} F^a \land F^d - \frac{\epsilon \beta^2}{\beta^1} F^e \land F^e - \frac{\epsilon \beta^2}{\beta^1} F^f \land F^f \right],$$

$$C^2 \land \left[ \frac{-\epsilon \beta^1}{2 \beta^2} (F^b \land F^b) + \frac{\epsilon \beta^1}{\beta^2} (F^e \land F^c) \right],$$

$^{11}$For simplicity we have set $\tilde{\epsilon} = 1$.  

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As in the four stack SM [20], or the five stack SM [21], the RR scalar $B_2^0$ does not couple to any field $F^i$, as we have imposed the condition (2.11) which excludes the appearance of any exotic matter representations in the models. Note that these representations are not necessary in the SM based stack constructions. However, in the context of building a GUT brane model that eventually has to break to the SM they are welcome, as they become the reason for achieving the breaking to the SM [22].

A closer look at (3.6) reveals that there are two anomalous $U(1)$’s that become massive through their couplings to the RR fields. They are the model independent fields, $U(1)_b$ and the combination $9U(1)_a + U(1)_d + U(1)_e + U(1)_f$, which become massive through their couplings to the RR 2-form fields $B_2^1, B_2^2$ respectively. Also there is a model dependent, non-anomalous and massive $U(1)$ field coupled to $B_2^3$ RR field. Thus the two non-anomalous free combinations are $U(1)_c$ and $U(1)_a - 3U(1)_d - 3U(1)_e - 3U(1)_f$. In addition, we note that the mixed anomalies $A_{ij}$ are cancelled by the GS mechanism set by the couplings (3.6, 3.7).

The question we have to ask now, is how we can, from the general class of models, associated with the generic SM’s of tables (1) and (2), we can separate the subclass of models associated with just the SM hypercharge assignment at low energies. The generalized Green-Schwarz mechanism that cancels the non-abelian anomalies of the $U(1)$’s to the non-abelian gauge fields involves couplings of closed RR string modes to the $U(1)$ field strengths \(^{12}\) in the form

$$\sum_a f_a^i B_a \wedge tr(F_i).$$

Finally, the mixture of couplings in the form

$$A^{ik} + \sum_a f_a^i g_a^k = 0$$

(3.10)

cancels all non-abelian $U(1)$ gauge anomalies. That means, as was argued in [20], that if we want to keep some $U(1)$ massless we have to keep it decoupled from some closed

\(^{12}\)In addition, to the couplings of the Poincare dual scalars $\eta_a$ of the fields $B_a$,

$$\sum_a g_a^k \eta_a tr(F^k \wedge F^k).$$

(3.8)
string mode couplings that can make it massive, that is
\[
\sum_a \left( \frac{1}{6} f_a^o - \frac{1}{2} f_a^c - \frac{1}{2} f_a^d - \frac{1}{2} f_a^e - \frac{1}{2} f_a^f \right) = 0 .
\] (3.11)

In conclusion, the combination of the \(U(1)\)'s which remains light at low energies is
\[
3(n_a^2 + n_d^2 + n_e^2 + n_f^2) \neq 0, \quad Q' = n_c^1(Q_a - 3Q_d - 3Q_e - 3Q_f) - \frac{3\beta^2 \tilde{\epsilon}(n_a^2 + n_d^2 + n_e^2 + n_f^2)}{2\beta_1} Q_c.
\] (3.12)

The subclass of tadpole solutions of (3.12) having the SM hypercharge assignment at low energies is exactly the one, where the combination (3.12) is proportional to (2.18). That is
\[
n_c^1 = \frac{\tilde{\epsilon} \beta_2}{2\beta_1}(n_a^2 + n_d^2 + n_e^2 + n_f^2).
\] (3.13)

Summarizing, we have found that as long as (3.13) holds, we can identify \(Q'\) as the hypercharge generator, which gives at the chiral fermions of table (1) their correct SM hypercharge assignments. Moreover, there are two extra anomaly free \(U(1)\)'s beyond the hypercharge combination, which read
\[
Q^{(5)} = \frac{\tilde{\epsilon}}{2\beta_1}(n_a^2 + n_d^2 + n_e^2)(Q_a - 3Q_d - 3Q_e - 3Q_f) + 28n_c^1Q_c \\
Q^{(6)} = Q_e - Q_f
\] (3.14)

In the next section, we will break these \(U(1)\) symmetries by requiring the intersections where the right handed neutrino is localized, to preserve \(N = 1\) supersymmetry. A comment is in order. The \(U(1)\) combinations \(Q_d - Q_e, Q_d - Q_f\) are anomaly free, that is we could have chosen either of them to be the \(Q^{(6)}\) generator. Then the only difference with the present choice (3.14) would be a different constraint on the RR tadpole cancellation conditions, once we will later impose \(N = 1\) on an intersection.

Let us summarize. Up to this point the gauge group content of the model includes, beyond \(SU(3) \otimes SU(2) \otimes U(1)_Y\), the additional \(U(1)\) symmetries, \(Q^{(5)}, Q^{(6)}\) under which some of the chiral SM particles of table (1) gets charged. The extra \(U(1)\) symmetries will be broken by imposing some open string sectors to respect some amount of SUSY. In the latter case the immediate effect on obtaining just the SM at low energies will be two additional linear conditions on the RR tadpole solutions of table (2). We note that when \(n_c^1 = 0\), it is possible to have massless in the low energy spectrum both the \(U(1)\) generators, \(Q_c\), and the B-L generator \((1/3)(Q_a - 3Q_d - 3Q_e - 3Q_f)\) as long as \(n_c^1 = 0, n_a^2 = -n_d^2 - n_e^2 - n_f^2\).
4 Higgs mechanism on open string sectors

The mechanism of electroweak symmetry breaking at the string theory level between intersecting branes is not well understood but it is believed to take place either by using open string tachyons [9, 3, 20, 21, 22] between paralleled branes or using brane recombination [25]. As the nature of the latter procedure is topological, it cannot be described using field theoretical methods. In this work, we will follow the former method and we leave the latter method for some future study.

4.1 The angle structure

In the previous sections we have detailed the appearance in the R-sector of open string excitations with $I_{ab}$ massless chiral fermions in the D-brane intersections that transform under the bifundamental representations $(N_a, \bar{N}_b)$. However, in backgrounds with intersecting branes, besides the actual presence of massless fermions at each intersection, we have evident the presence of an equal number of massive scalars (MS), in the NS-sector, in exactly the same representations as the massless fermions [20]. The mass of the these MS is of order of the string scale. In some cases, it is possible that some of those MS may become tachyonic, triggering a potential that looks like the Higgs potential of the SM, especially when their mass, that depends on the angles between the branes, is such that is decreases the world volume of the 3-cycles involved in the recombination process of joining the two branes into a single one [28].

The models that we are describing, are based on orientifolded six-tori on type IIA strings. In those configurations the bulk has $\mathcal{N} = 4$ SUSY. Lets us now give some details about the open string sector of the models. In order to describe the open string spectrum we introduce a four dimensional twist [13, 20] vector $v_\theta$, whose I-th entry is given by $\vartheta_{ij}$, with $\vartheta_{ij}$ the angle between the branes $i$ and $j$-branes. After GSO projection the states are labeled by a four dimensional twisted vector $r + v_\theta$, where $\sum_I r^I =$odd and $r_I \in \mathbb{Z}, \mathbb{Z} + \frac{1}{2}$ for NS, R sectors respectively. The Lorentz quantum numbers are denoted by the last entry. The mass operator for the states reads:

$$\alpha'M^2_{ij} = \frac{Y^2}{4\pi^2\alpha'} + N_{bos}(\vartheta) + \frac{(r + v)^2}{2} - \frac{1}{2} + E_{ij}, \quad (4.1)$$

where $E_{ij}$ the contribution to the mass operator from bosonic oscillators, and $N_{asc}(\vartheta)$ their number operator, with

$$E_{ij} = \sum_I \frac{1}{2}|\vartheta_I|(1 - |\vartheta_I|), \quad (4.2)$$
and $Y$ measures the minimum distance between branes for minimum winding states.

If we represent the twisted vector $r + \nu$, by $(\vartheta_1, \vartheta_2, \vartheta_3, 0)$, in the NS open string sector, the lowest lying states are given \(^{13}\) by:

<table>
<thead>
<tr>
<th>State</th>
<th>Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(-1 + \vartheta_1, \vartheta_2, \vartheta_3, 0)$</td>
<td>$\alpha' M^2 = \frac{1}{2}(-\vartheta_1 + \vartheta_2 + \vartheta_3)$</td>
</tr>
<tr>
<td>$(\vartheta_1, -1 + \vartheta_2, \vartheta_3, 0)$</td>
<td>$\alpha' M^2 = \frac{1}{2}(\vartheta_1 - \vartheta_2 + \vartheta_3)$</td>
</tr>
<tr>
<td>$(\vartheta_1, \vartheta_2, -1 + \vartheta_3, 0)$</td>
<td>$\alpha' M^2 = \frac{1}{2}(\vartheta_1 + \vartheta_2 - \vartheta_3)$</td>
</tr>
<tr>
<td>$(-1 + \vartheta_1, -1 + \vartheta_2, -1 + \vartheta_3, 0)$</td>
<td>$\alpha' M^2 = 1 - \frac{1}{2}(\vartheta_1 + \vartheta_2 + \vartheta_3)$</td>
</tr>
</tbody>
</table>

Also the angles at the thirteen different intersections can be expressed in terms of the tadpole solutions parameters. Let us define the angles:

$$\tilde{\theta}_1 = \frac{1}{\pi} \tan^{-1} \frac{\beta_1 R_2^{(1)}}{n_1 R_1^{(1)}}, \quad \theta_2 = \frac{1}{\pi} \tan^{-1} \frac{\beta_2 R_2^{(2)}}{n_2 R_1^{(2)}}, \quad \tilde{\theta}_3 = \frac{1}{\pi} \tan^{-1} \frac{R_2^{(3)}}{6 R_1^{(3)}},$$

$$\theta_1 = \frac{1}{\pi} \tan^{-1} \frac{\beta_1 R_2^{(1)}}{n_1 R_1^{(1)}}, \quad \theta'_2 = \frac{1}{\pi} \tan^{-1} \frac{\beta_2 R_2^{(2)}}{n_2 R_1^{(2)}}, \quad \theta_3 = \frac{1}{\pi} \tan^{-1} \frac{R_2^{(3)}}{2 R_1^{(3)}},$$

$$\tilde{\theta}_2 = \frac{1}{\pi} \tan^{-1} \frac{\beta_2 R_2^{(2)}}{n_2 R_1^{(2)}}, \quad \tilde{\theta}_3 = \frac{1}{\pi} \tan^{-1} \frac{R_2^{(3)}}{n_3 R_1^{(2)}},$$

where $R_{1,2}^{(i)}$ are the compactification radii for the three $i = 1, 2, 3$ tori, namely projections of the radii onto the $X_{1,2}^{(i)}$ directions when the NS flux B field, $b^i$, is turned on and we have chosen for convenience $\epsilon = \tilde{\epsilon} = 1$.

At each of the thirteen non-trivial intersections we have the presence of four states $t_{i}, i = 1, \cdots, 4$, associated to the states (4.3). Hence we have a total of fifty two different massive scalars, with lowest lying spectrum, in the model \(^{14}\).

The following mass relations hold between the different intersections of the model:

$$m_{ab}^2(t_2) + m_{ac}^2(t_3) = m_{cd}^2(t_2) + m_{cd}^2(t_3) = m_{cd}^2(t_2) + m_{cd}^2(t_3)$$

$$m_{ab}^2(t_2) + m_{ab}^2(t_3) = m_{ab}^2(t_2) + m_{ab}^2(t_3) = m_{bd}^2(t_2) + m_{bd}^2(t_3)$$

$$m_{be}^2(t_2) + m_{be}^2(t_3) = m_{be}^2(t_2) + m_{be}^2(t_3) = m_{bf}^2(t_2) + m_{bf}^2(t_3)$$

$$m_{be}^2(t_1) + m_{be}^2(t_2) = m_{bf}^2(t_1) + m_{bf}^2(t_2) = m_{bf}^2(t_1) + m_{bf}^2(t_2)$$

\(^{13}\)we assumed $0 \leq \vartheta_i \leq 1$.

\(^{14}\)In figure one, we can see the D6 branes angle setup in the present models.